Duration: **50 minutes**
Aids Allowed: **NONE** (in particular, no calculator)

Student Number:

Last (Family) Name(s): SOLUTIONS

First (Given) Name(s): SAMPLE

Tutorial Section: **(circle one)**	BA 2165 Steve	BA 2159 Alex	BA 2175 Anatoliy

*Do **not** turn this page until you have received the signal to start.*
(In the meantime, please fill out the identification section above, and read the instructions below *carefully*.)

This term test consists of 4 questions on 5 pages (including this one), printed on one side of the paper. *When you receive the signal to start, please make sure that your copy of the test is complete and write your student number where indicated at the bottom of every page* (except page 1).

Answer each question directly on the test paper, in the space provided, and use the reverse side of the pages for rough work. If you need more space for one of your solutions, use the reverse side of a page and *indicate clearly the part of your work that should be marked.*

Good Luck!

Marking Guide

1: ______/10
2: ______/ 6
3: ______/10
4: ______/10

TOTAL: ______/36

Question 1. [10 MARKS]

Consider the following statement:

(S) When A or B is true, so is C.

Part (a) [2 MARKS]

What, if anything, can be concluded from (S) if A is true?

SAMPLE SOLUTION

We first note that (S) means $(A \vee B) \Rightarrow C$.

Following the implication in (S), we know that C is true. The implication says nothing about whether B is true or false.

Part (b) [2 MARKS]

What, if anything, can be concluded from (S) if C is true?

SAMPLE SOLUTION

Nothing! The implication says nothing about A or B (C could be true without either A or B necessarily being true).

Part (c) [3 MARKS]

Express symbolically a statement equivalent to (S), but without using implication ($\Rightarrow$).

SAMPLE SOLUTION

We use the implication rule to get an equivalent formula using or:
$\neg(A \vee B) \vee C$

Part (d) [3 MARKS]

Express symbolically a statement equivalent to (S), but without using disjunction ($\vee$).

SAMPLE SOLUTION

We use DeMorgan's laws to replace the or with an and: $\neg(\neg A \wedge \neg B) \Rightarrow C$
Or we use our answer from (c) + DeMorgan's law: $(\neg A \wedge \neg B) \vee C$
Or we use the contrapositive + DeMorgan's law: $\neg C \Rightarrow (\neg A \wedge \neg B)$

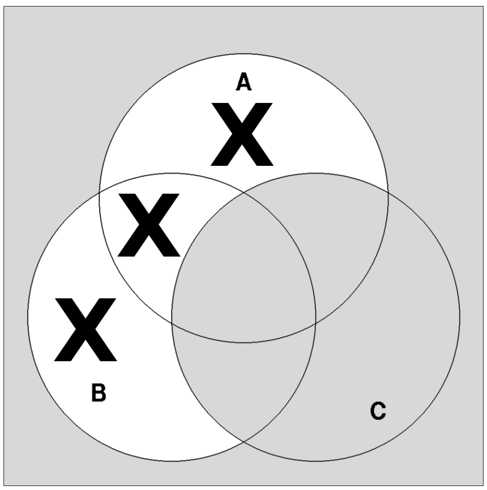
Question 2. [6 MARKS]

Let U be some universe containing sets A, B and C. Let $A(x)$ mean $x \in$ A, $B(x)$ mean $x \in$ B, and $C(x)$ mean $x \in$ C.

For each of the following statements, draw a Venn diagram with overlapping circles for A, B and C, making 8 regions. Shade in all the regions where the statement is true, and put an **X** in each region where the statement is false.

When A or B is true, so is C.

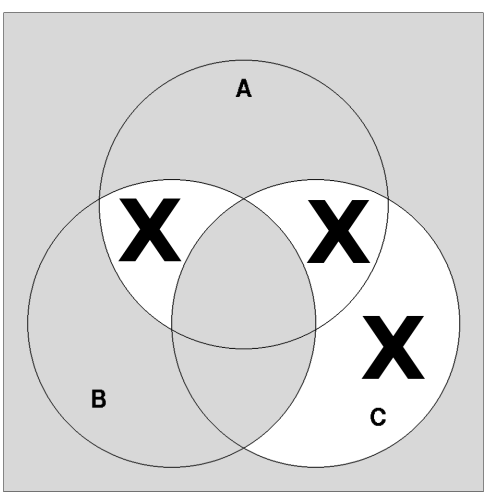
SAMPLE SOLUTION



$\forall x \in U, \big(A(x) \vee \neg B(x)\big) \Rightarrow \big(B(x) \Leftrightarrow C(x)\big)$

SAMPLE SOLUTION

An X should appear in each of the three regions $A \cap B - C$ and $A \cap C - B$ and $C - A - B$.



Question 3. [10 MARKS]

Let S be the set of Hollywood movie stars, and let $a(x, y)$ mean "movie star x admires movie star y." Express each of the following statements symbolically, matching the English form as closely as possible.

Part (a) [2 MARKS]

Some movie star admires himself/herself.

SAMPLE SOLUTION

$\exists x \in S, a(x, x)$

Part (b) [3 MARKS]

Some movie stars have no admirers.

SAMPLE SOLUTION

$\exists x \in S, \neg \exists y \in S, a(y, x)$
or
$\exists x \in S, \forall y \in S, \neg a(y, x)$

Part (c) [2 MARKS]

All movie stars admire Sidney Poitier.

SAMPLE SOLUTION

$\forall x \in S, a(x, SP)$
where SP stands for Sidney Poitier, a movie star ($SP \in S$).

Part (d) [3 MARKS]

Every movie star is admired by another movie star.

SAMPLE SOLUTION

$\forall x \in S, \exists y \in S, x \neq y \wedge a(y, x)$

Question 4. [10 MARKS]

Consider the following statement about sequences of natural numbers $a_0, a_1, a_2, \dots$
(recall that $\mathbb{N} = \{0, 1, 2, \dots\}$):

(T) $\forall i \in \mathbb{N}, \exists k \in \mathbb{N}, (a_i < i \Rightarrow a_k > i)$

Part (a) [2 MARKS]

Write the negation of (T) symbolically, moving negations inside as much as possible.

SAMPLE SOLUTION

$\exists i \in \mathbb{N}, \forall k \in \mathbb{N}, a_i < i \wedge a_k \leq i$

Part (b) [6 MARKS]

For each of the following sequences, state whether (T) is true or false. Justify your claim, using an example or counterexample when appropriate.

$1, 2, 3, 4, 5, 6, 7, \dots$

SAMPLE SOLUTION

True. Given any element $i \in \mathbb{N}$, $a_i < i$ is false. Hence, no matter which element $k \in \mathbb{N}$ we pick, the implication is true. Thus (T) is true (the universally quantified implication is vacuously true).

$2, 2, 2, 2, 2, 2, 2, \dots$

SAMPLE SOLUTION

False. We will show the negation of (T) is true (thus showing that (T) itself must be false). Pick $i = 3$ (or any value greater than 3). Then no matter what value we choose for k, both $a_i < i$ is true (since $a_i = 2$ always) and $a_k \leq i$ (since $a_k = 2$ always).

Part (c) [2 MARKS]

Write the converse of (T) symbolically, moving negations inside as much as possible.

SAMPLE SOLUTION

$\forall i \in \mathbb{N}, \exists k \in \mathbb{N}, a_k > i \Rightarrow a_i < i$