Duration: **50 minutes**
Aids Allowed: **NONE** (in particular, no calculator)

Student Number: |_|_|_|_|_|_|_|_|_|

Last (Family) Name(s): ______________________________

First (Given) Name(s): ______________________________

Tutorial Section: **(circle one)**	BA 1210 Anthony To	BA 1220 Periklis Papakonstantinou

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*Do **not** turn this page until you have received the signal to start.*
(In the meantime, please fill out the identification section above, and read the instructions below *carefully*.)

This term test consists of 3 questions on 5 pages (including this one), printed on one side of the paper. *When you receive the signal to start, please make sure that your copy of the test is complete and write your student number where indicated at the bottom of every page* (except page 1).

Answer each question directly on the test paper, in the space provided, and use the reverse side of the pages for rough work. If you need more space for one of your solutions, use the reverse side of a page and *indicate clearly the part of your work that should be marked.*

In your answers, you may use without proof any result or theorem covered in lectures, tutorials, assignments, or in the textbook, as long as you give a clear statement of the result(s)/theorem(s) you are using. You must justify all other facts required for your solutions.

If you are unable to answer a question (or part of a question), you will get 20% of the marks for the question (or part of the question) if you state clearly that you do not know how to answer. Note that you will *not* get those marks if your answer contains contradictory statements (such as "I don't know" followed or preceded by parts of a solution that have not been crossed off).

Marking Guide

1: ______/ 8

2: ______/ 8

3: ______/14

TOTAL: ______/30

Good Luck!

Question 1. [8 MARKS]

Part (a) [2 MARKS]

Complete the following definition (check all that are true): "*NP* is defined to be the set of…

- ☐ …nondeterministic Turing machines that run in polynomial time."
- ☐ …languages that are not decidable in polynomial time."
- ☐ …all sets of strings decidable in nondeterministic polynomial time."
- ☐ …problems that can be solved in deterministic polynomial time."

Indicate whether each statement below is true or false (circle the appropriate answer), and briefly justify.

Part (b) [2 MARKS]

For all languages A, if $A \in P$, then $A \in NP$.

TRUE FALSE I DON'T KNOW

Justification:

Part (c) [2 MARKS]

For all languages A, if $SAT \leq_p A$, then A is NP-complete.

TRUE FALSE I DON'T KNOW

Justification:

Part (d) [2 MARKS]

For all languages A and B, if $A \leq_p B$ and $B \leq_p A$, then $A \in P$ iff $B \in P$.

TRUE FALSE I DON'T KNOW

Justification:

Question 2. [8 MARKS]

For each language below, circle the **smallest** class that the language belongs to and justify your answer.

Part (a) [4 MARKS]

TAUT $= \{\langle F \rangle : F$ is a Boolean formula that is always true (for any truth assignment of its variables)$\}$ belongs to:

P NP $coNP$ NONE OF THESE CLASSES I DON'T KNOW

Justification:

Part (b) [4 MARKS]

NOKCLIQUE $= \{\langle G \rangle : G$ is an undirected graph that contains **no** clique with exactly k vertices$\}$ (k is a constant independent of G) belongs to:

P NP $coNP$ NONE OF THESE CLASSES I DON'T KNOW

Justification:

Question 3. [14 MARKS]

Give a detailed proof that the following language is *NP*-complete. A significant portion of your mark will be based on the structure of your answer, so write your proof carefully. In your answer, you may use any of the known *NP*-complete languages listed at the bottom of the last page.

TSP = { $\langle G, k \rangle$: G is a complete weighted undirected graph (every edge has a "cost" $c(e)$)
that contains a simple cycle visiting every vertex of G such that
the sum of the weights on the edges of the cycle is at most k }

A *complete graph* has an edge between each pair of vertices.

[*More space on the next page...*]

Question 3. (continued)

Total Marks = 30

List of known *NP*-complete languages.

- 3SAT = $\{ \langle F \rangle : F$ is a propositional formula in 3-CNF that is satisfiable, *i.e.*, such that there exists an assignment of truth values to the variables of F that makes F true $\}$
- VertexCover = $\{ \langle G, k \rangle : G$ is an undirected graph that contains a vertex cover of size k, *i.e.*, a subset of k vertices C such that every edge of G has at least one endpoint in C $\}$
- SubsetSum = $\{ \langle S, t \rangle : S$ is a set of positive integers $\{x_1, \ldots, x_n\}$ and t is a positive integer such that there is some subset of S whose sum is exactly t, *i.e.*, $\exists S' \subseteq S$, $\sum_{x \in S'} x = t$ $\}$
- Clique = $\{ \langle G, k \rangle : G$ is an undirected graph that contains a clique of size k, *i.e.*, a subset of k vertices C such that G contains all of the edges between any two vertices of C $\}$
- HamPath = $\{ \langle G, s, t \rangle : G$ is an undirected graph that contains a Hamiltonian path from s to t, *i.e.*, a simple path from s to t that includes every vertex of G $\}$
- HamCycle = $\{ \langle G \rangle : G$ is an undirected graph that contains a Hamiltonian cycle, *i.e.*, a simple cycle that includes every vertex of G $\}$