Learning distributed representations for statistical language modelling

Overview

1. Discrete data and distributed representations

2. Language modelling
   - Factored RBM language model
   - Log-bilinear language model
   - Hierarchical log-bilinear language model

Discrete data

- Discrete data: datapoints with discrete-valued attributes

- When such datapoints are high-dimensional, regression / classification / density estimation is hard:
  - Amounts to estimating entries of an exponentially large table
    - Attributes correspond to table dimensions
    - Attribute values correspond to indices for the dimensions
  - Data sparsity: little or no data available for most entries
  - No a priori smoothness constraint on table entries
  - No general way to generalize to new table entries

Distributed representations

- Observation: making a model less local often improves generalization.
  - In a continuous space: average over datapoints near the point of interest.
  - In a discrete space: not clear what to average over.
    - What does "near" mean?
    - No general concept of distance / neighbourhood.

- Working with smooth functions over continuous spaces results in automatic smoothing.
  - Similar inputs produce similar outputs

- Idea: map discrete attributes to real-valued vectors and learn a smooth function that maps the vectors to the desired output values.
  - Learn the attribute mapping *jointly* with the function.
  - Automatic generalization!

Statistical language modelling

- Goal: Model the joint distribution of words in a sentence.
- Such a model can be used to
  - predict the next word given several preceding ones
  - arrange bags of words into sentences
  - assign probabilities to documents
- Applications: speech recognition, machine translation, information retrieval.
- Most statistical language models are based on the Markov assumption:
  - The distribution of the next word depends on only n words that immediately precede it.
  - This assumption is clearly wrong but useful – it makes the task much more tractable.

n-gram models

- n-gram models are simply conditional probability tables for $P(w_n|w_{1:n-1})$.
  - w_n is the word to be predicted (the *next* word)
  - words $w_{1:n-1} = w_1, \ldots, w_{n-1}$ are called the *context*

- n-gram models are estimated by counting the number of occurrences of each possible word n-tuple and normalizing.
  - smoothing the estimates is essential for good performance
  - many different smoothing methods exist

- n-gram models are the most widely used statistical language models due to their simplicity and excellent performance.

- Curse of dimensionality: the number of model parameters is exponential in n.

Neural language models

- Several neural probabilistic language models based on distributed representations have been proposed.
- Common approach:
  - Represent each word with a real-valued feature vector
  - Represent the context by the sequence of the context word feature vectors
  - Train a neural network to output the distribution for the next word from the context representation
  - Learn word feature vectors jointly with other neural net parameters
- Neural language models can outperform n-gram language models, especially when little training data is available.
- Main drawback: very long training and testing times.

Conditional RBM language model

- Use a restricted Boltzmann machine to model $P(w_n | w_{1:n-1})$
  - Capture the interaction between w_n and $w_{1:n-1}$ through a vector of latent variables.
  - Represent words using low-dimensional real-valued vectors.
    - R_w is the feature vector for word w.

- Energy function:

$$E(w_n, h; w_{1:n-1}) = -\sum_{i=1}^{n} R_{w_i} W_i h$$

  - h is the vector of latent variables
  - W_i is the interaction matrix between the feature vector for w_i and the latent variables.
  - Normalization is done only over w_n.

- Both inference and prediction take time linear in the number of latent variables.

Log-bilinear model

- The log-bilinear (LBL) model is perhaps the simplest neural language model.
- Given the context $w_{1:n-1}$, the LBL model first predicts the representation for the next word w_n by linearly combining the representations of the context words:

$$\hat{r} = \sum_{i=1}^{n-1} C_i r_{w_i}$$

  - r_w is the real-valued vector representing word w
- Then the distribution for the next word is computed based on the similarity between the predicted representation and the representations of all words in the vocabulary:

