

# Machine Learning I

## MATH60629A

Modern Generative Models  
— Week #11



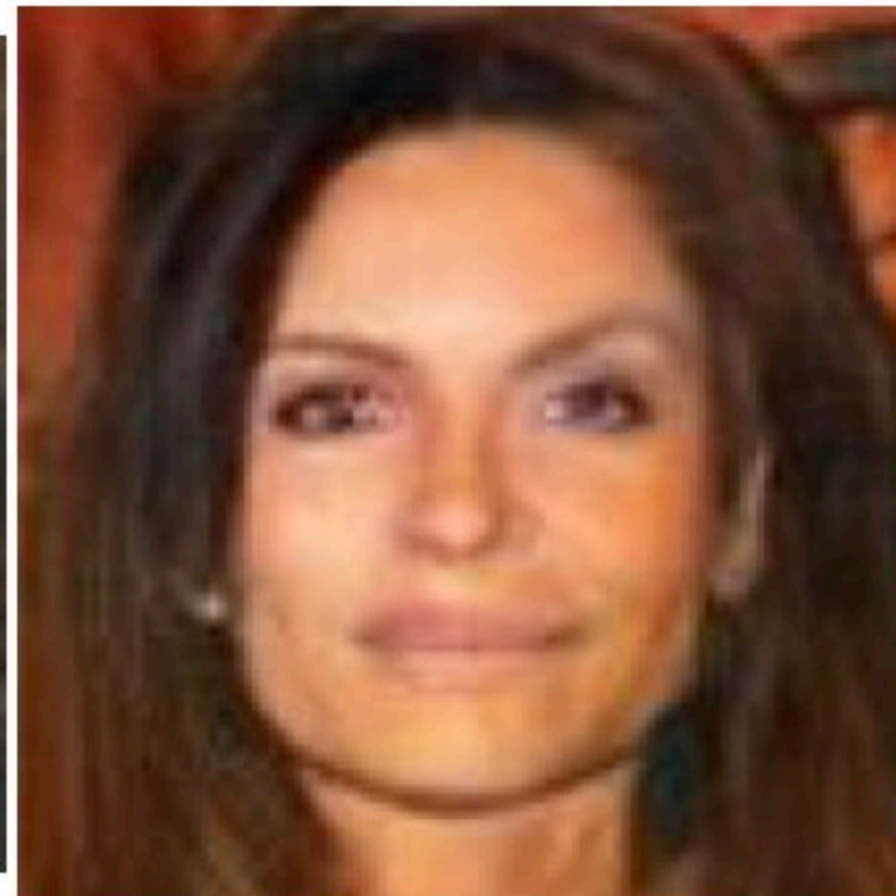
# Rapid evolution of (image) generation capability



2014



2015



2016



2017

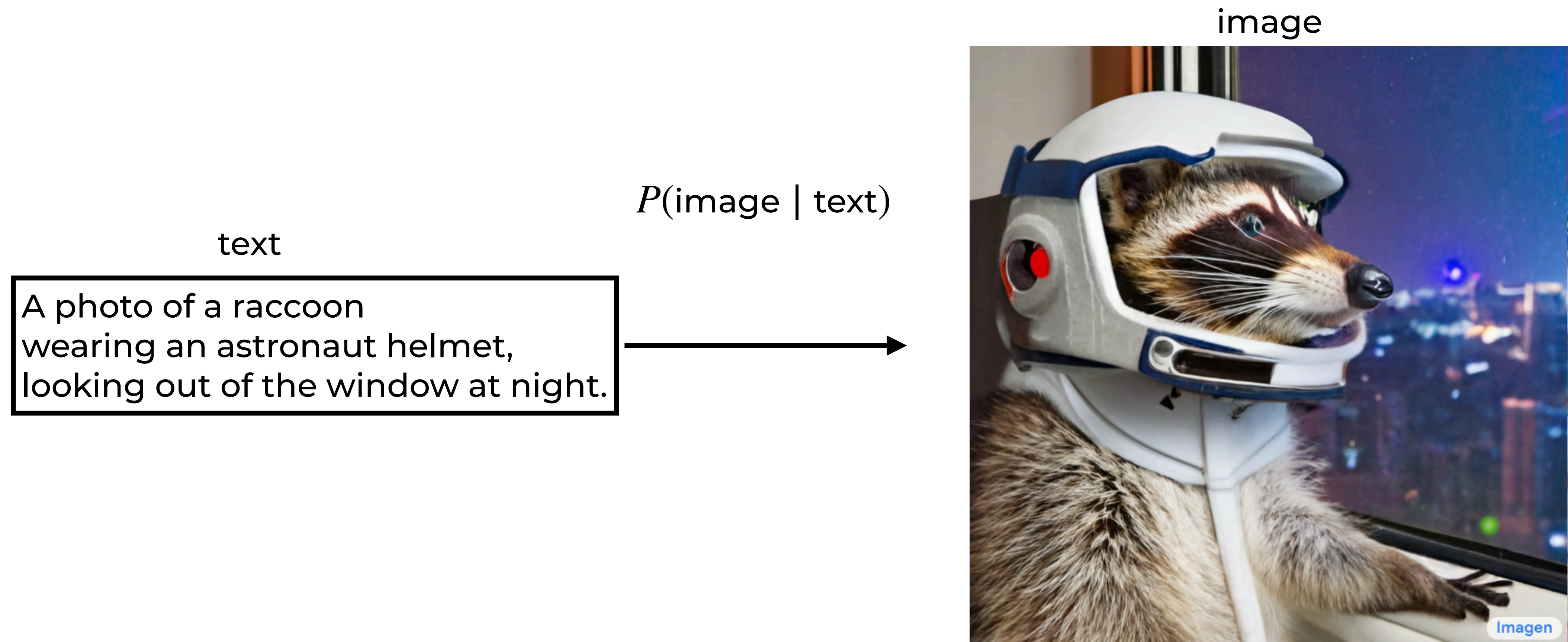


2018

$P(\text{image})$



# Conditional Generation



# Why generate images?



- It used to be a tougher question to answer
- To use wherever images are used (visualizations, video games, presentations, ads, etc...)
- Human-in-the-loop design

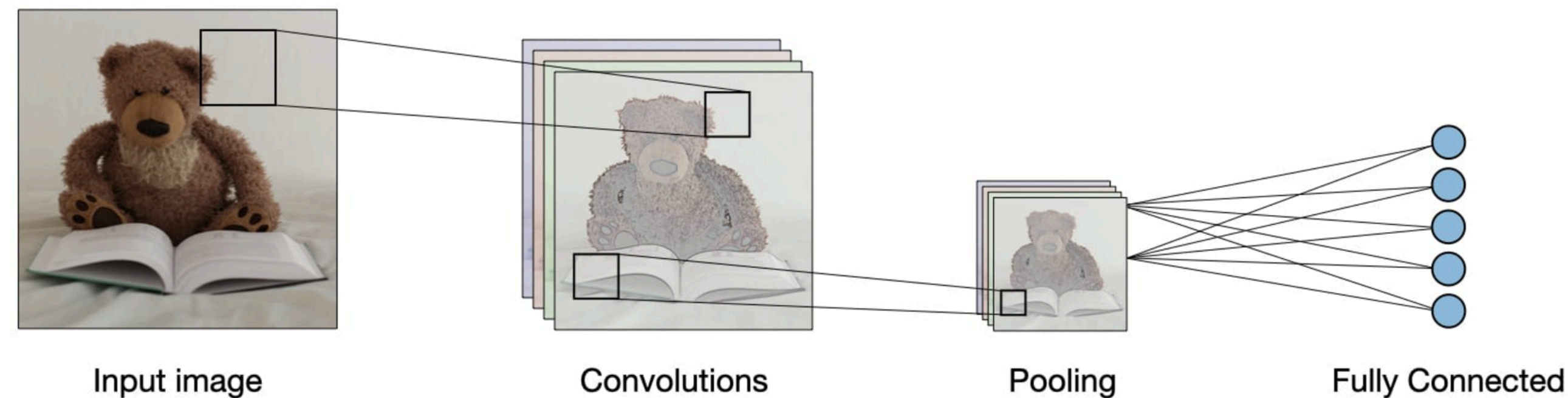


# Today's Plan\*

- “Predict” an image
  - Generative Models
    - Images ( $P(x)$ ):
      - Frameworks: Variational auto-encoders (VAEs), Generative Adversarial Networks (GANs)
    - Images conditioned on text ( $P(x | y)$ ) :
      - Dall-E 2, Imagen
- \* Like last week, the slides are mostly from David Berger

# Convolution Neural Network (CNN) — Recall

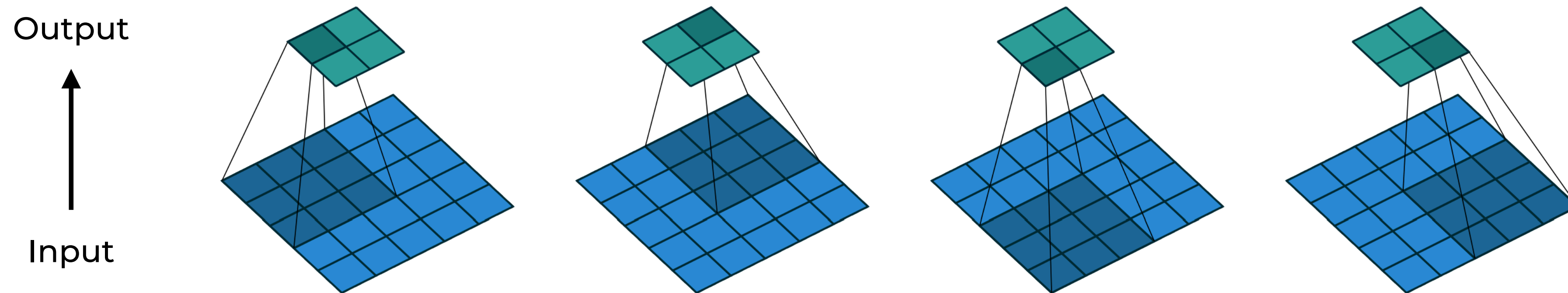
- Séries of layers (blocs): convolutions + pooling
- Each layer reduces the dimensionality of the representation





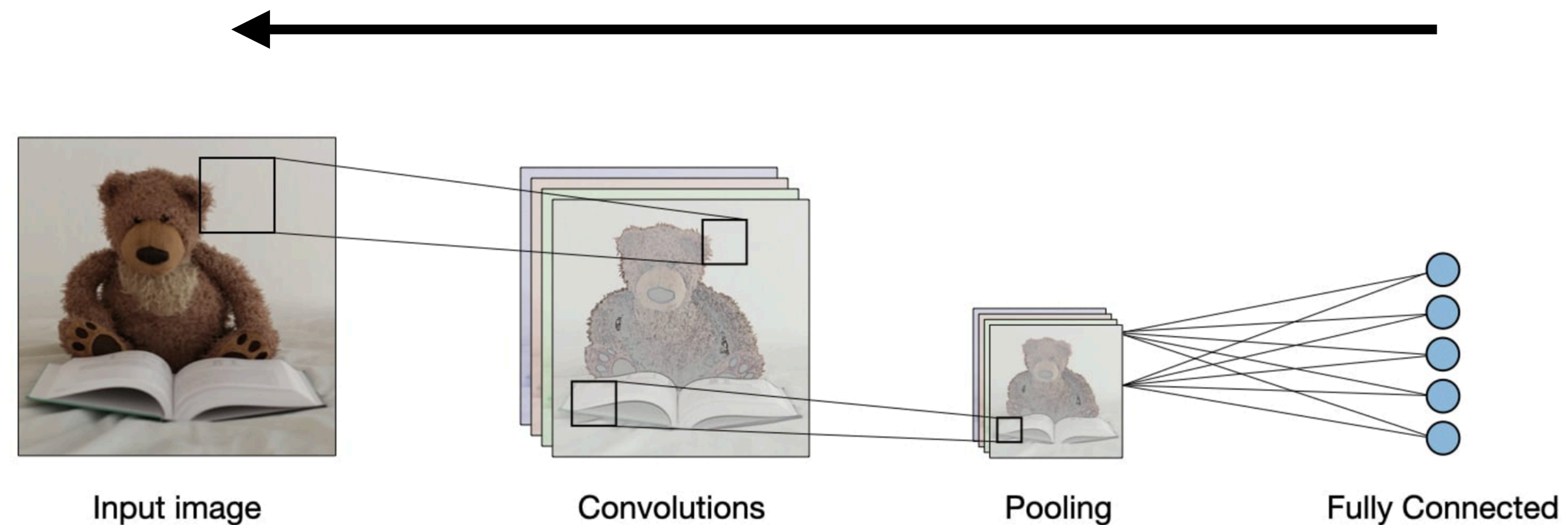
# Convolution

- Input: 5 x 5
- Filter (kernel): 3 x 3. Stride 2.
- Output: 2 x 2



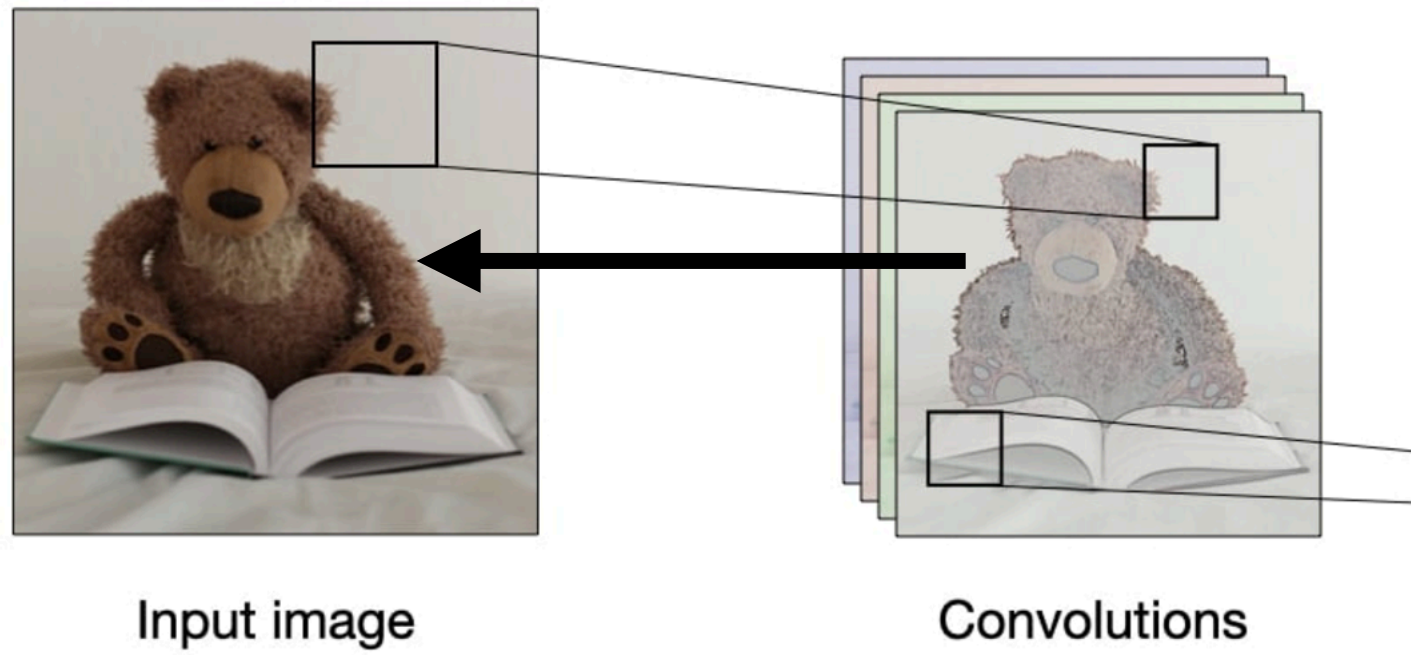
# “Reverse” a convolution network

- Each layers increases the dimensionality of the incoming representation



- (Note: this is in fact the same operation as done by backdrop in a CNN.)





# Interpolation

## 1. Repetition

### Nearest Neighbor

1	2
3	4

Input: 2 x 2



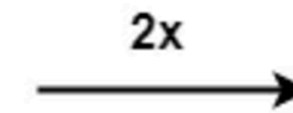
1	1	2	2
1	1	2	2
3	3	4	4
3	3	4	4

Output: 4 x 4

## 2. (Bi-)linear interpolation

10	20
30	40

2x2



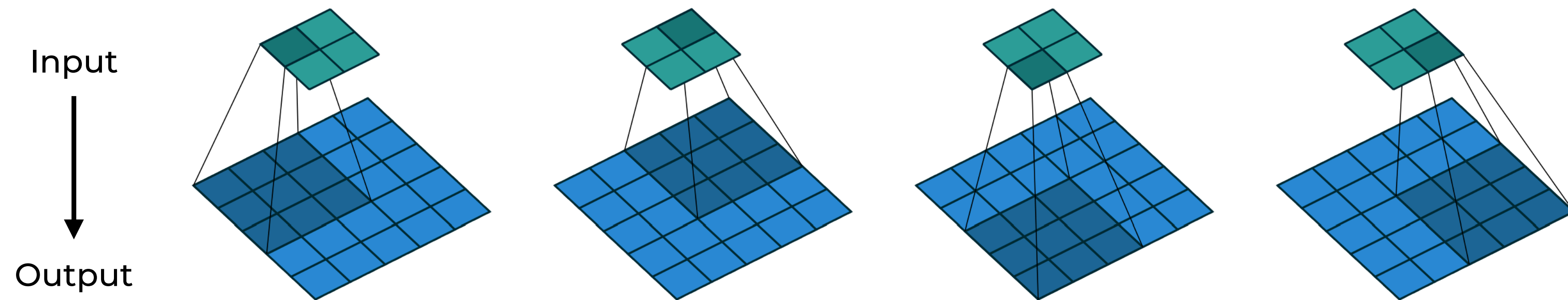
10	12	17	20
15	17	22	25
25	27	32	35
30	32	37	40

4x4

- No parameters to learn

# Transposed Convolution

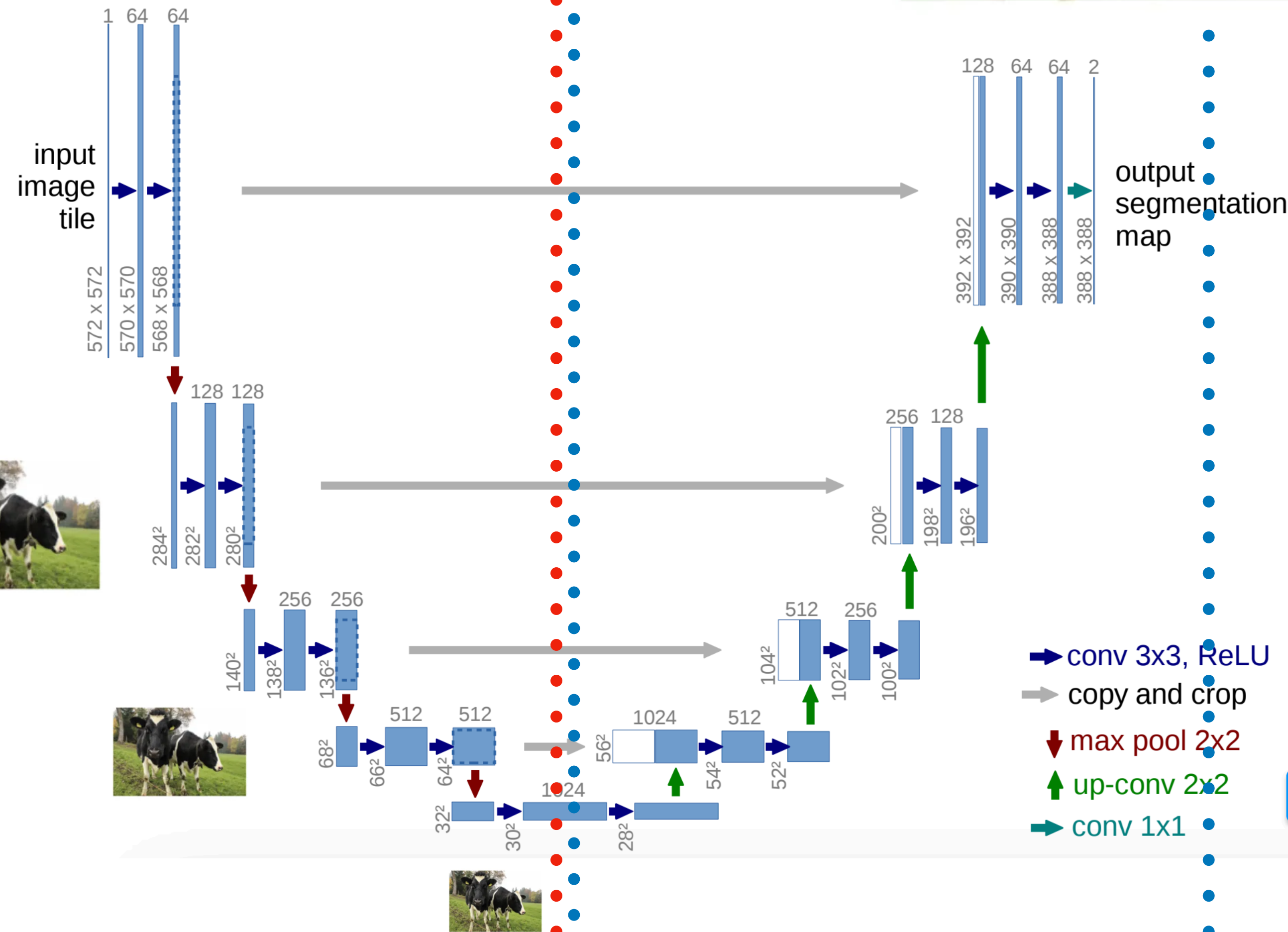
- Useful to generate images or increase the resolution of an image (like in movies)
- Input:  $2 \times 2$
- Filter:  $3 \times 3$ . Stride 1.
- Output:  $3 \times 3$



