Due: By 6:00pm on Thursday, June 8. **Worth**: 10%

1. [15 marks]

   Write a *high-level description*, an *implementation description*, and a *formal description* (as defined near the top of page 157 of the textbook) for a Turing machine that decides the following language:

   $$L_1 = \{\mathtt{a}^i\mathtt{b}^k : i \leq 2k \text{ and } k \leq 2i\}.$$

   For example, `aaabbb` $\in L_1$ and `aaaabb` $\in L_1$, but `aabbbbb` $\notin L_1$ because $5 \not\leq 2 \cdot 2$, and `abba` $\notin L_1$ because it does not have the correct format.

   For your machine, use the alternate convention mentionned in class (where the initial configuration is "$\sqcup\ q_0\ w$", *i.e.*, the input w is written starting on the second cell of the tape with the leftmost cell blank, and the head starts on the first input symbol). Write your formal description in the style of the "extra example of Turing machine description" posted on the course website, *i.e.*, using a transition table with states grouped by purpose, with short comments to explain the purpose of each group of states and relate them to your implementation description. (Your presentation of your Turing machine will be marked, so write it such that the TA can easily understand it and verify its correctness.)

2. [15 marks]

   A *Turing machine with multiple tracks* is similar to an ordinary Turing machine, but contains k independent tracks on the tape, for some fixed $k \geq 1$. Each track is able to store a separate character from the tape alphabet, thus it is like there are k symbols stored within each cell. The tape is initailly filled with blanks except for the portion containing the input, which is writen in the first track. When reading or writing a cell, the read-write head reads or writes all tracks simultaneously (thus the transition function has the form $\delta : Q \times T^k \to Q \times T^k \times \{\mathrm{L}, \mathrm{R}\}$).

   Show that this type of Turing machine recognizes the class of Turing-recognizable languages.

3. [15 marks]

   Prove that the following language is recognizable:

   $$L_3 = \{\langle M, k\rangle : M \text{ is a Turing machine that accepts some string of length } k\}.$$

4. [15 marks]

   Suppose A and B are two decidable languages. Prove that the following language is decidable:

   $$L_4 = \{ab : a \in A, b \notin B\}.$$