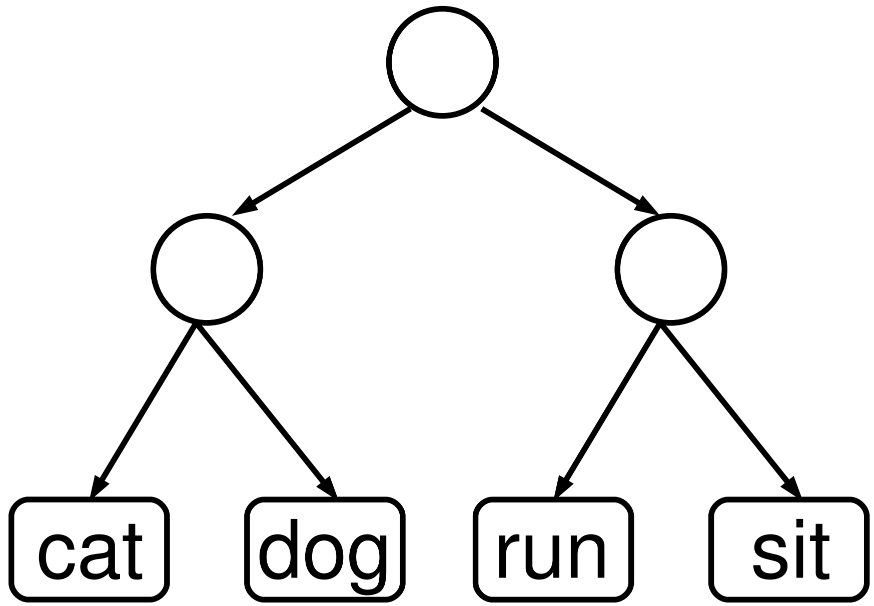
$$P(w_n = w | w_{1:n-1}) = \frac{\exp(\hat{r}^T r_w)}{\sum_j \exp(\hat{r}^T r_j)}.$$

Faster models through structured vocabulary

- Computing the probability of the given next word requires considering all N words in the vocabulary.
  - Need to consider all words because the word space is unstructured.

- Idea: Organize words in the vocabulary into a binary tree and exploit its structure to speed up normalization (Morin and Bengio, 2005).
  - Construct a binary tree over words
    - words are associated with leaf nodes
    - one word per leaf
  - Replace the N-way decision by a sequence of $O(\log N)$ binary decisions for predicting the next word.
    - Can achieve an exponential speedup if the tree is balanced!

Tree-based factorization



- To define a distribution over leaf nodes:
  - Specify the probability of taking the left branch at each non-leaf node.
  - The probability of a leaf node is the product of probabilities of the left/right decisions that lead from the root node to the leaf node.

Constructing trees over words

- The approach of Morin and Bengio:
  - Start with the WordNet IS-A hierarchy (which is a DAG)
  - Manually select one parent node per word
  - Use clustering to make the resulting tree binary
  - Use the Neural Probabilistic Language Model for making the left/right decisions

- Drawbacks:
  - Tree construction process uses expert knowledge
  - The resulting model does not work as well as its non-hierarchical counterpart

- Our approach:
  - Construct the word tree from data alone (no experts needed)
  - Allow each word to occur more than once in the tree
  - Use the simplified log-bilinear language model for making the left/right decisions

Hierarchical log-bilinear model

- Let d be the binary code that encodes the sequence of left-right decisions in the tree that lead to word w.

- Each non-leaf node in the tree is given a feature vector.
  - Used for discriminating the words in the left subtree from those in the right subtree.

- The probability of taking the left branch at i^{th} node in the sequence is

$$P(d_i = 1 | q_i, w_{1:n-1}) = \sigma(\hat{r}^T q_i),$$

  - $\hat{r}$ is computed as in the LBL model
  - q_i is the feature vector for the node

- The probability of w being the next word is

$$P(w_n = w | w_{1:n-1}) = \prod_i P(d_i | q_i, w_{1:n-1}).$$

Data-driven tree construction

- We would like to cluster words based on the distribution of contexts in which they occur.

- This distribution is hard to estimate and work with due to the high dimensionality of the space of contexts.
  - same difficulties as with estimating n-gram models

- To avoid this problem, we represent contexts using distributed representations and cluster words based on their *expected* predicted representation.

- Constructing a tree over words:
  1. Train a model using a (balanced) random tree over words.
  2. Extract the word representations from the trained model.
  3. Perform hierarchical clustering on the extracted representations.

Hierarchical clustering

- Hierarchical top-down clustering of feature vectors:
  - At each level, fit a mixture of two Gaussians with spherical covariances using EM to the current group of word representations.
  - Assign words to mixture components based on the component responsibilities.

- We considered several splitting rules:
  - BALANCED: Sort the responsibilities and make the split to ensure a balanced tree.
  - ADAPTIVE: Assign the word to the component with the greater responsibility.
  - ADAPTIVE(ϵ): Assign the word to a component if its responsibility for the word is at least 0.5-ϵ.

Dataset and evaluation

- APNews dataset:
  - collection of Associated Press news stories (16 million words)
- Preprocessing (Bengio et al.):
  - convert all words to lower case
  - map all rare words and proper nouns to special symbols
  - just under 18000 words in the vocabulary
- Models were compared based on the perplexity they assigned to the test set.
- Perplexity is the geometric average of $\frac{1}{P(w_n|w_{1:n-1})}$.