- If we write down a convolution as a matrix operation (by vectorizing the image), then the reverse operation is multiplying by the transpose (hence the name)



# U-Nets



## Encoder-Decoder Architecture

- Encoder: Obtain a representation of the image (classic CNN without the classification output)
- Decoder: From the representation, obtain an image
- Connections between the encoder and decoder allow for precise localization
- up-conv -> transposed convolution (for example)
- Proposed for segmentation
- Has become standard for image generation

# Generative models

- A U-Net can obtain a representation from an image, but it does not have a probabilistic interpretation
- A generative model is a method for parametrizing  $P(x)$  — unsupervised



# Why are generative models (often) probabilistic?

- Allows different types of evaluation
  - For example, the probability of an image according to the model:  $P_{\theta}(x_{\text{new}})$
- Can obtain samples  $x \sim P_{\theta}(x)$
- Quantifies uncertainty
- It has been popular recently to parametrize distributions with neural networks (think of a softmax layer) — these are not always proper generative models

# Auto-Encoder (AE)



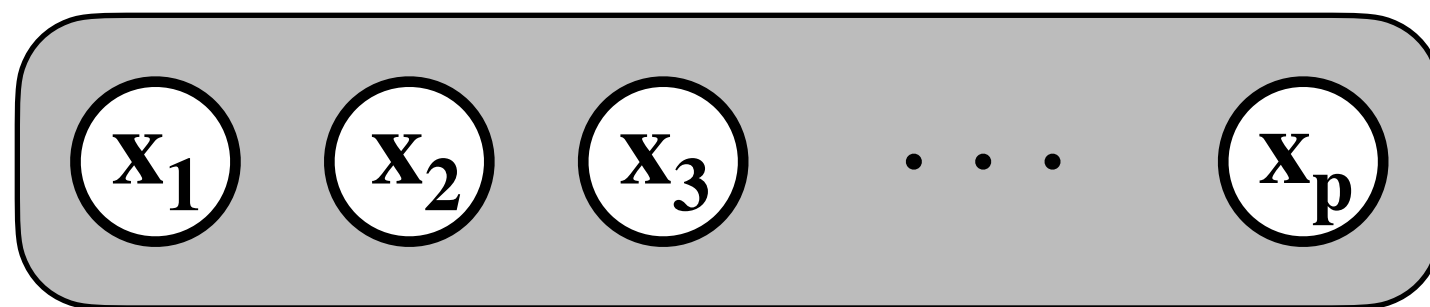
# Auto-encoder - Recall

For an AE with a single hidden layer:

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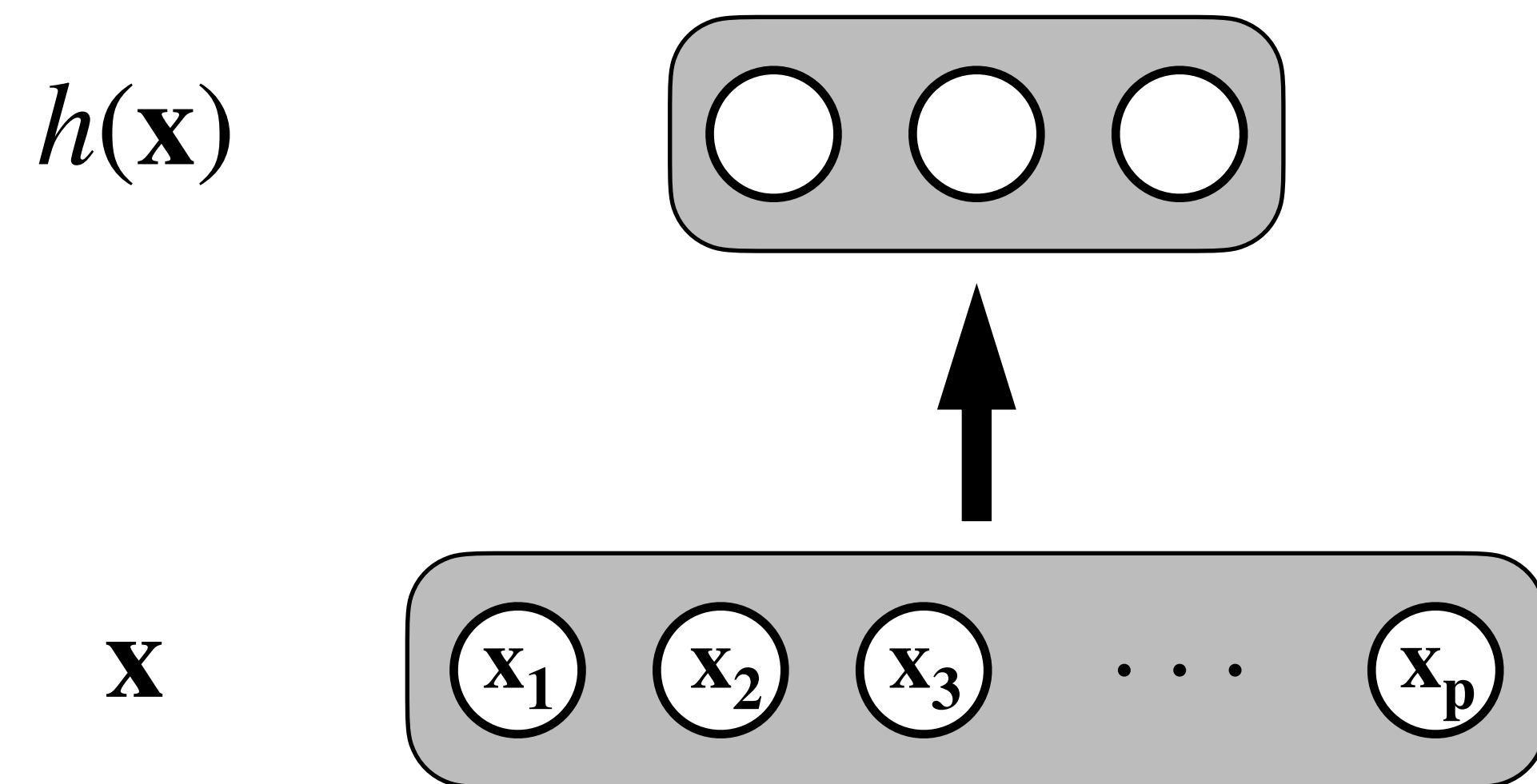
For an AE with a single hidden layer:

**X**



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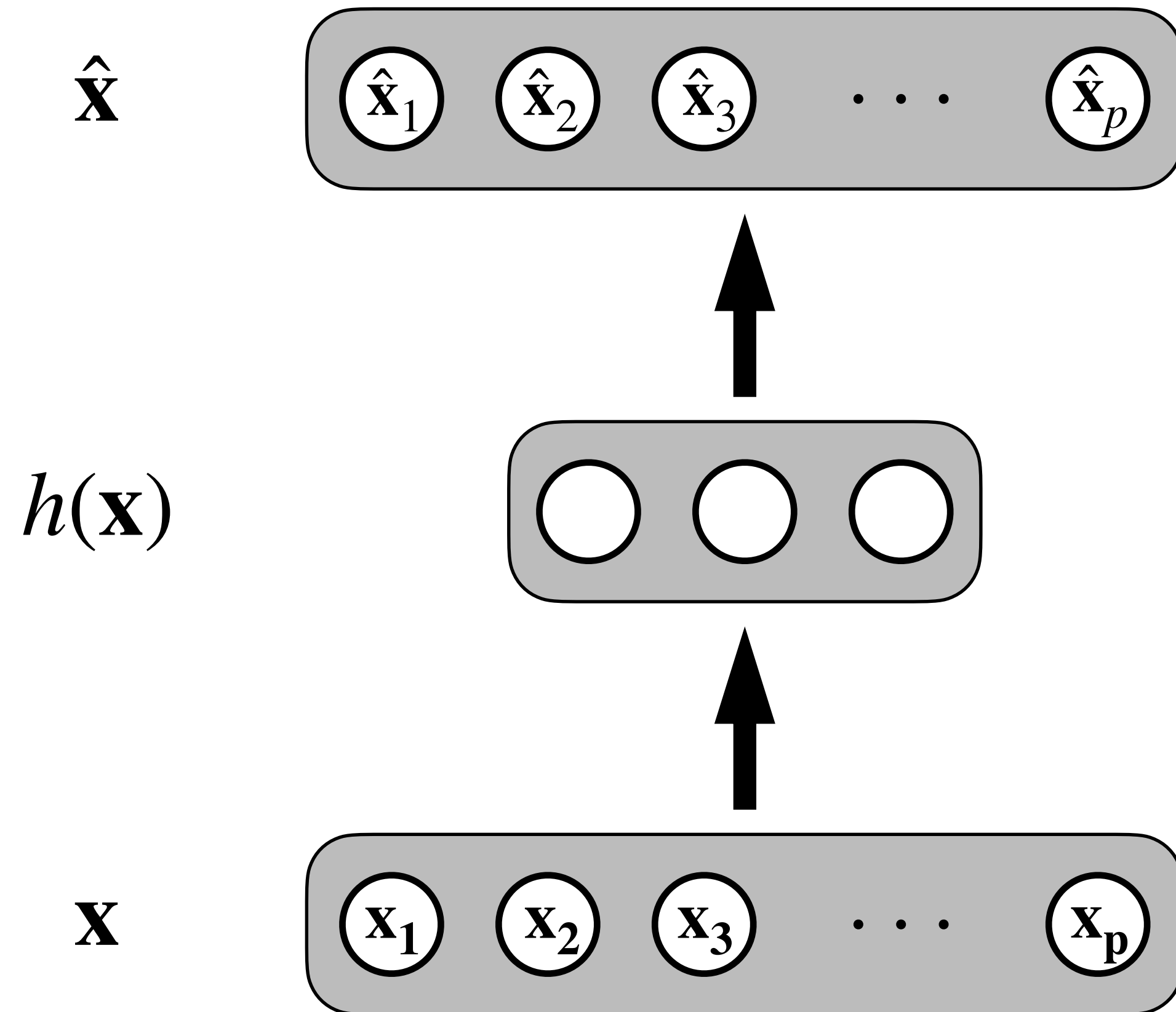
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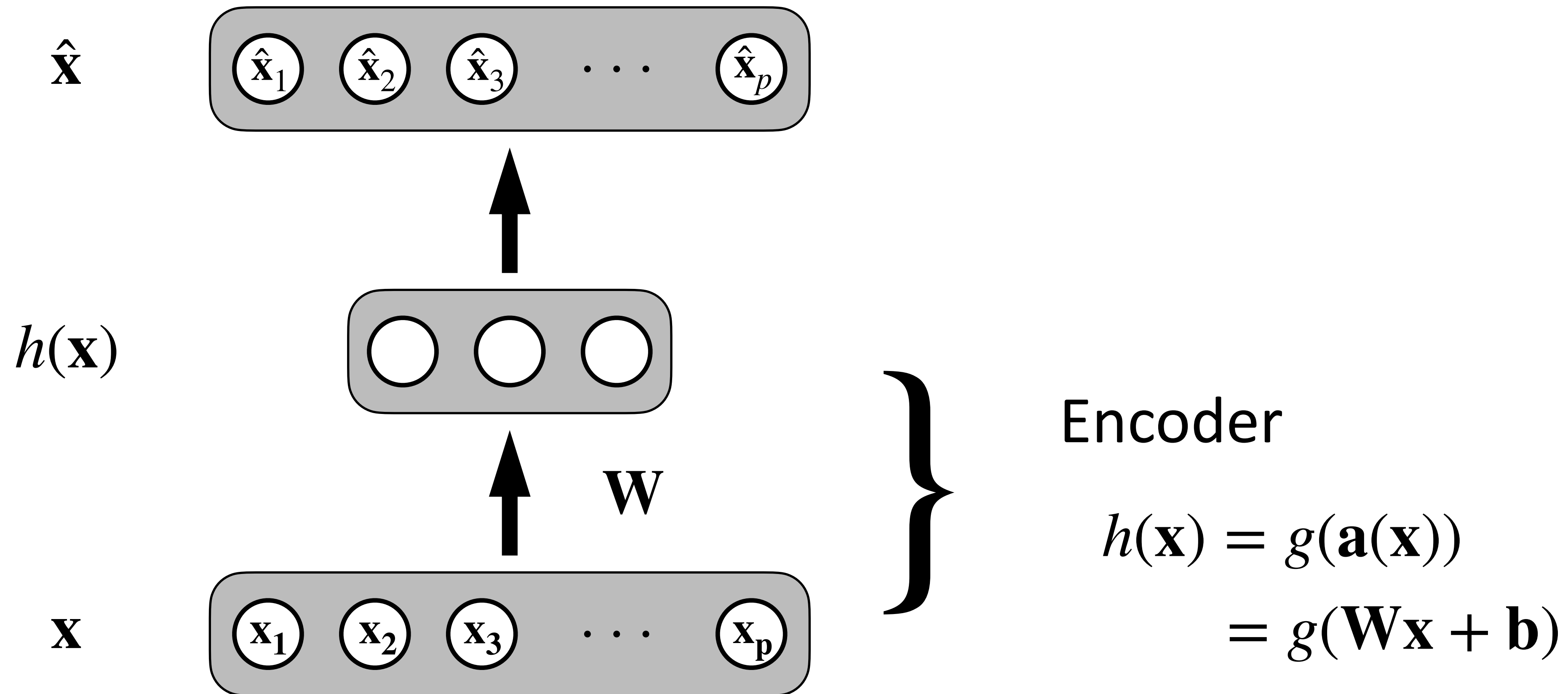
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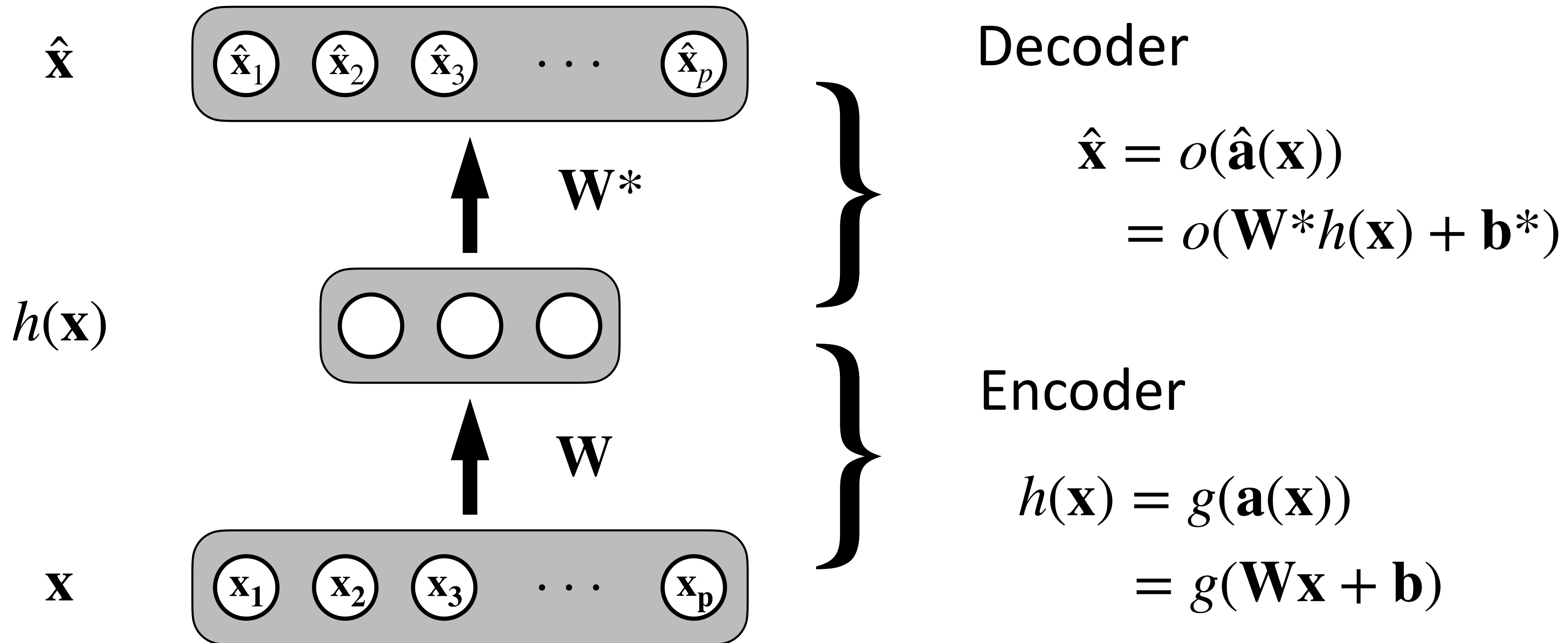
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# Variational Auto-encoder (VAE)

\* Understanding these models precisely requires concepts that are beyond our class. Here, we aim for familiarization with the terms and an intuitive understanding.

# Variational Auto-Encoder - Motivation

Idea:

- The data are generated conditioned on a random variable (Z):

$$\mathbb{P}_{\theta} (\mathbf{x} \mid \mathbf{z})$$

You can think of Z has an embedding.  
VAEs will learn a prior  $P(z)$  and  
a posterior  $P(z|x)$  over it.