Model evaluation (I)

- Preliminary comparison:
  - 10M training set, 0.5M validation set, 0.5M test set
  - Feature-based models have 100D feature vectors.
  - FRBMs have 1000 hidden units.
  - KNn is a Kneser-Ney back-off n-gram model.

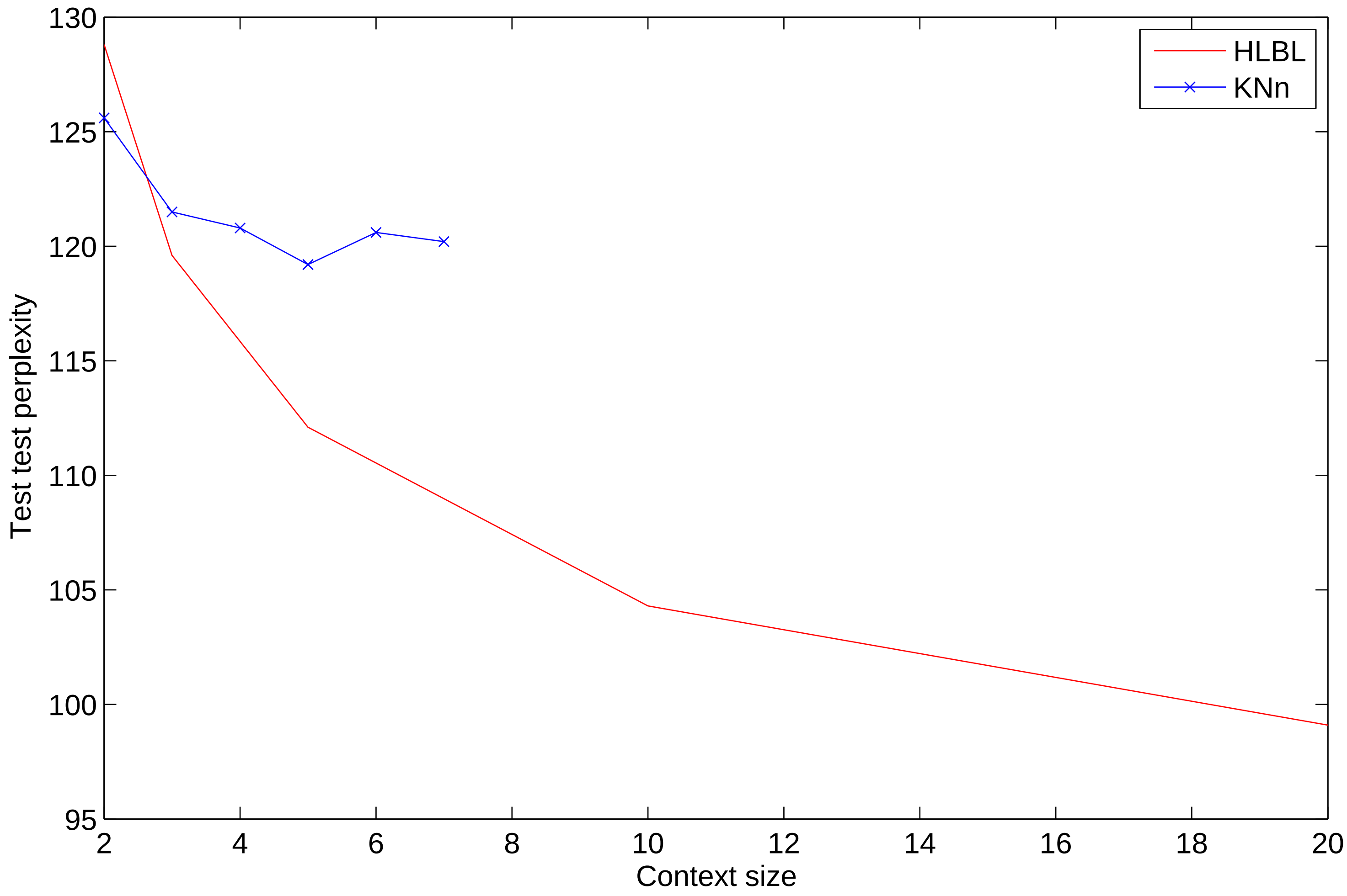
Model type	Context size	Model test perplexity	Mixture test perplexity
FRBM	2	169.4	110.6
Temporal FRBM	2	127.3	95.6
Log-bilinear	2	132.9	102.2
Log-bilinear	5	124.7	96.5
Back-off GT3	2	135.3	–
Back-off KN3	2	124.3	–
Back-off GT6	5	124.4	–
Back-off KN6	5	116.2	–

Model evaluation (II)

- Final comparison:
  - 14M training set, 1M validation set, 1M test set
  - (H)LBL used 100D feature vectors and a context size of 5.
  - KNn is an interpolated Kneser-Ney n-gram model.

Model type	Tree generating algorithm	Test perplex.	Mixture perplex.	Fitted mix. perplexity	Minutes per epoch
HLBL	RANDOM	151.2	107.2	106.0	4
HLBL	BALANCED	131.3	99.9	99.7	4
HLBL	ADAPTIVE	127.0	98.3	98.2	4
HLBL	ADAPTIVE(0.25)	124.4	97.5	97.4	6
HLBL	ADAPTIVE(0.4)	123.3	97.2	97.1	7
HLBL	ADAPTIVE(0.4) $\times$ 2	115.7	95.3	95.3	16
HLBL	ADAPTIVE(0.4) $\times$ 4	112.1	94.4	94.3	32
LBL	–	117.0	94.0	94.0	6420
KN2	–	174.2	–	–	–
KN3	–	125.6	–	–	–
KN6	–	119.2	–	–	–

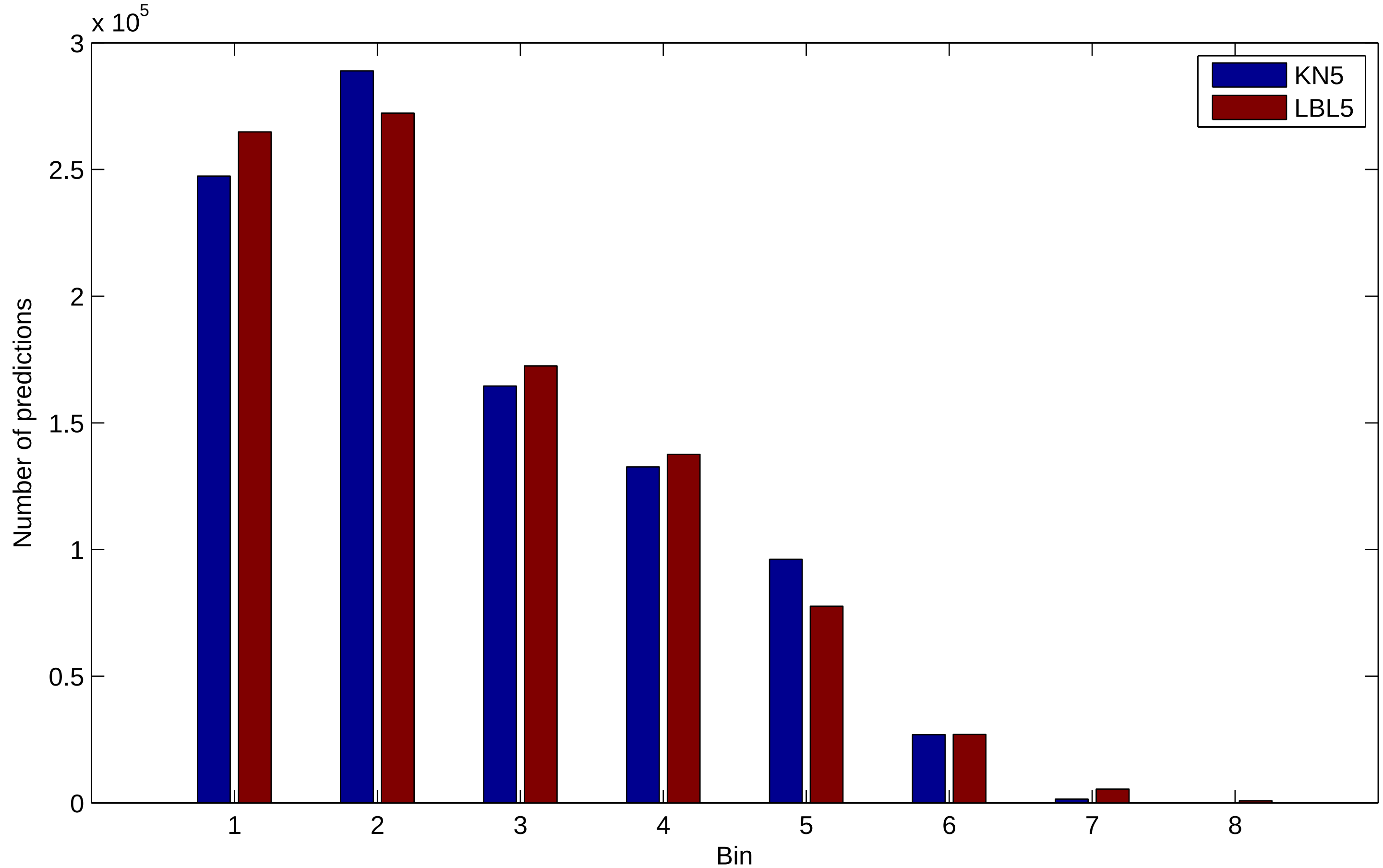
The effect of the context size



- The HLBL models were based on the ADAPTIVE(0.4) $\times$ 4 tree.
- KNn is an interpolated modified Kneser-Ney n-gram model.

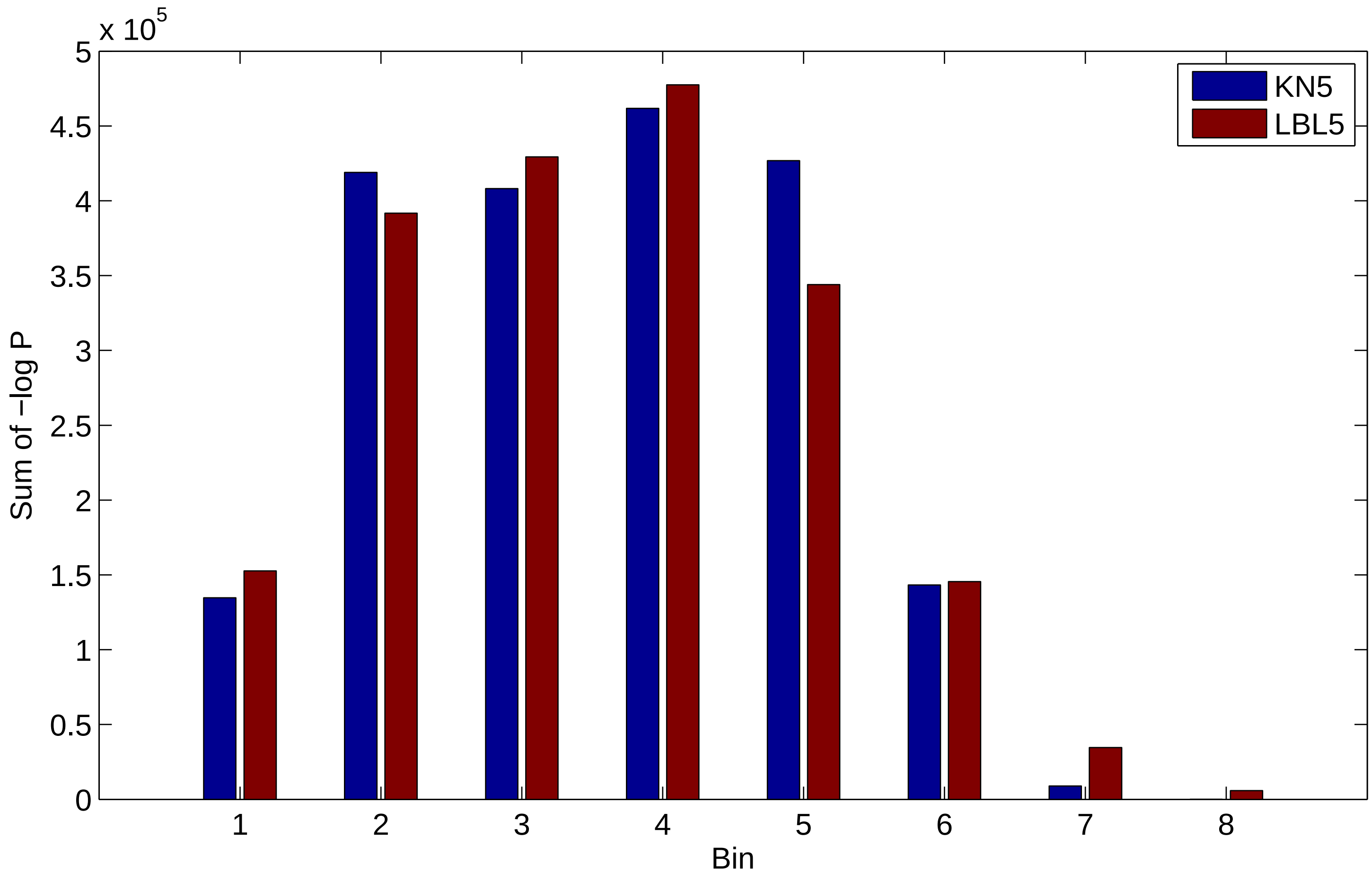
THE END

Log-prob contributions: 5-gram vs. LBL (I)