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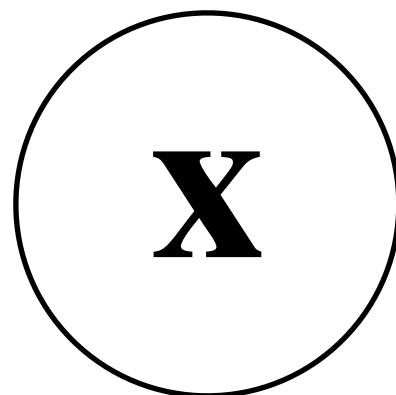
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Graphical representation





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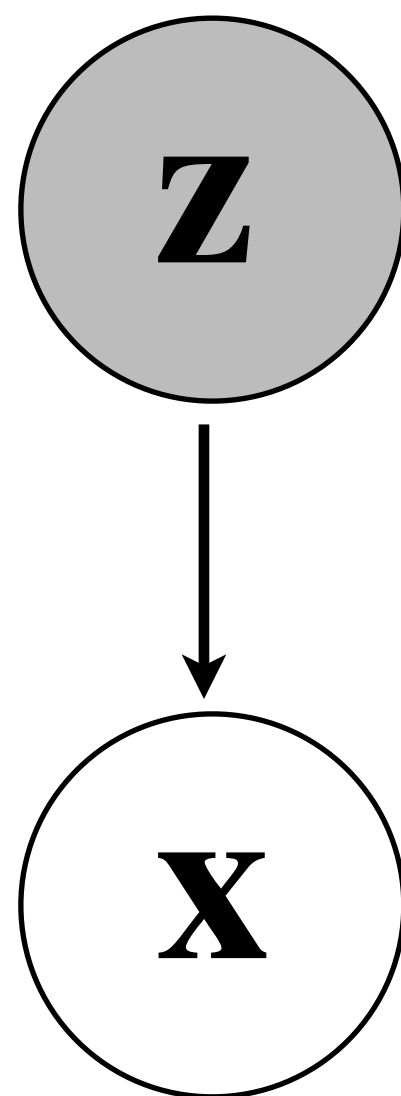
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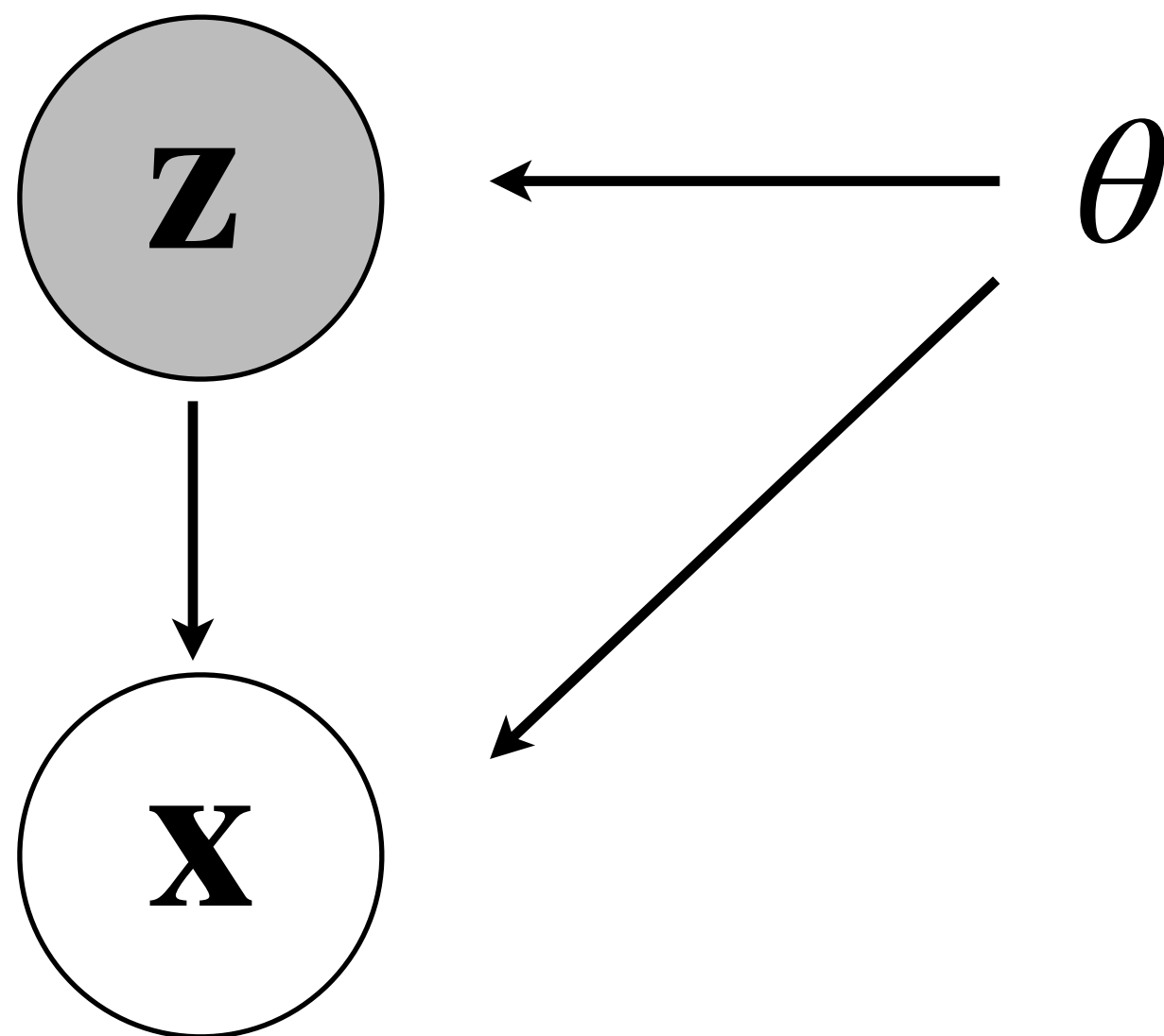
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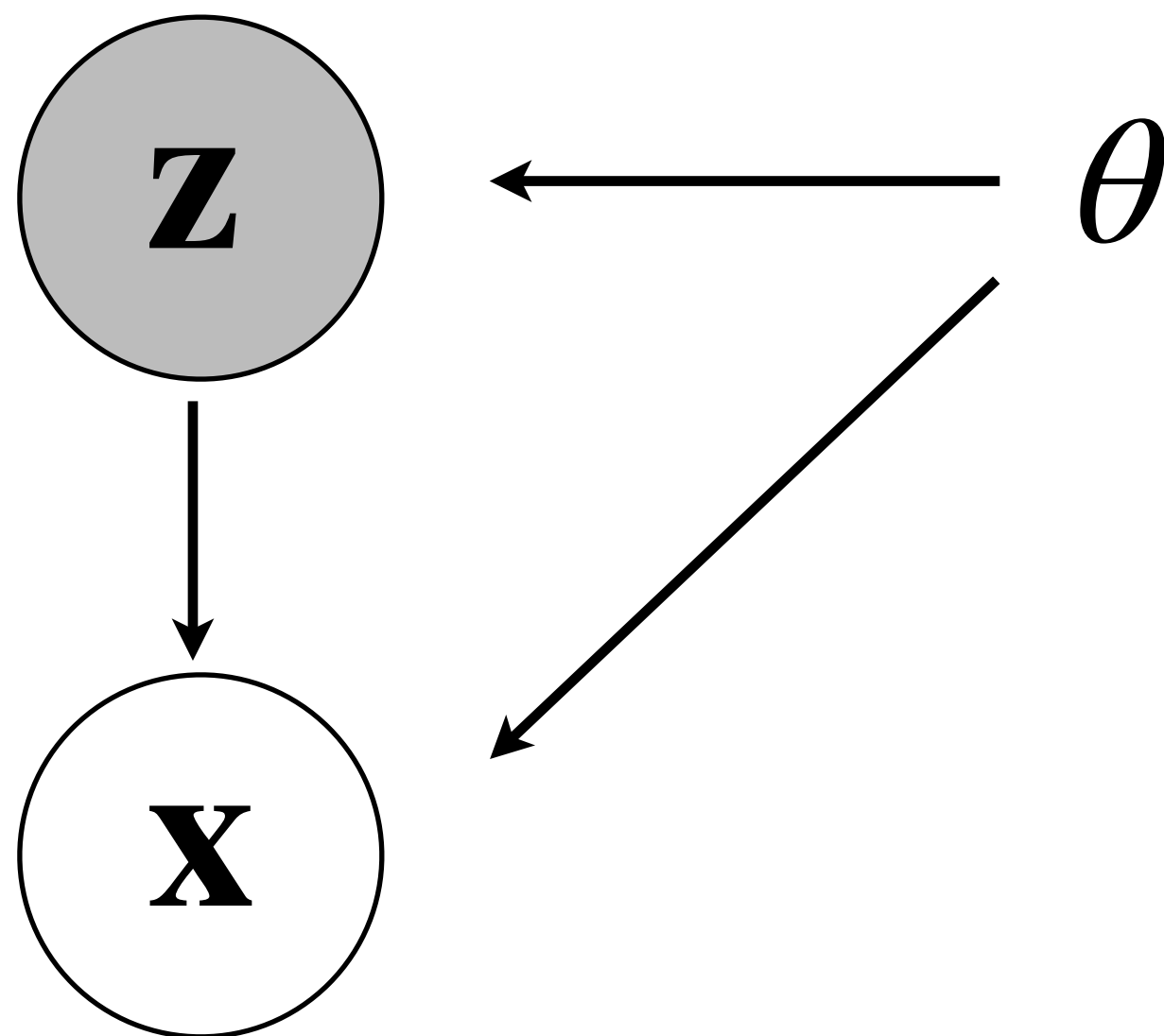
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Question:

- How do we learn such a distribution?



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Partial answer:

- A “good” model should maximize the likelihood of the data  $P(x)$
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Idea:

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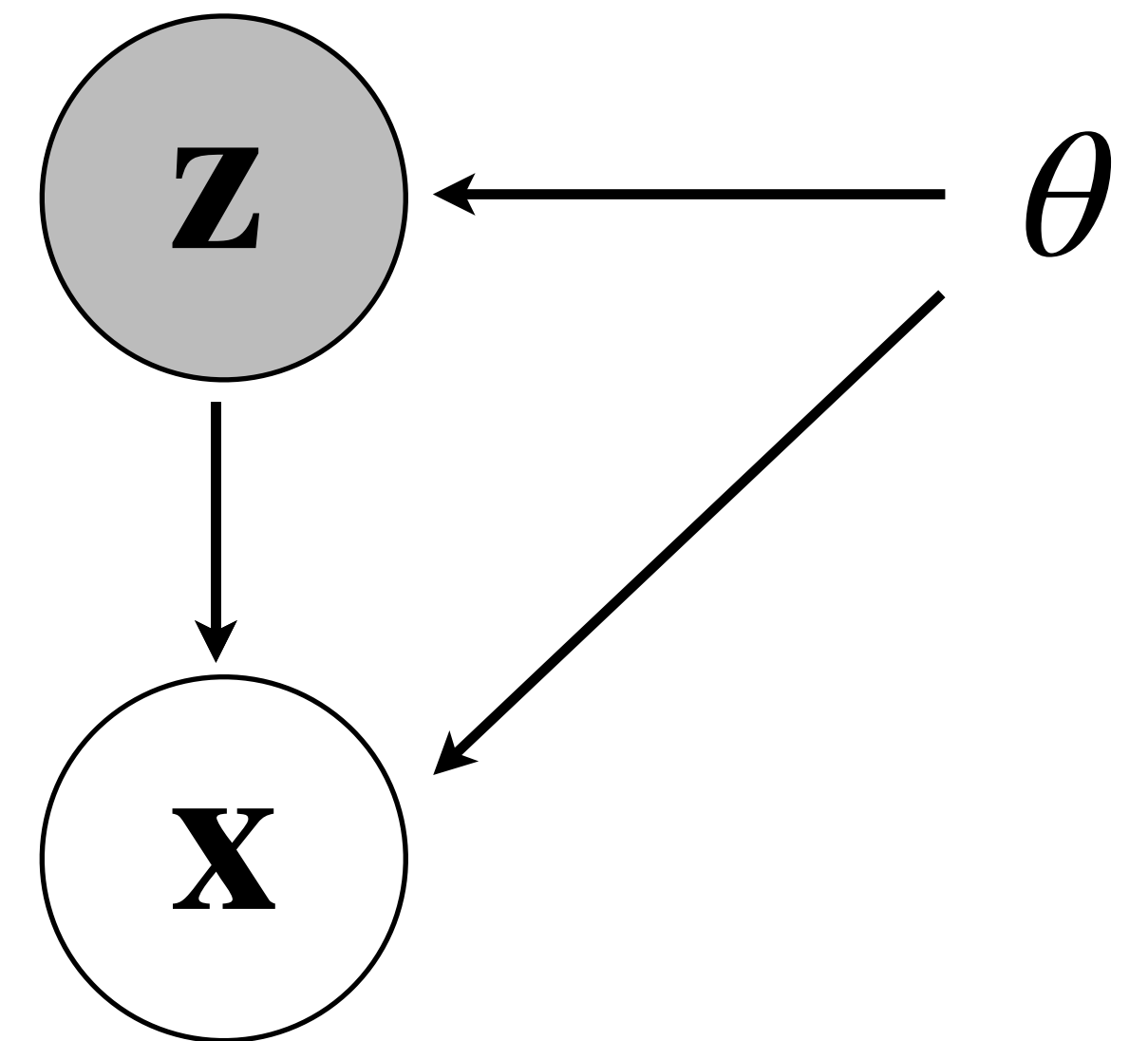
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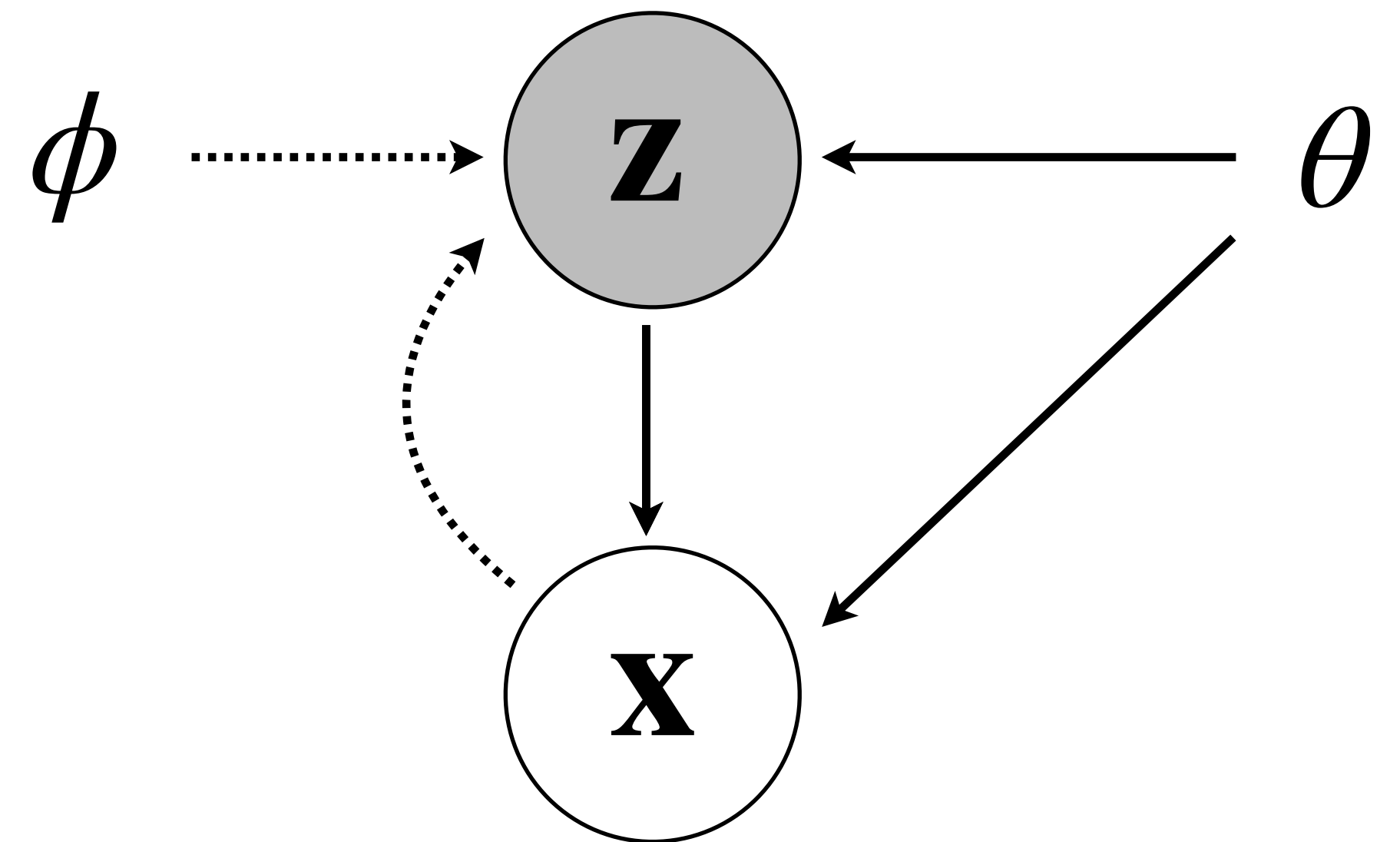
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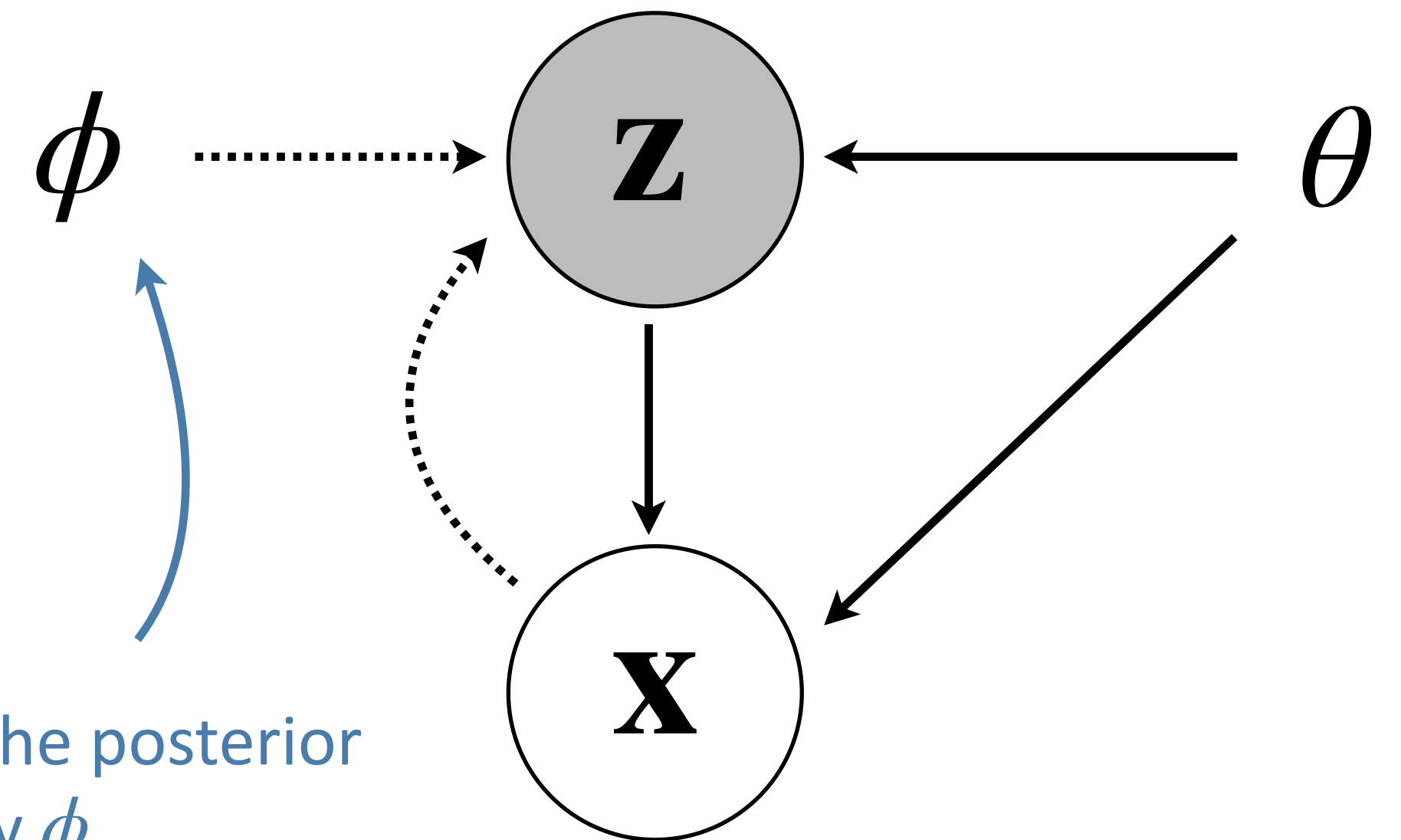
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The approximation of the posterior would be parametric by  $\phi$

# Variational Auto-Encoder - Formalism

We can obtain a bound on the log-likelihood of  $\mathbf{x}$ :

$$\log p_{\theta}(\mathbf{x}) \geq -\text{KL}\{q_{\phi}(\mathbf{z} \mid \mathbf{x}) \parallel q_{\phi}(\mathbf{z})\} + \log p_{\theta}(\mathbf{x} \mid \mathbf{z}),$$



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Question:

- How do we do this in practice?