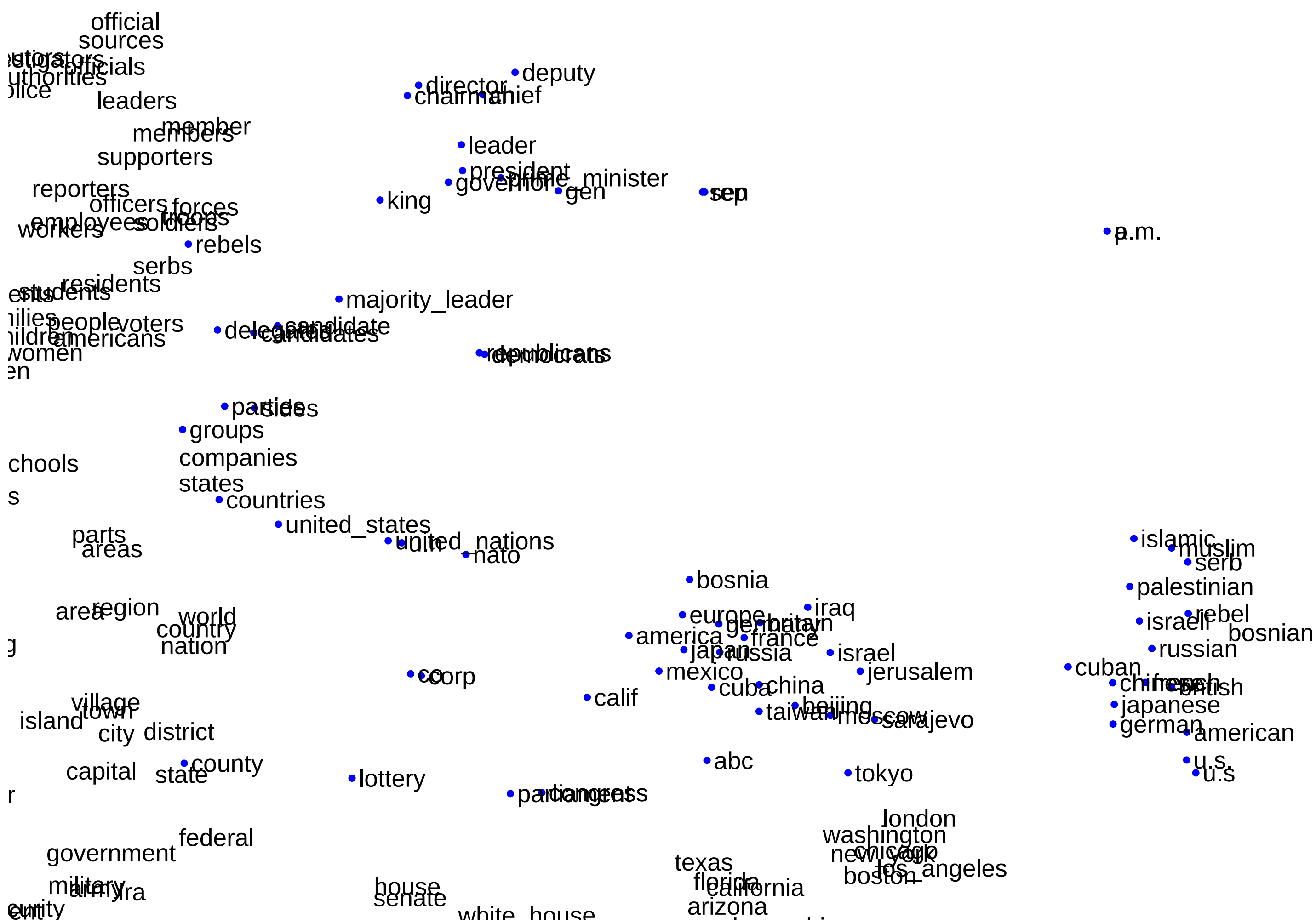
Number of predictions ($P(w_n|w_{1:n-1})$) on the test set as a function of the their magnitude. Bin i (for $i = 1, ..., 7$) contains predictions between 10^{-i} and 10^{-i+1}. Bin 8 contains predictions smaller than 10^{-7}.

Log-prob contributions: 5-gram vs. LBL (II)



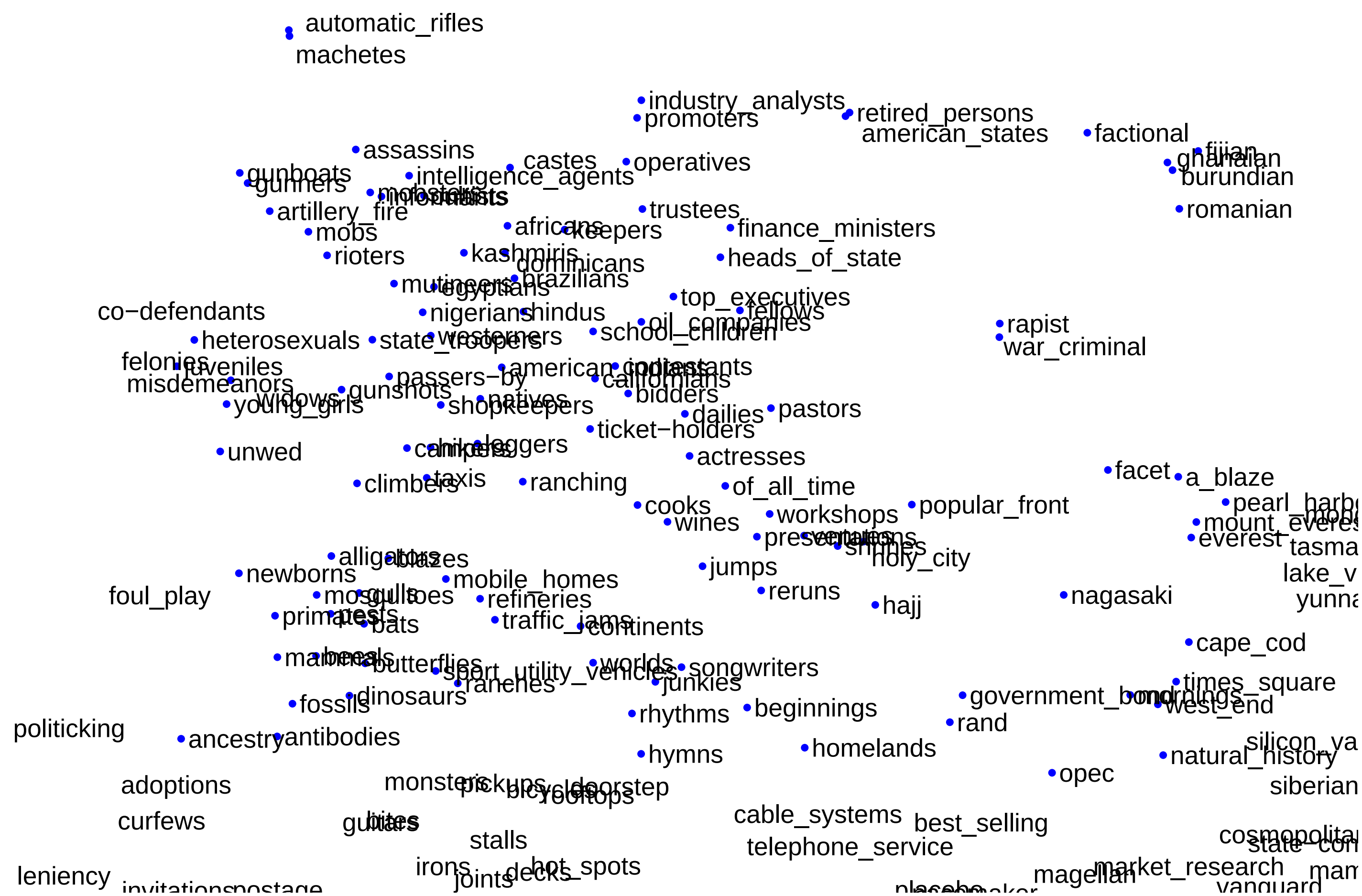
Contribution to the negative log-probability of the test set as a function of the prediction magnitude. Bin i (for $i = 1, ..., 7$) contains predictions between 10^{-i} and 10^{-i+1}. Bin 8 contains predictions smaller than 10^{-7}.

t-SNE embedding of LBL feature vectors (I)