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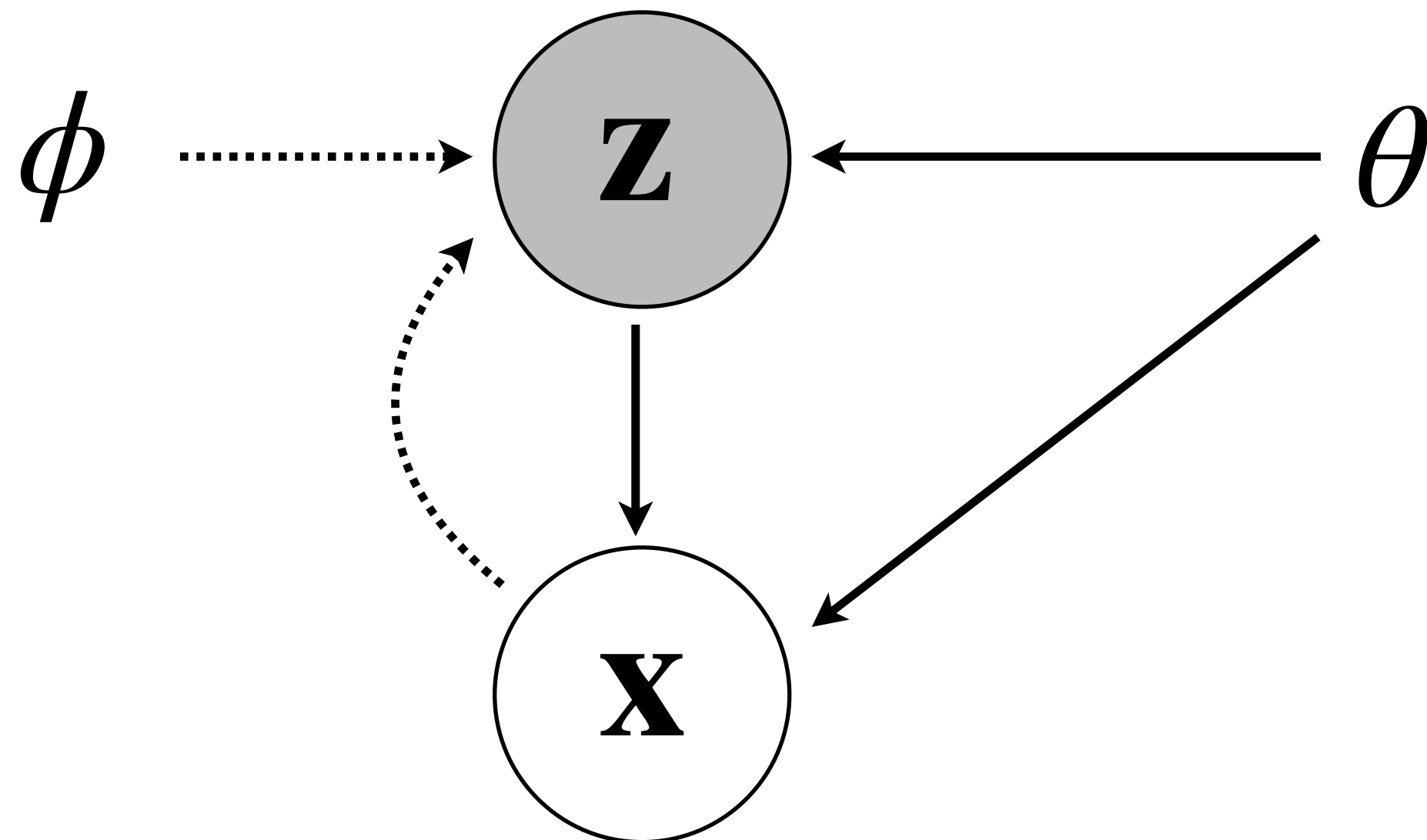
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Graphical view:

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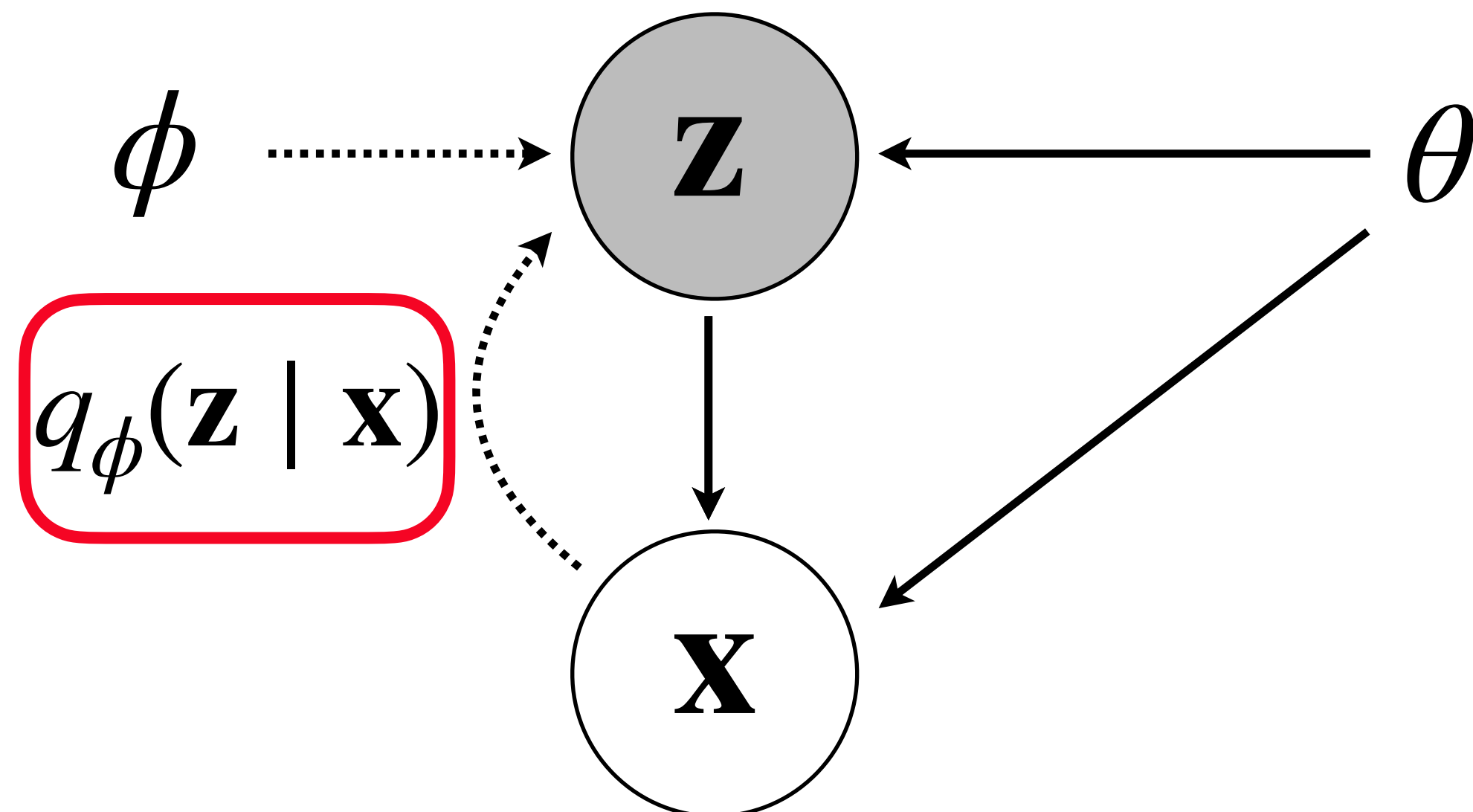
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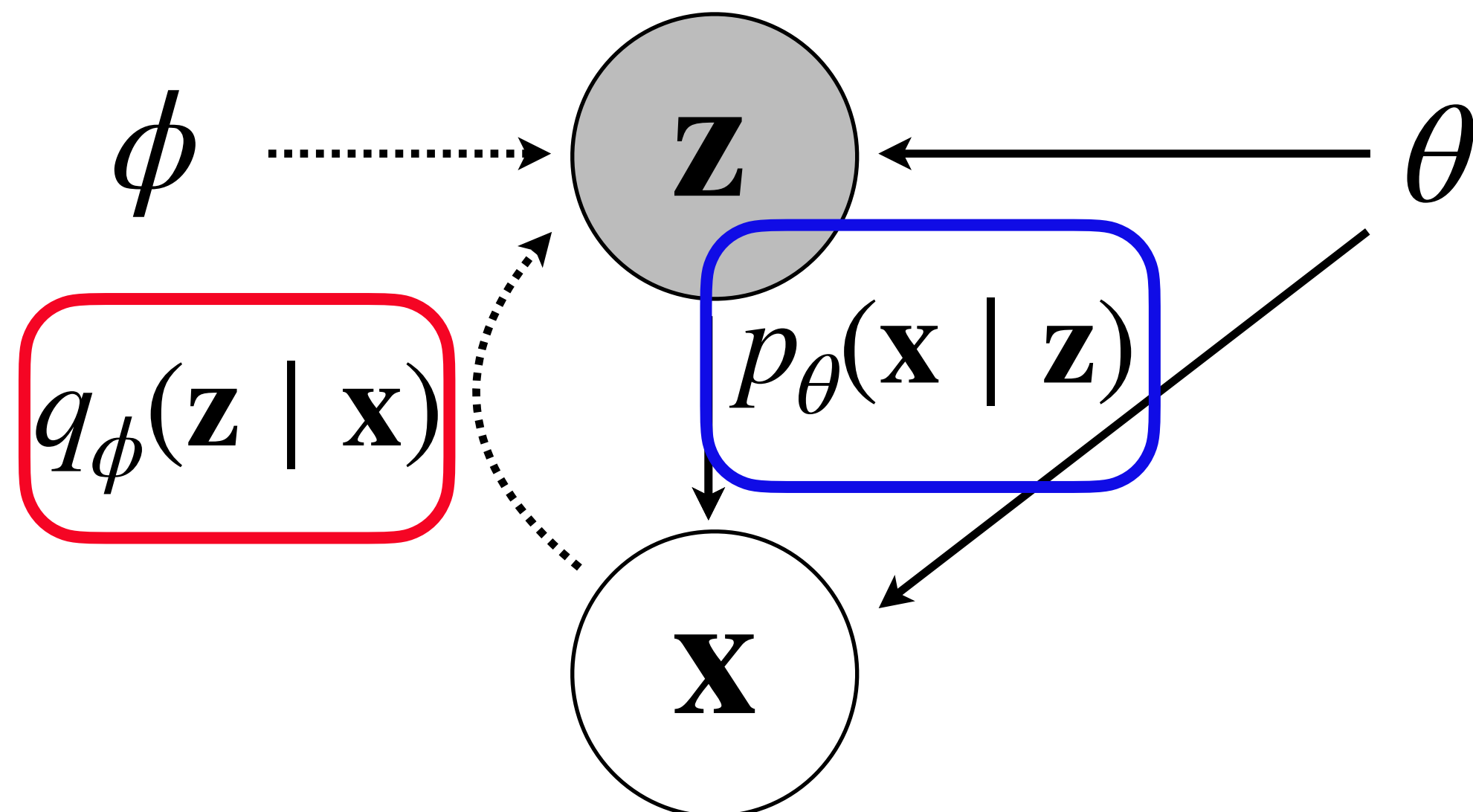
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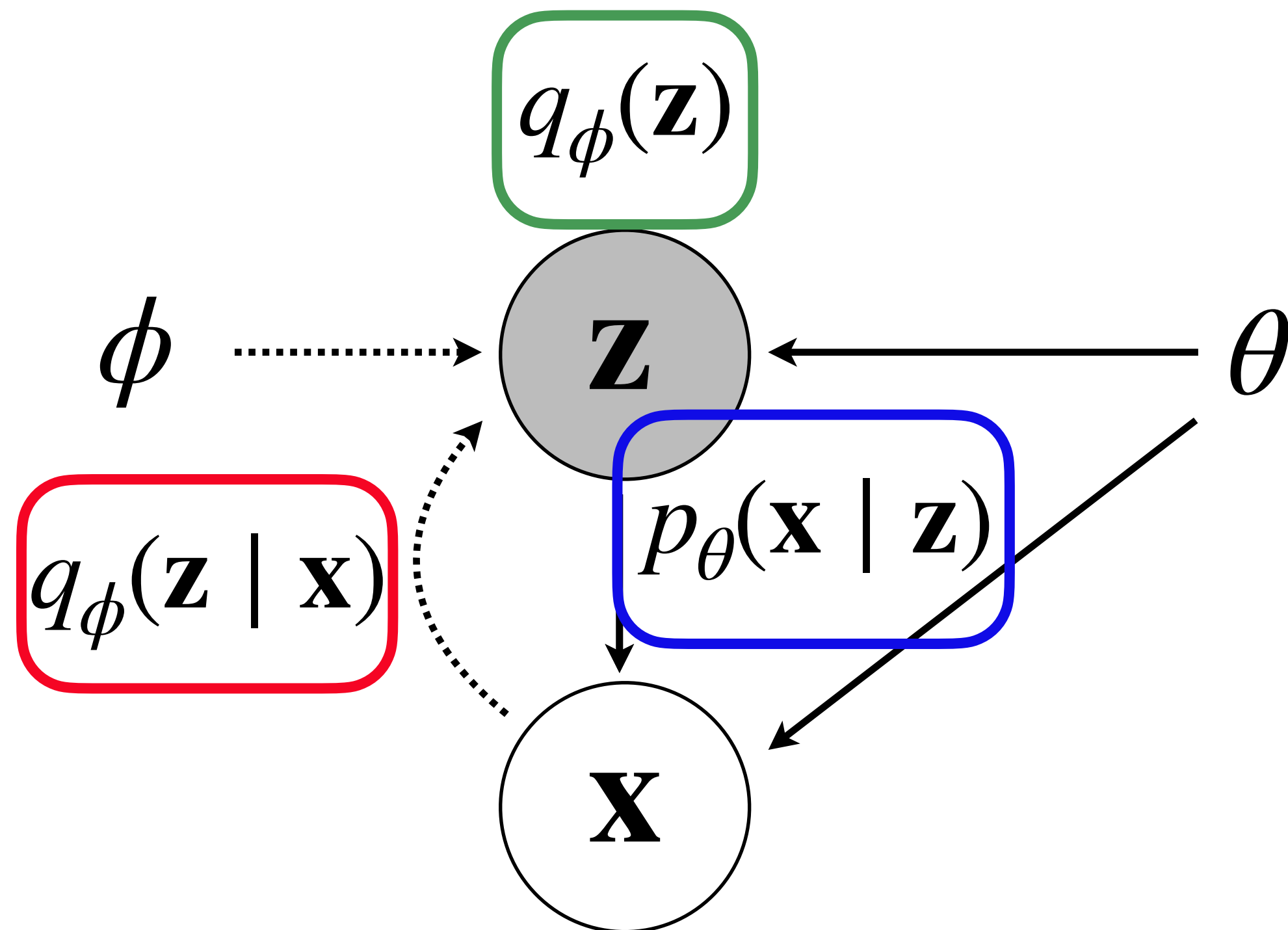




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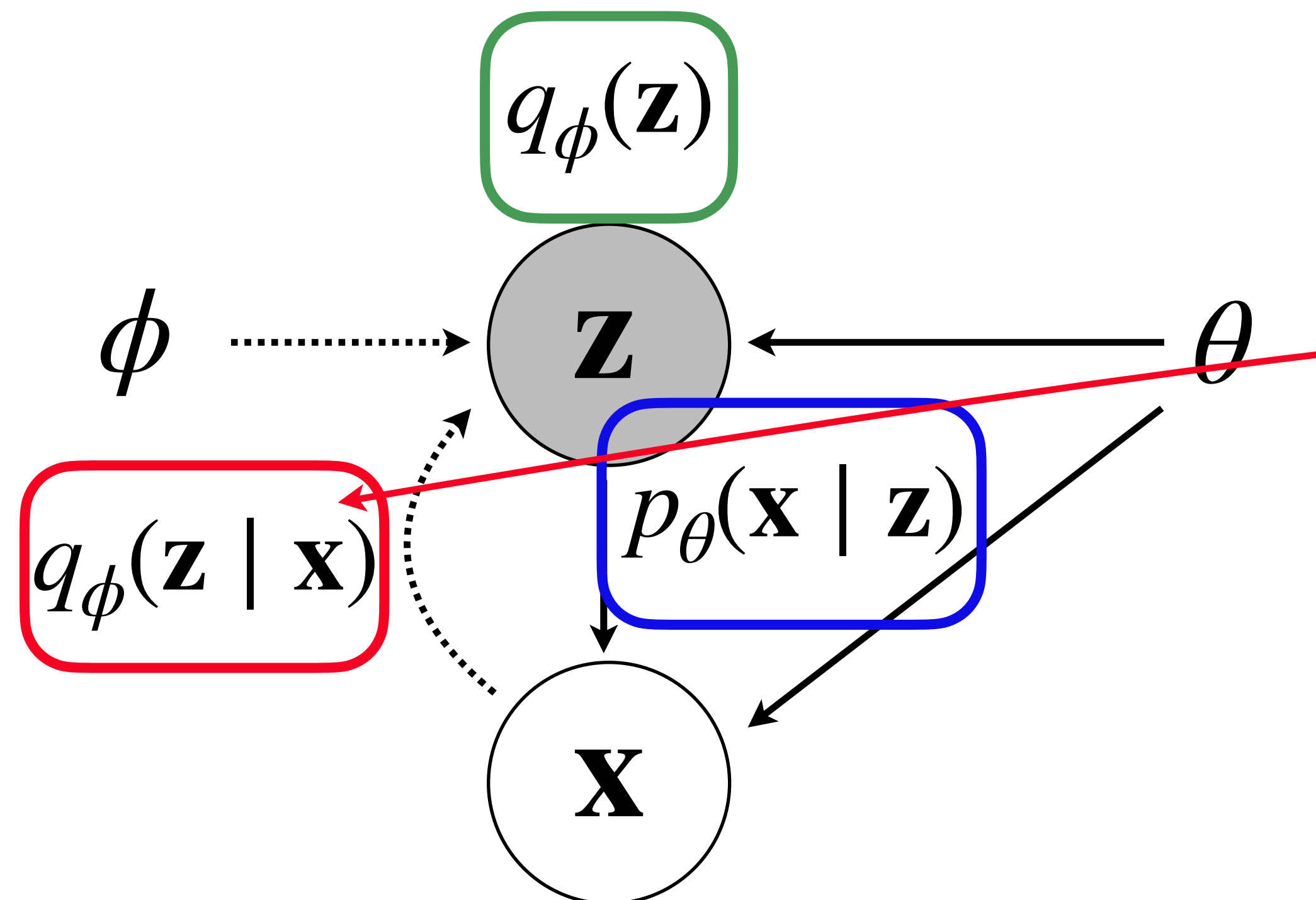
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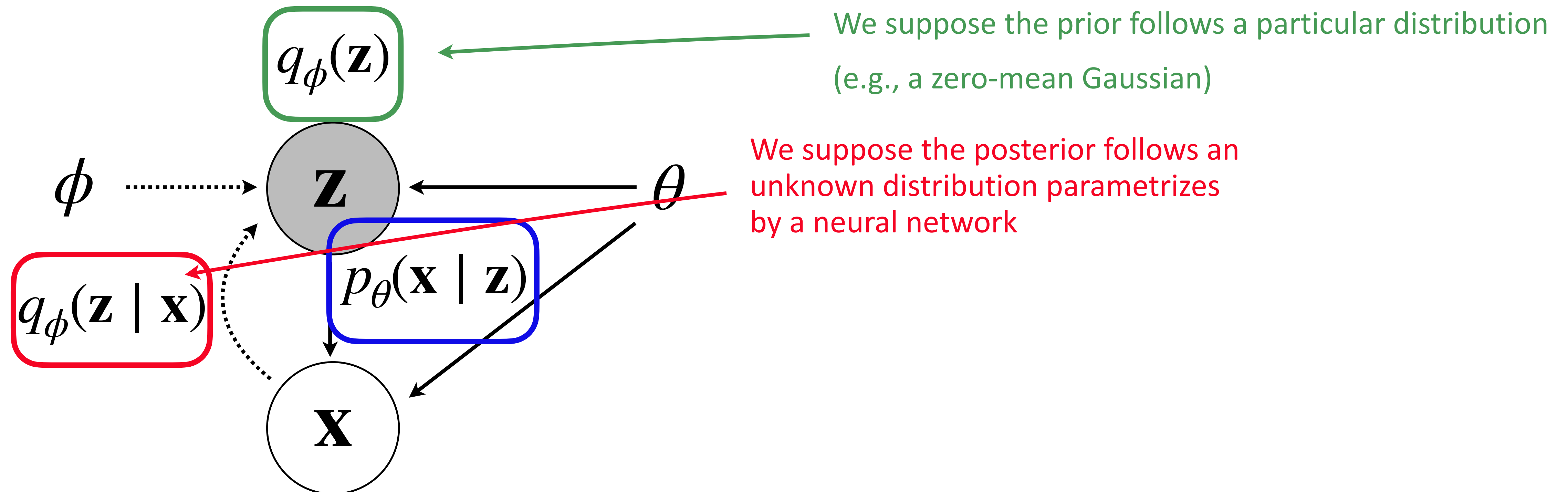


We suppose the posterior follows an unknown distribution parametrizes by a neural network

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# Variational Auto-encoders

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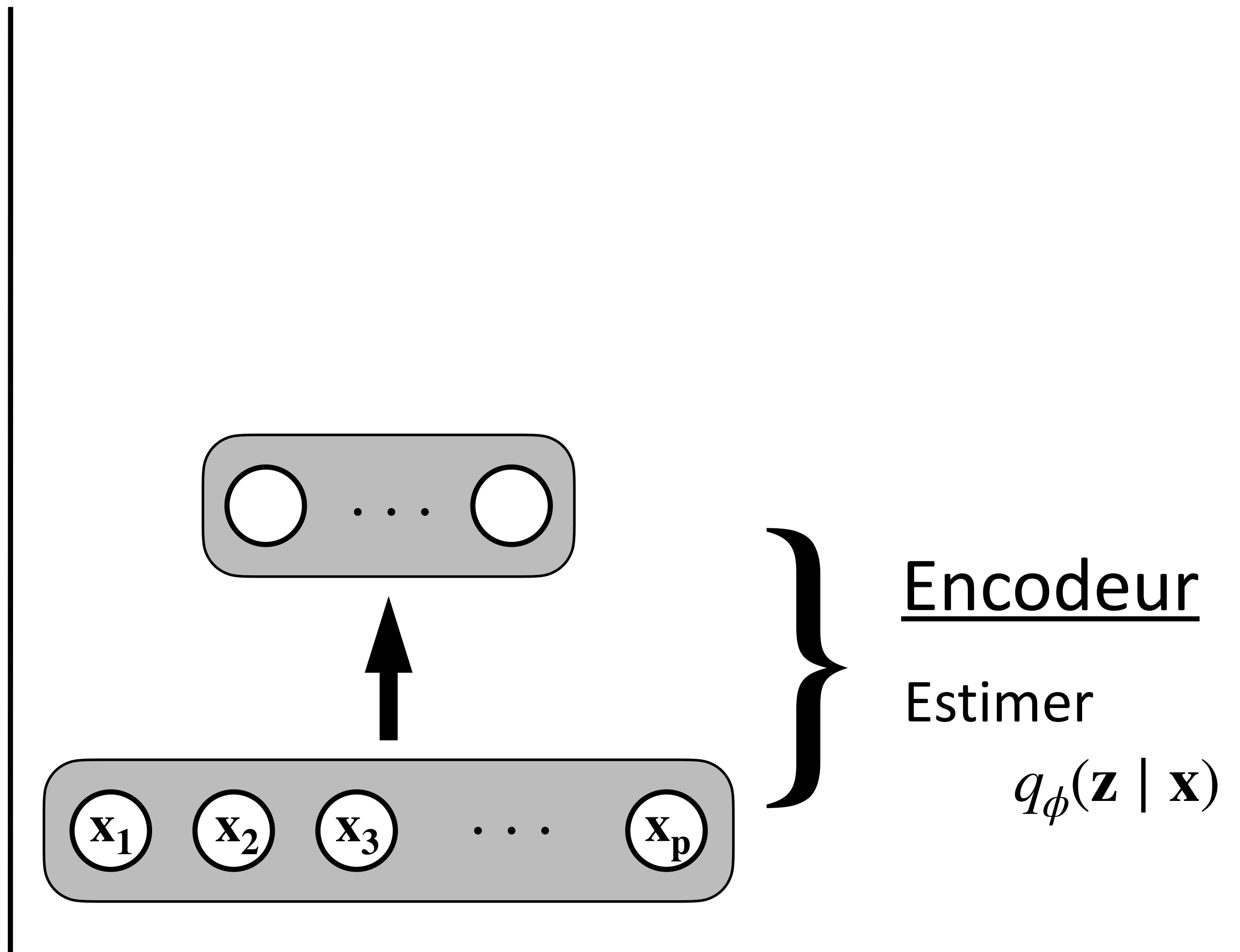
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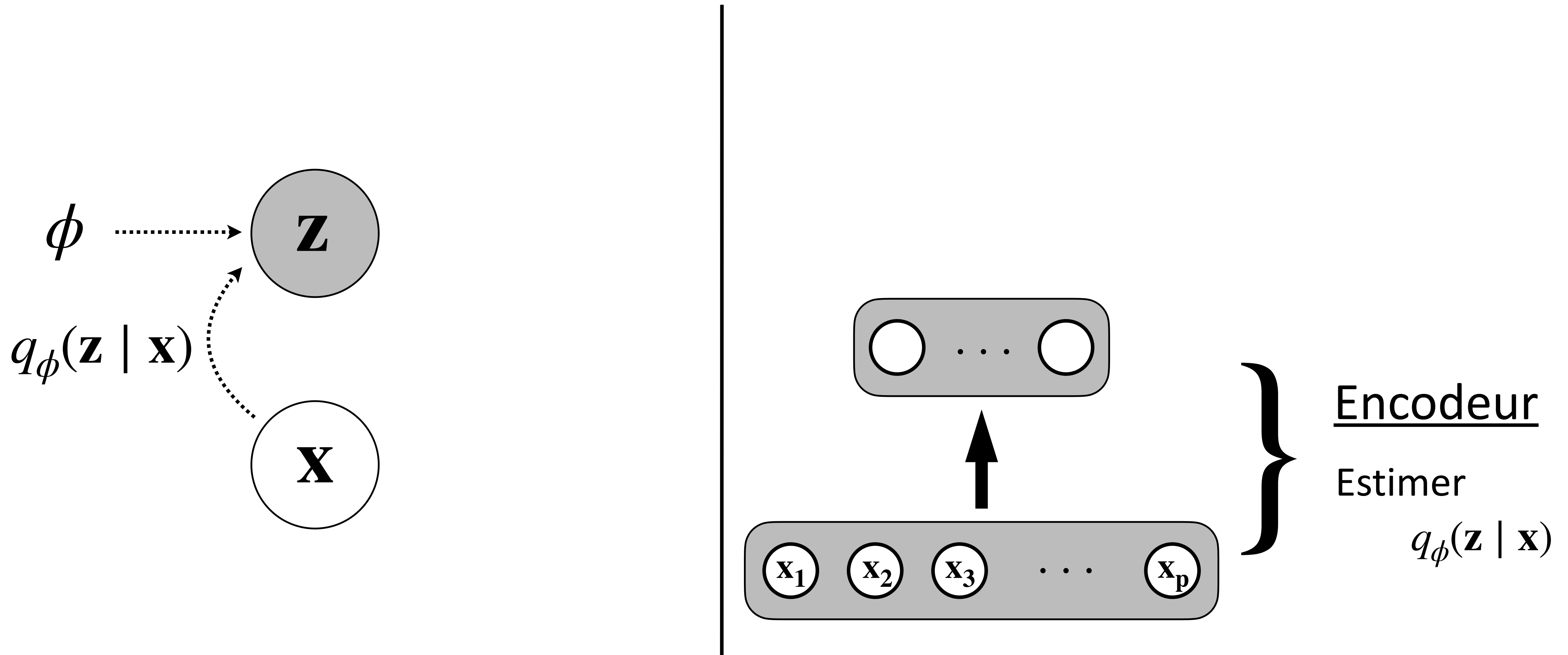
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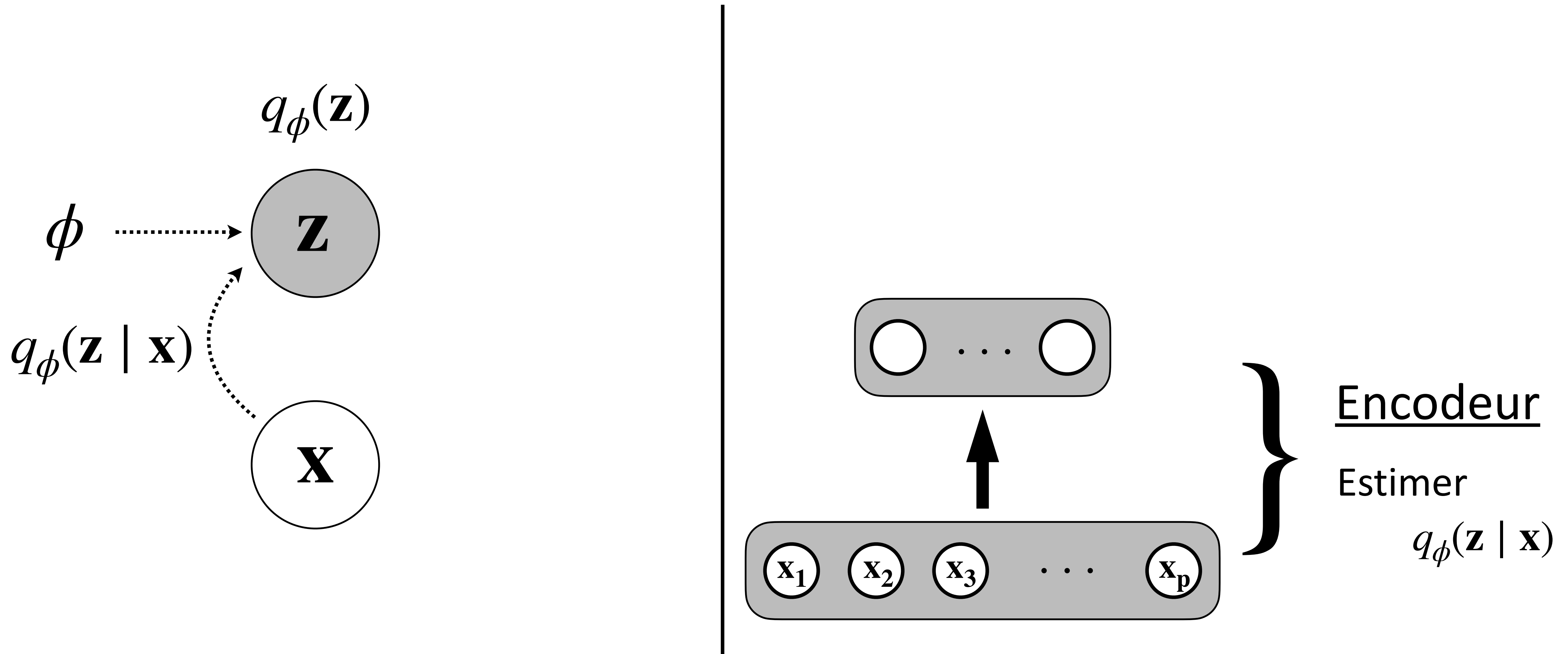
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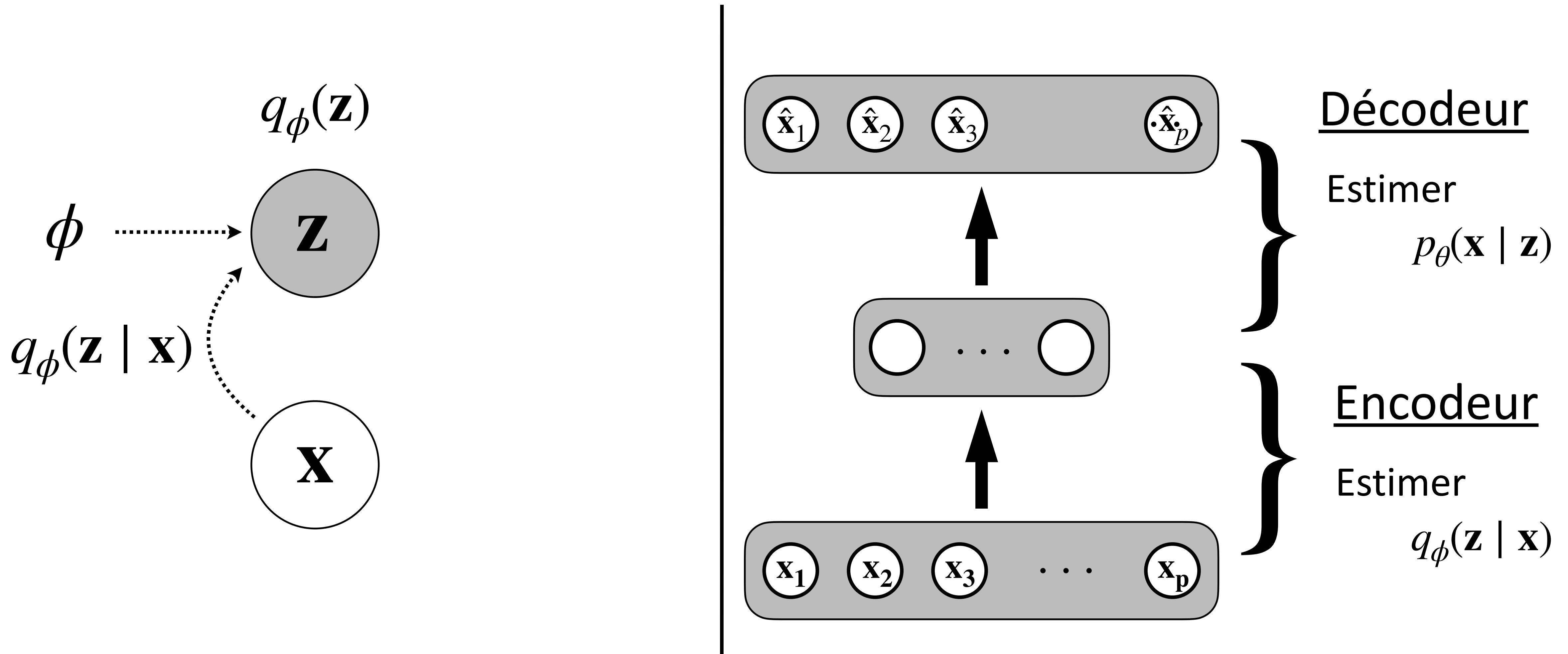
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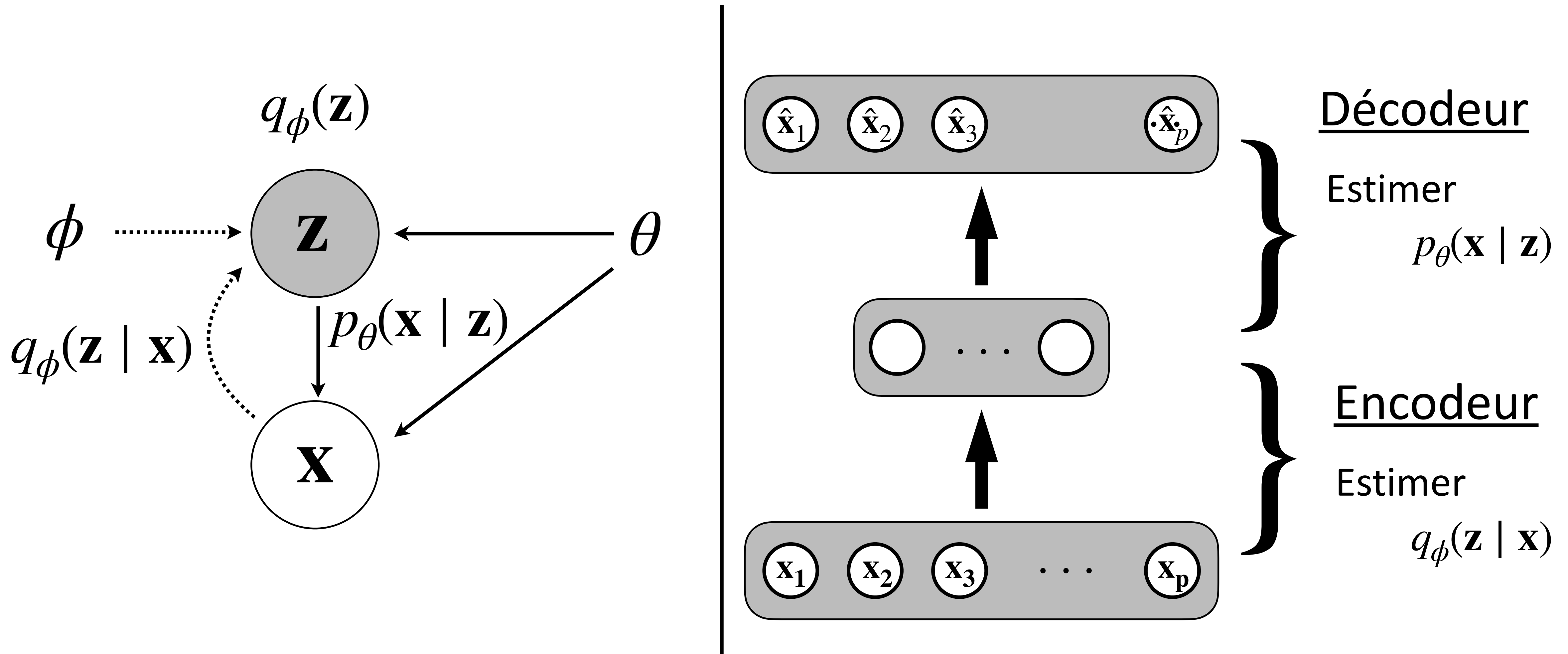
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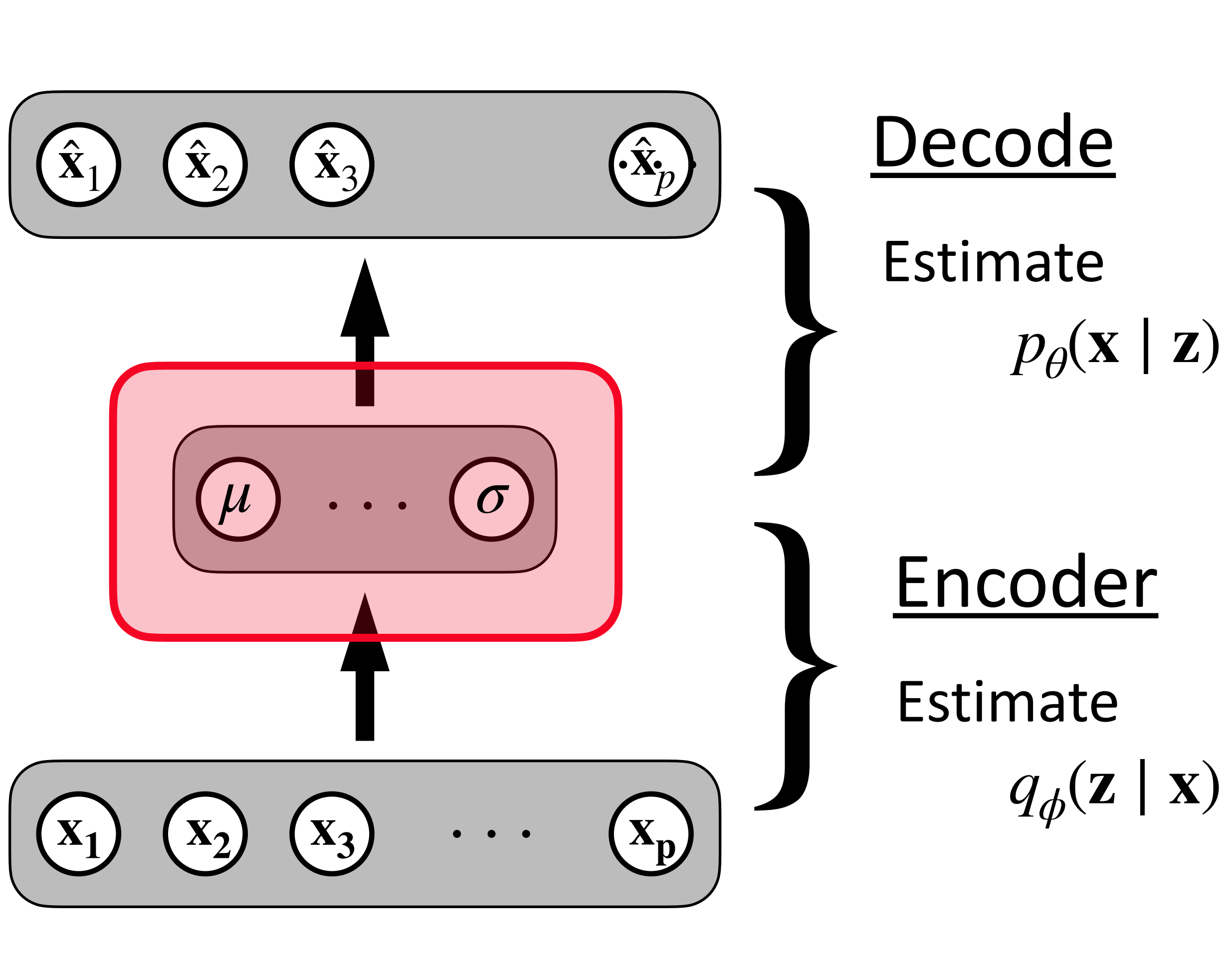