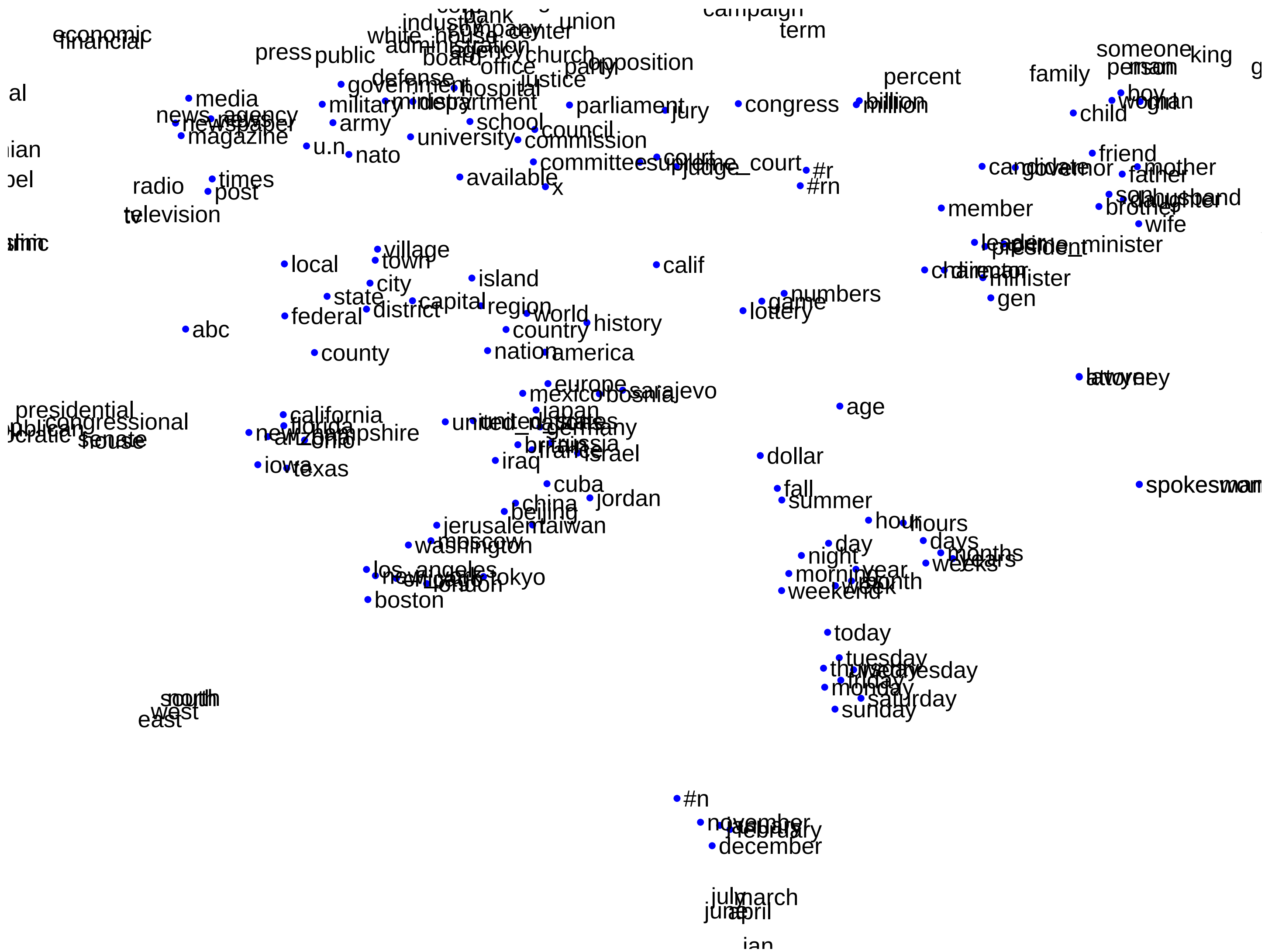
A fragment of a t-SNE embedding of the feature vectors (learned by an LBL model) of the most frequent 1000 words.

t-SNE embedding of LBL feature vectors (II)



A fragment of a t-SNE embedding of the feature vectors (learned by an LBL model) of the least frequent 1000 words.

t-SNE embedding of HLBL feature vectors (I)



A fragment of a t-SNE embedding of the feature vectors (learned by an HLBL model) of the most frequent 1000 words.

t-SNE embedding of HLBL feature vectors (II)

A fragment of a t-SNE embedding of the feature vectors (learned by an HLBL model) of the least frequent 1000 words.