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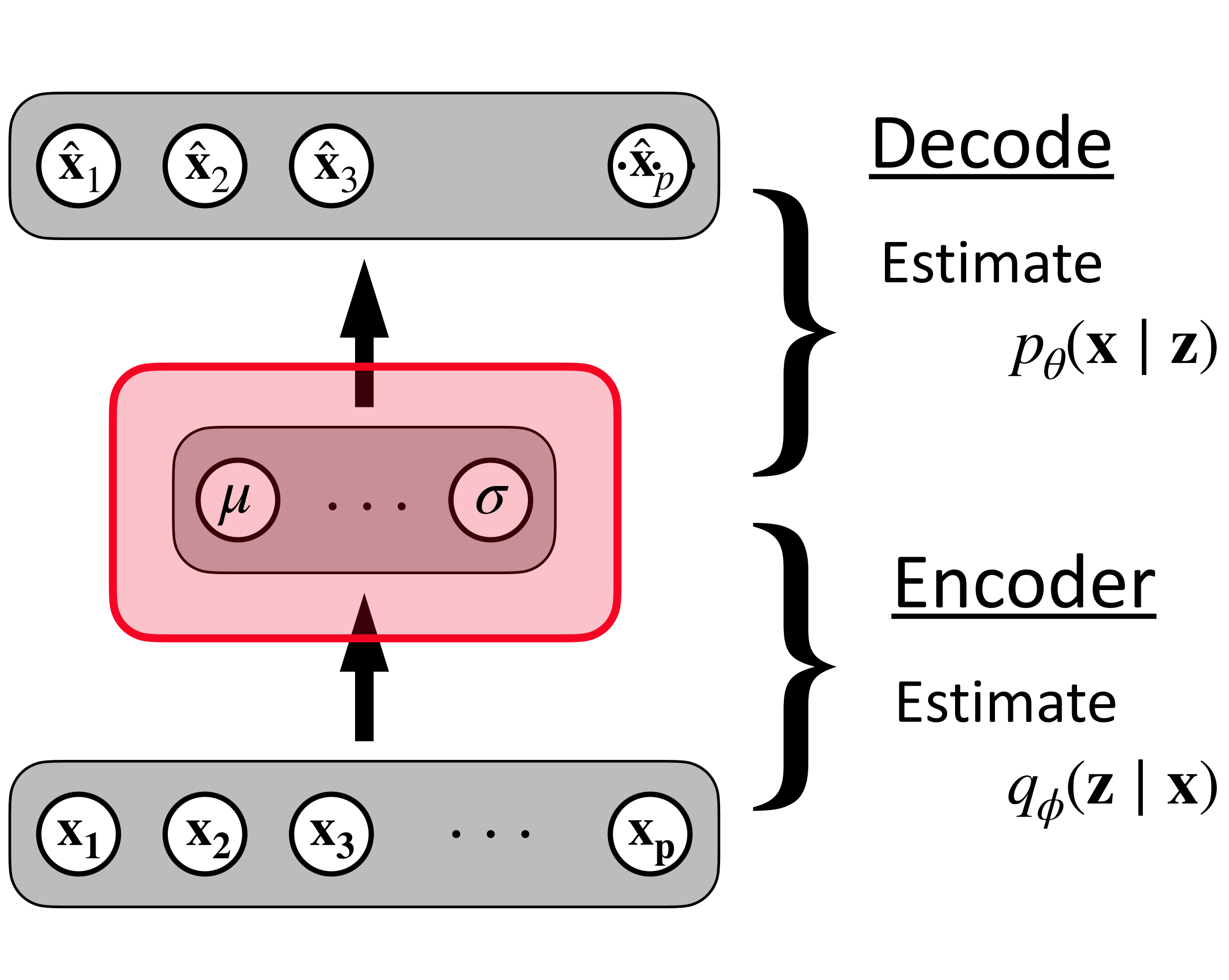


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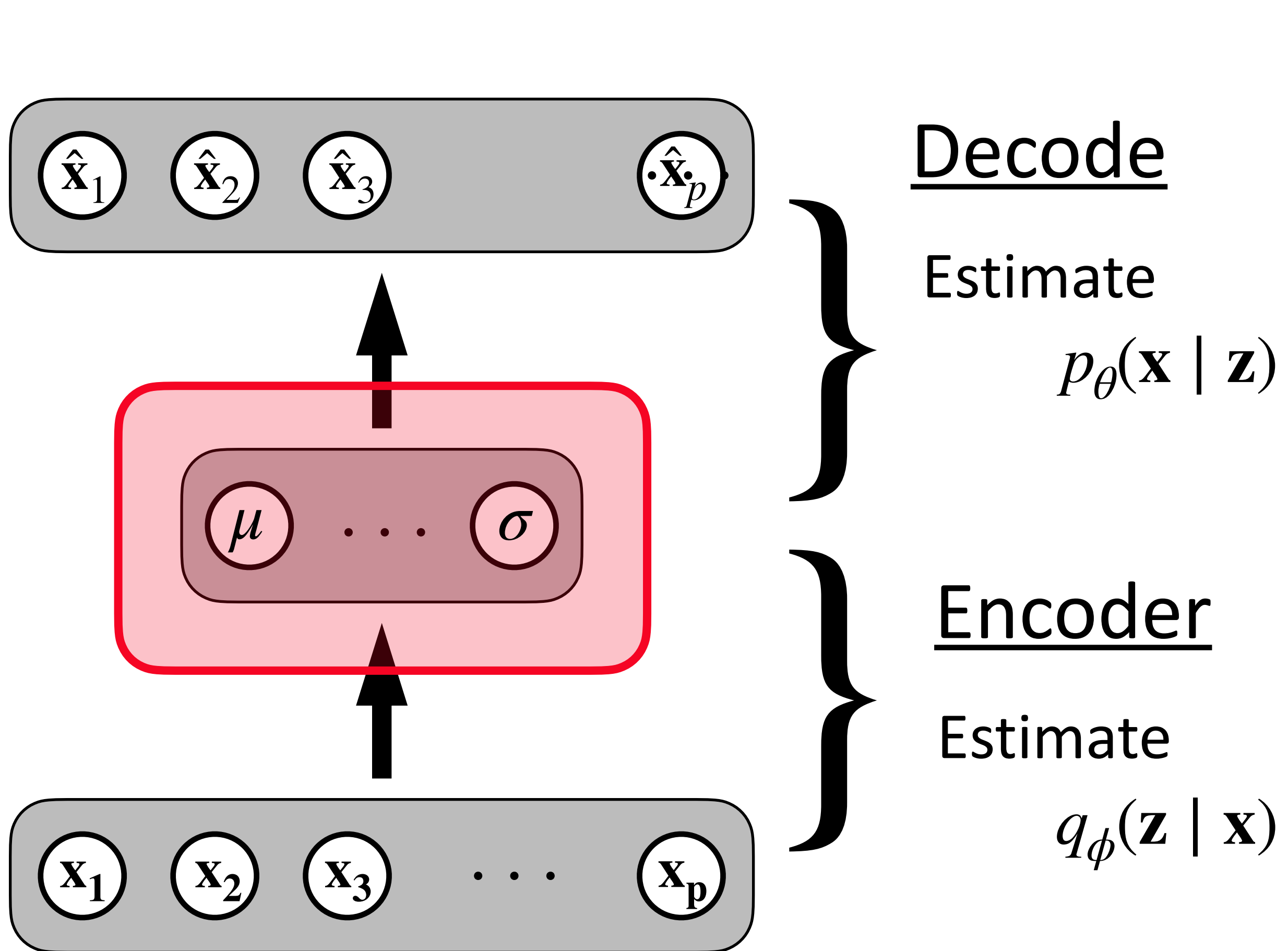
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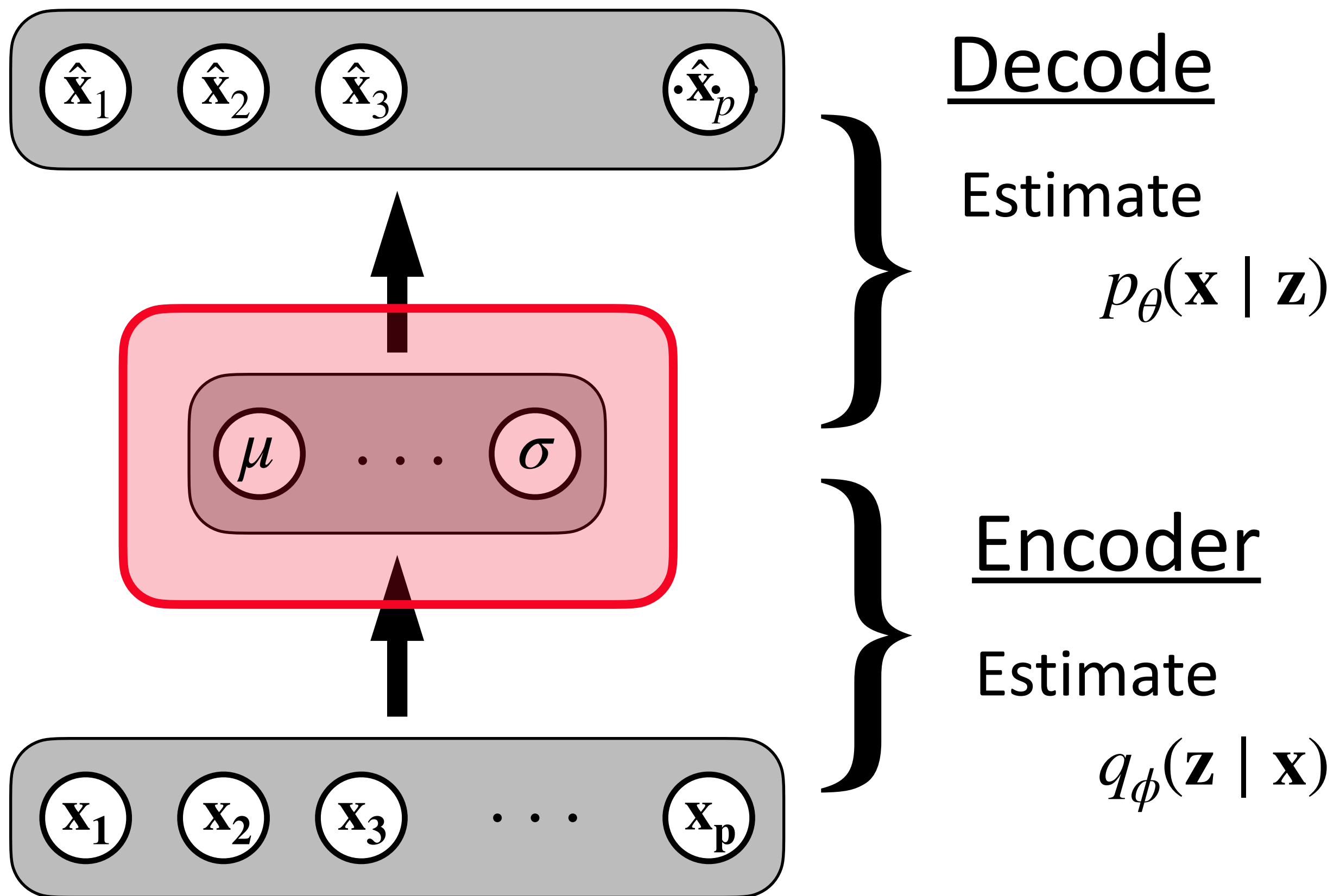


Suppose that  $q_{\phi}(\mathbf{z} \mid \mathbf{x}) \sim \mathcal{N}(\mu, \sigma^2)$ .



# Variational Auto-encoders

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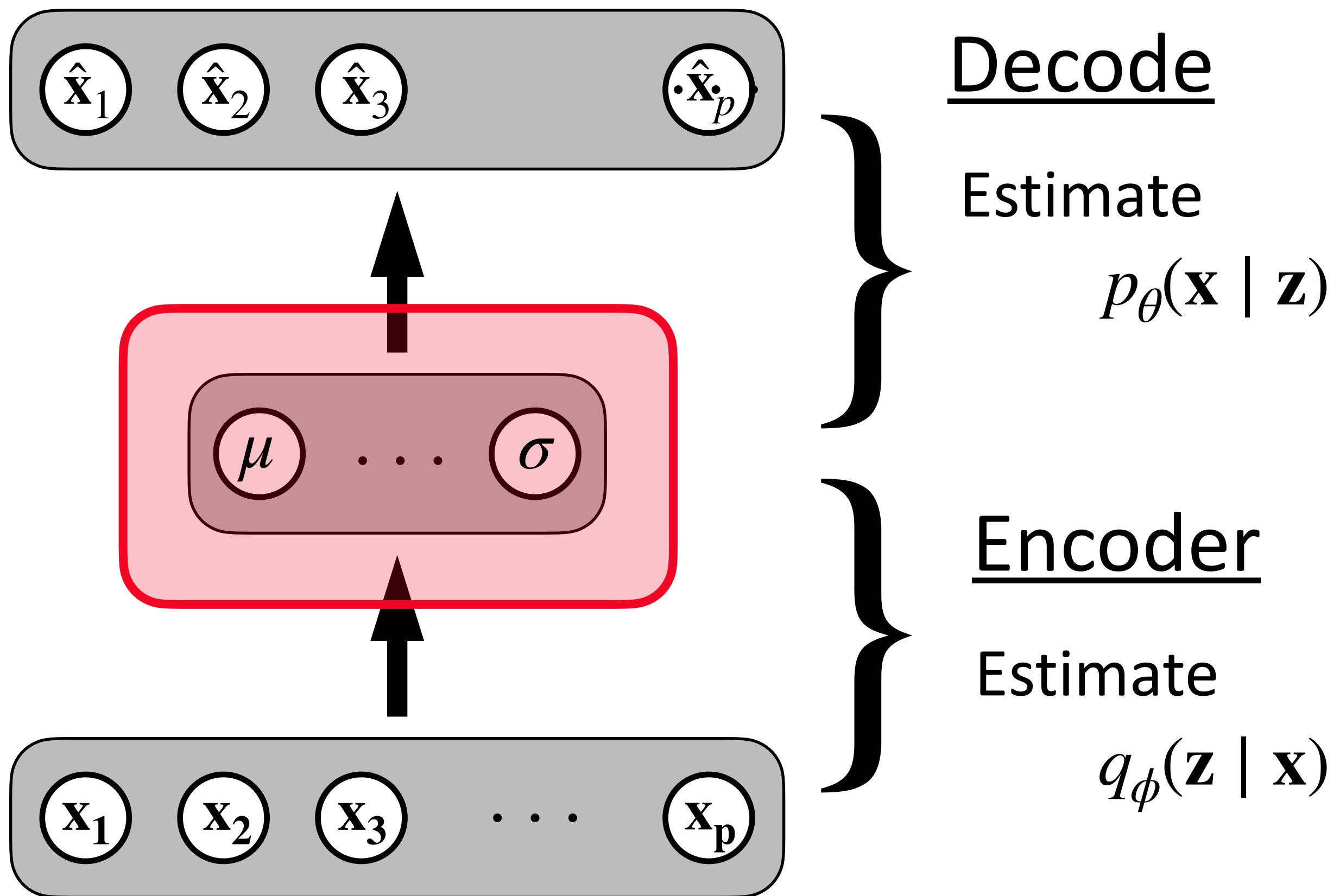


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Then we can write  $z = \mu + \sigma\epsilon$  où  $\epsilon \sim \mathcal{N}(0,1)$ .

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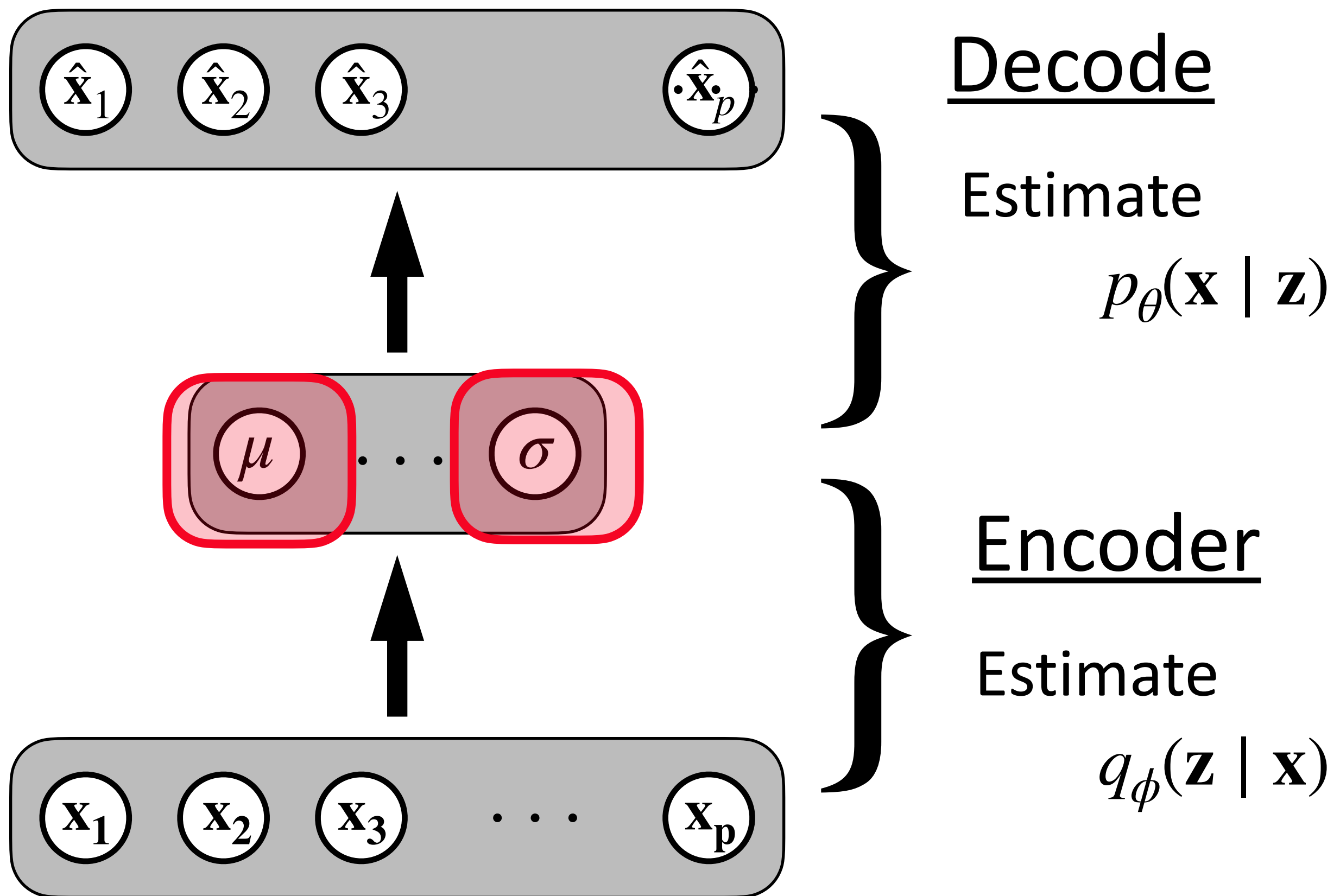
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Now, learning means estimating parameters  $\mu$  et  $\sigma$ !

# Variational Auto-encoders

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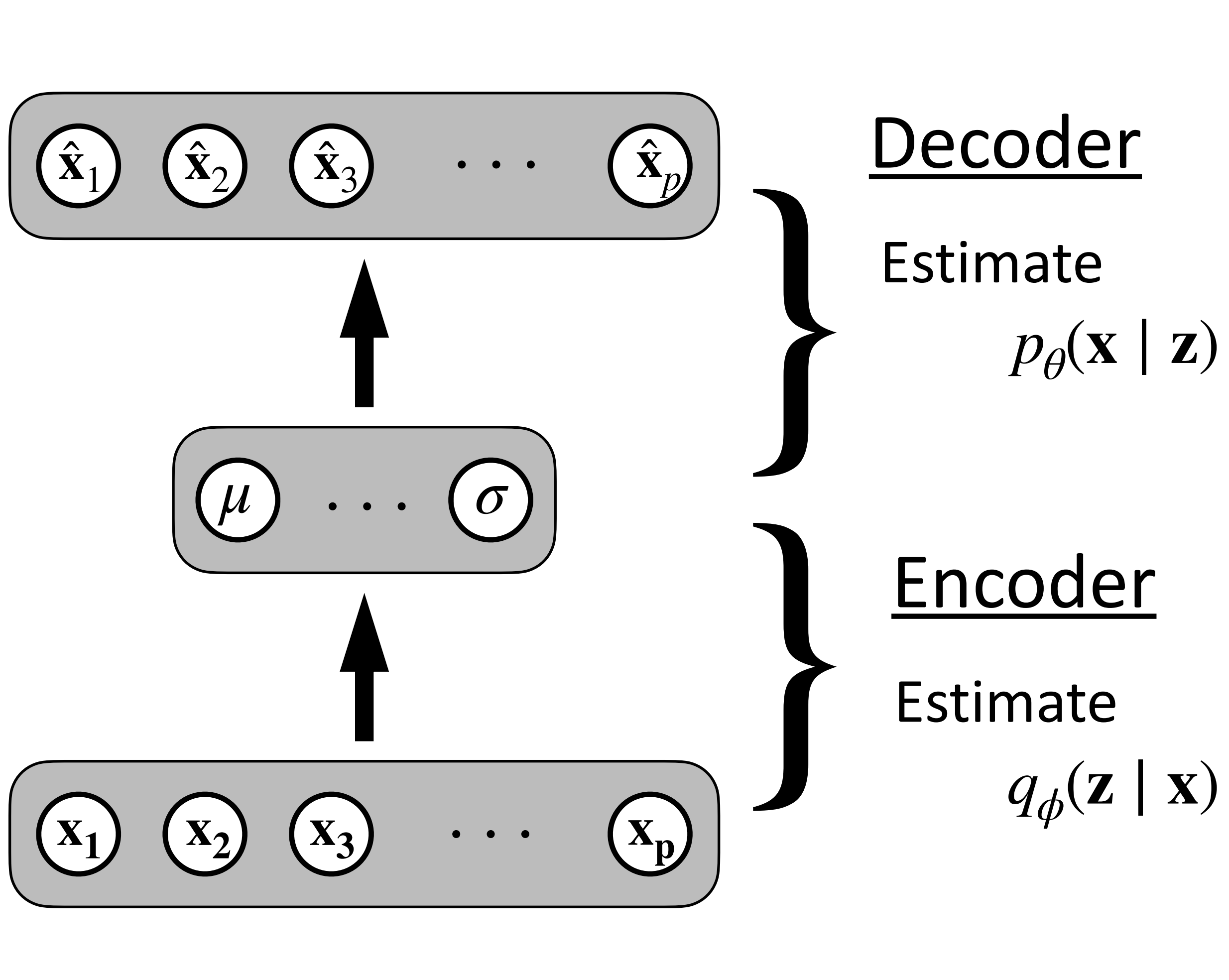


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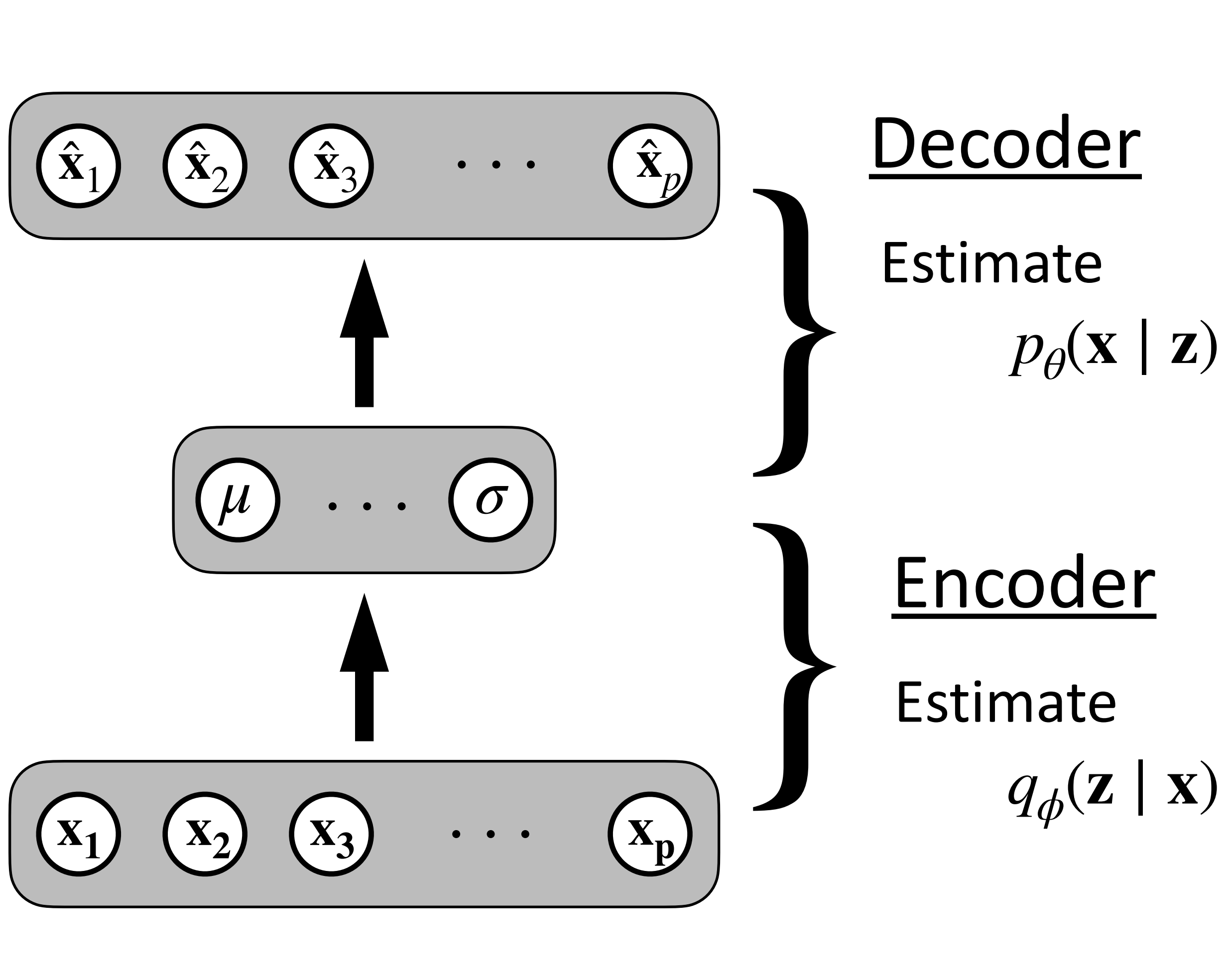
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# Variational Auto-encoders



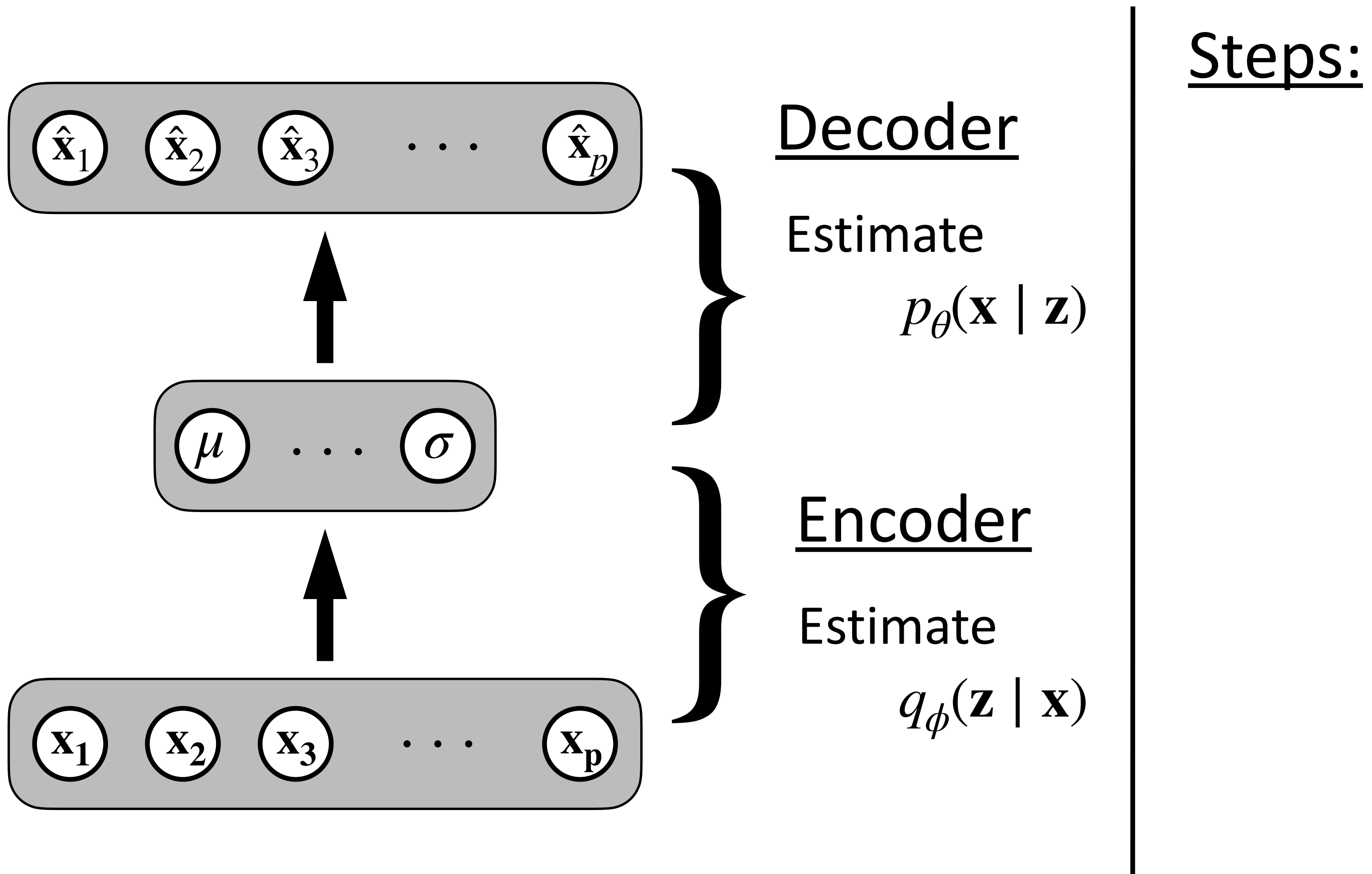
# Variational Auto-encoders

VAEs are a way to train a generative model



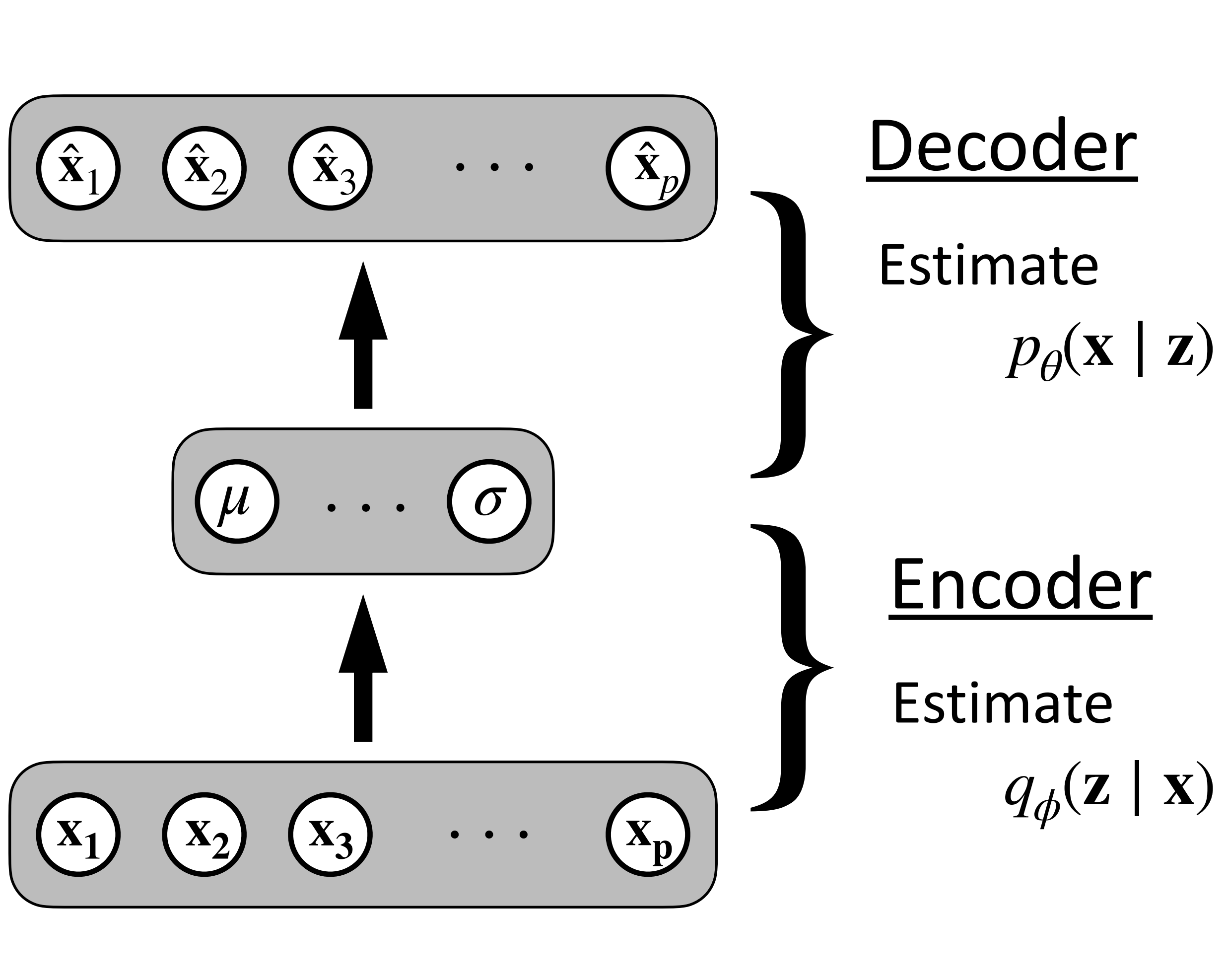
# Variational Auto-encoders

VAEs are a way to train a generative model



# Variational Auto-encoders

VAEs are a way to train a generative model

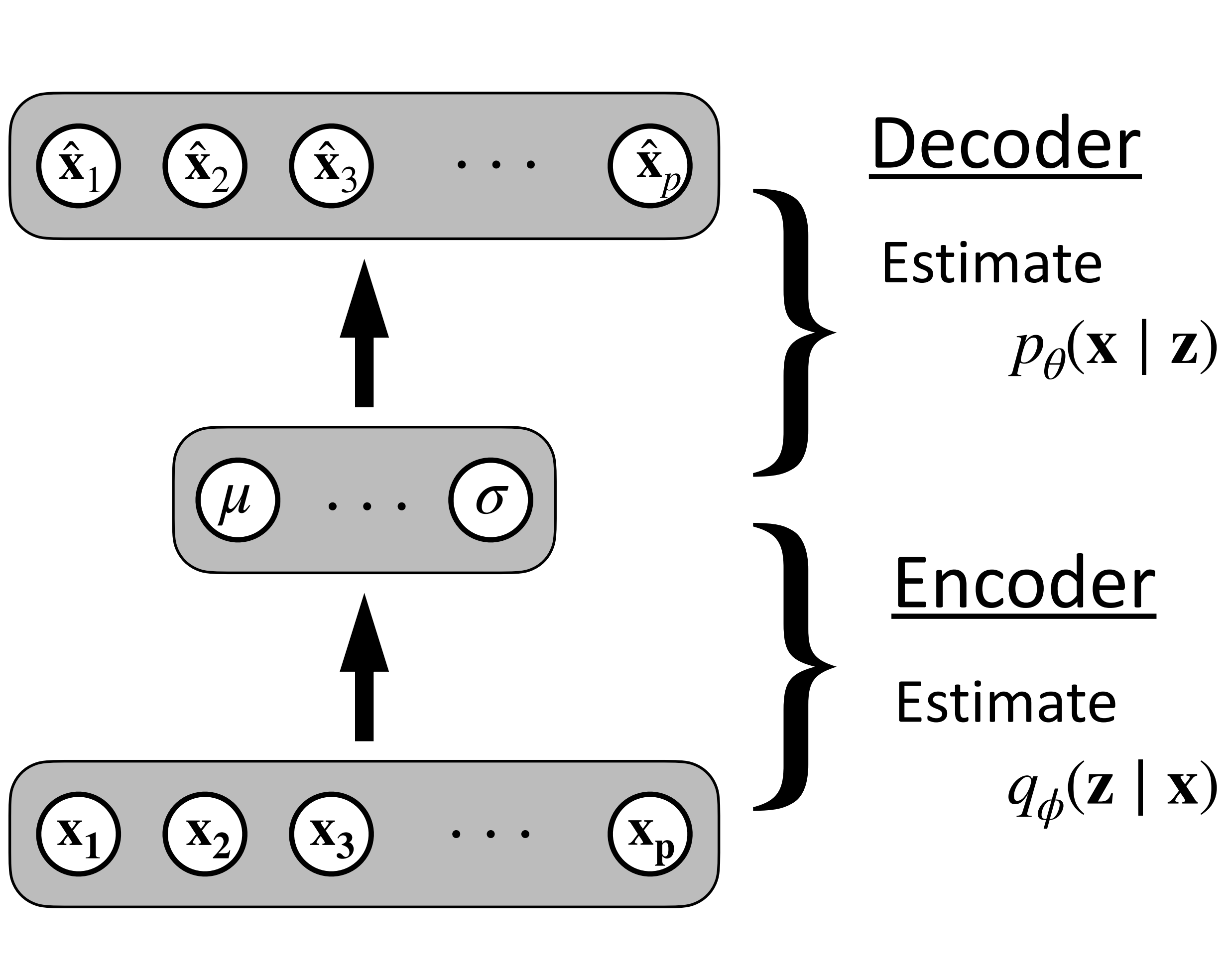


## Steps:

1. Train the model to learn:

# Variational Auto-encoders

VAEs are a way to train a generative model



## Steps:

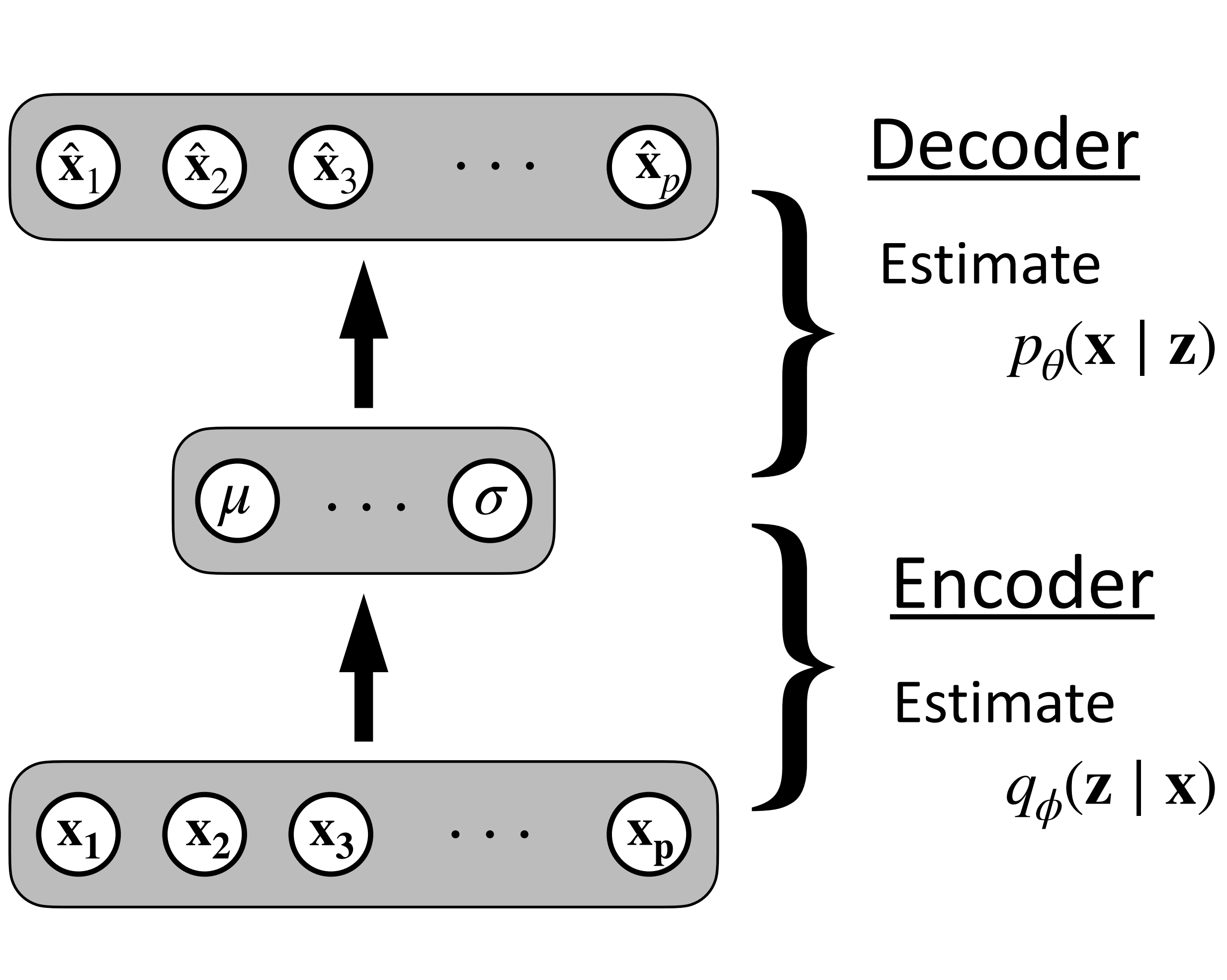
1. Train the model to learn:

- $p_{\theta}(\mathbf{x} \mid \mathbf{z})$



# Variational Auto-encoders

VAEs are a way to train a generative model



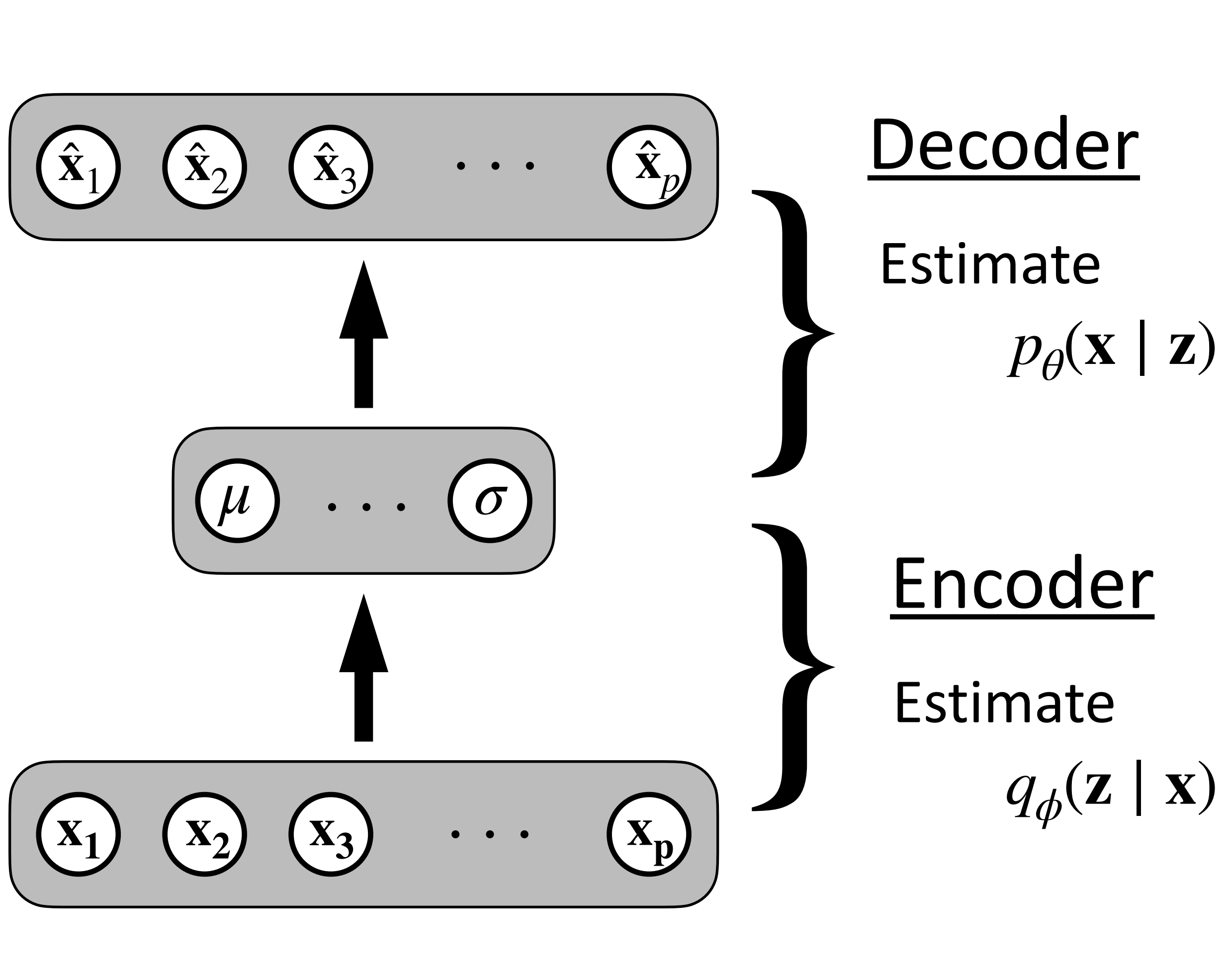
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# Variational Auto-encoders

VAEs are a way to train a generative model



## Steps:

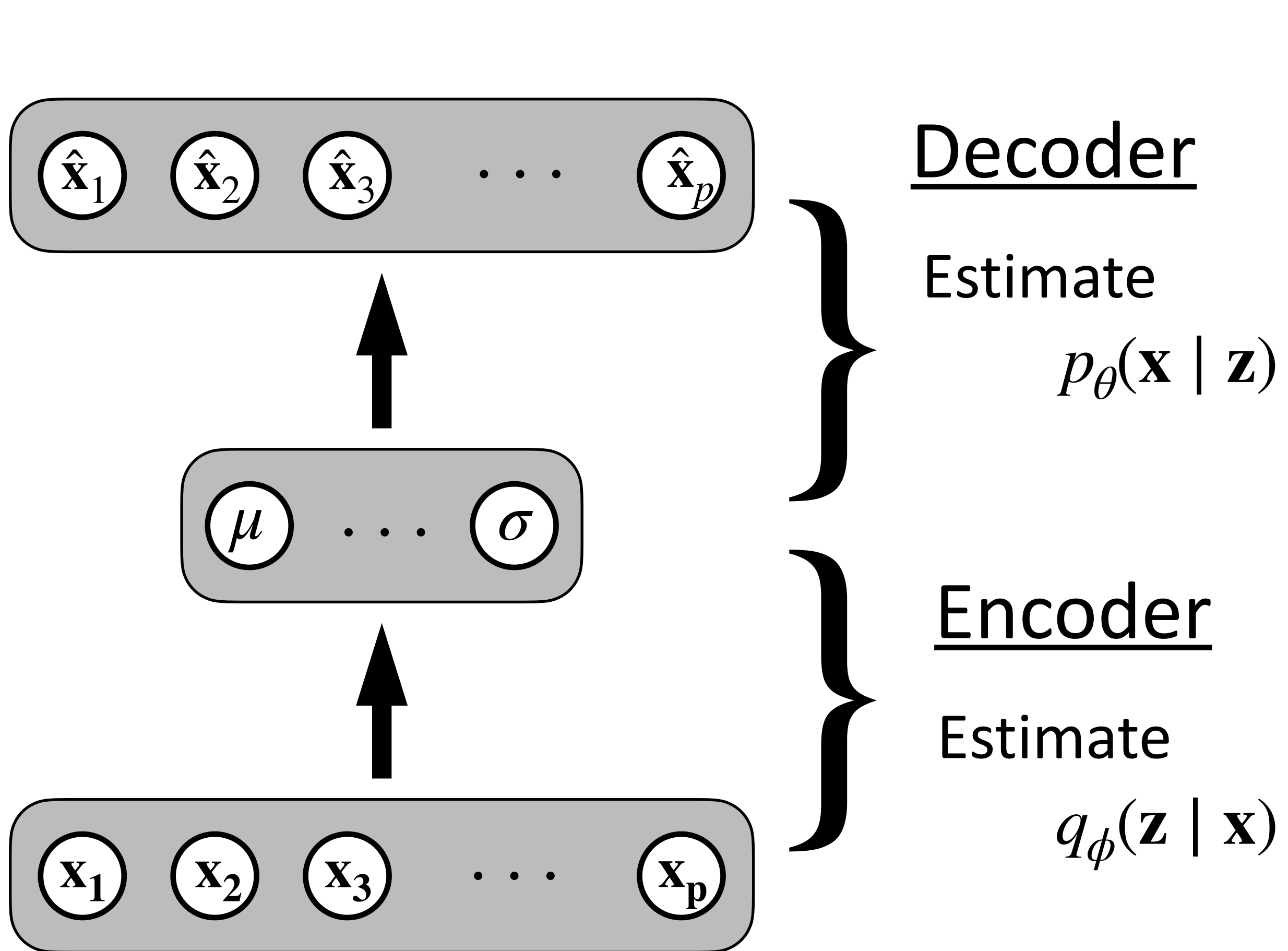
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# Variational Auto-encoders

VAEs are a way to train a generative model



## Steps:

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- $q_\phi(\mathbf{z} \mid \mathbf{x}) \sim \mathcal{N}(\mu, \sigma^2)$

2. Generate  $\epsilon$  et obtain  $\mathbf{z} \sim P(\mathbf{z})$ .

3. Obtain  $\mathbf{x}$  from  $p_\theta(\mathbf{x} \mid \mathbf{z})$ .

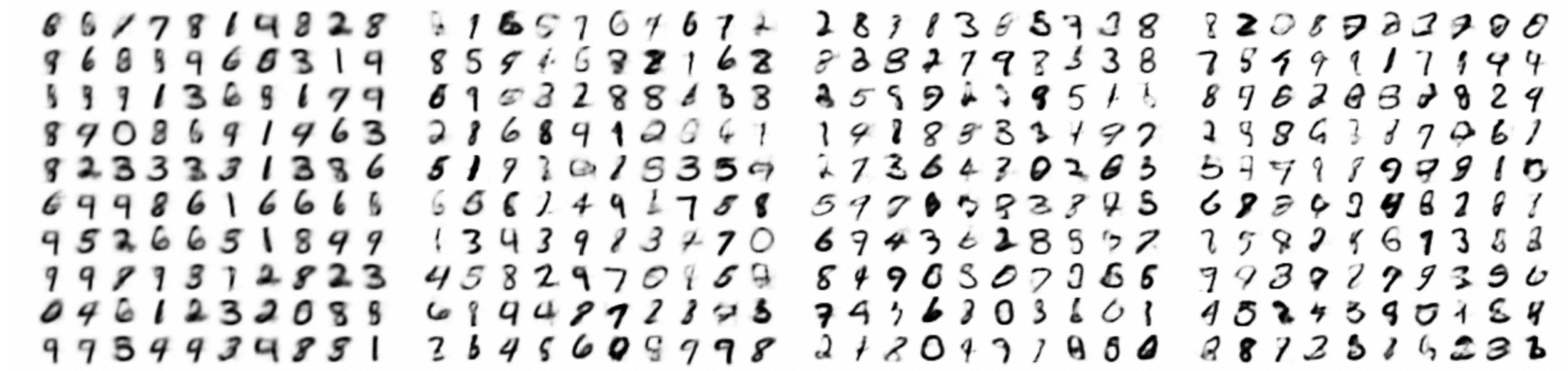
# Variational Auto-encoders

Example: Generating numbers (range 0—9)



# Variational Auto-encoders

Example: Generating numbers (range 0—9)



Multivariate Normal  
( $\mathbb{R}^2$ )

Multivariate Normal  
( $\mathbb{R}^5$ )

Multivariate Normal ( $\mathbb{R}^{10}$ )

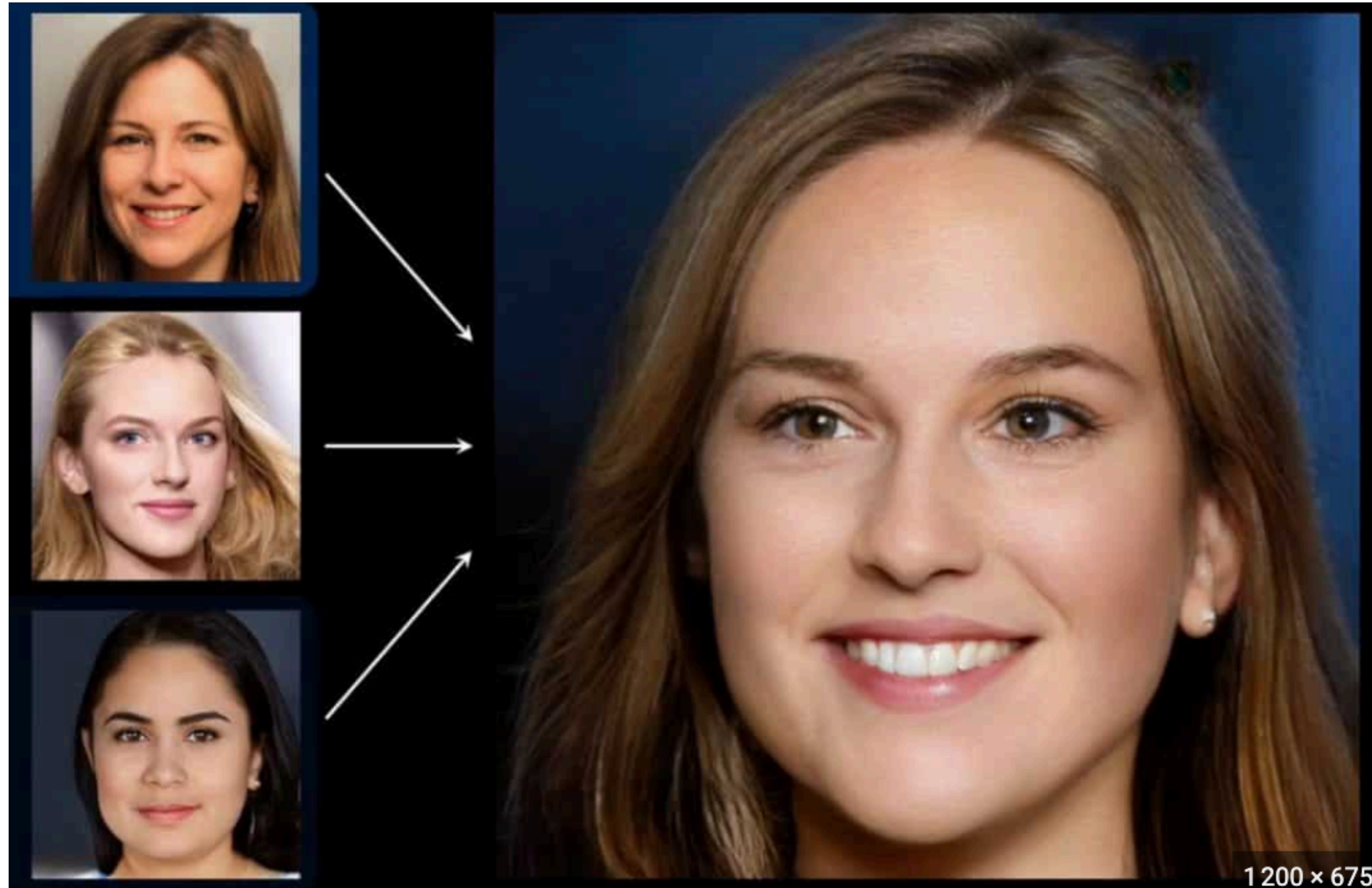
Multivariate Normal ( $\mathbb{R}^{20}$ )

# Generative Adversarial Networks (GANs)



# GANs - Introduction

Well-known for image generation



# Historical note

- Framework for learning a generative model (for example, an “inverted” CNN)
- Developed by researchers at Université de Montréal (2014)
- The first to generate high-quality complex images
- Have been (mostly?) replaced by diffusion models
- The study of GANs has provided insights into 2-player (min-max) optimization



# GANs — Intuition

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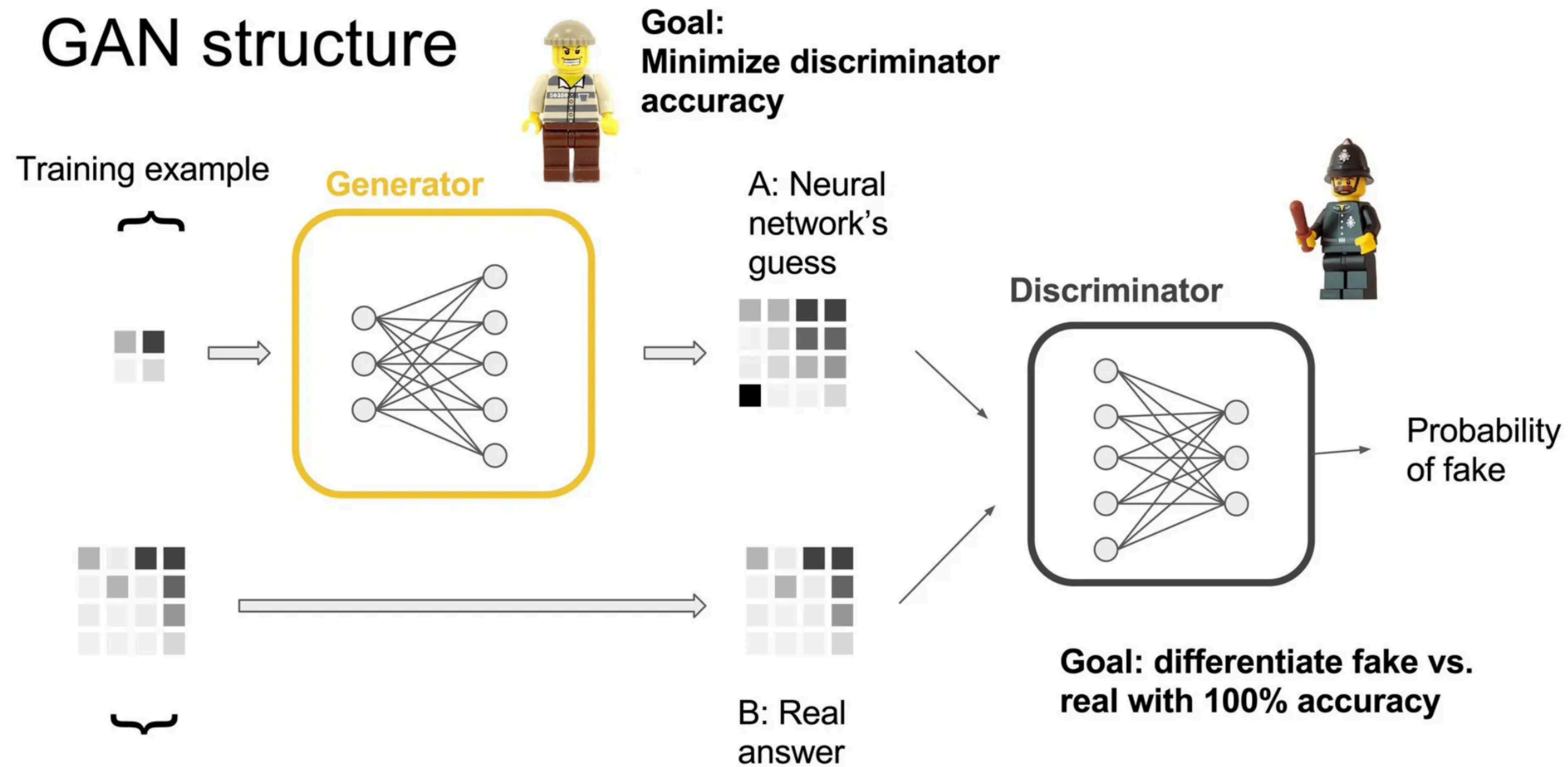
- Two players (each is a neural network)
  - Generator: The first player. It learns to generate a (good) image
  - Discriminator: The second player, learns to recognize (discriminate) good images from bad images

# GANs — Intuition

- Two players (each is a neural network)
  - Generator: The first player. It learns to generate a (good) image
  - Discriminator: The second player, learns to recognize (discriminate) good images from bad images
- The game (at each round):
  - The second player receives an image. It must determine whether the image comes from the generator or from the training data
  - Depending on the response, the players update (their weights)

# GANs — Introduction

Visually:



# GANs - formalism



The diagram illustrates the formalism of Generative Adversarial Networks (GANs). It features a light blue background with a white rounded rectangle on the left containing the equation  $x \sim \mathbb{P}_r$ , and the word "Data" below it. To the right of this is a large white rectangle, and further right is a large light blue rectangle. The white rounded rectangle has a thin black border.

$$x \sim \mathbb{P}_r$$

Data

# GANs - formalism

$$\mathbf{x} \sim \mathbb{P}_r$$

Data

$$\mathbf{z} \sim p(\mathbf{z})$$

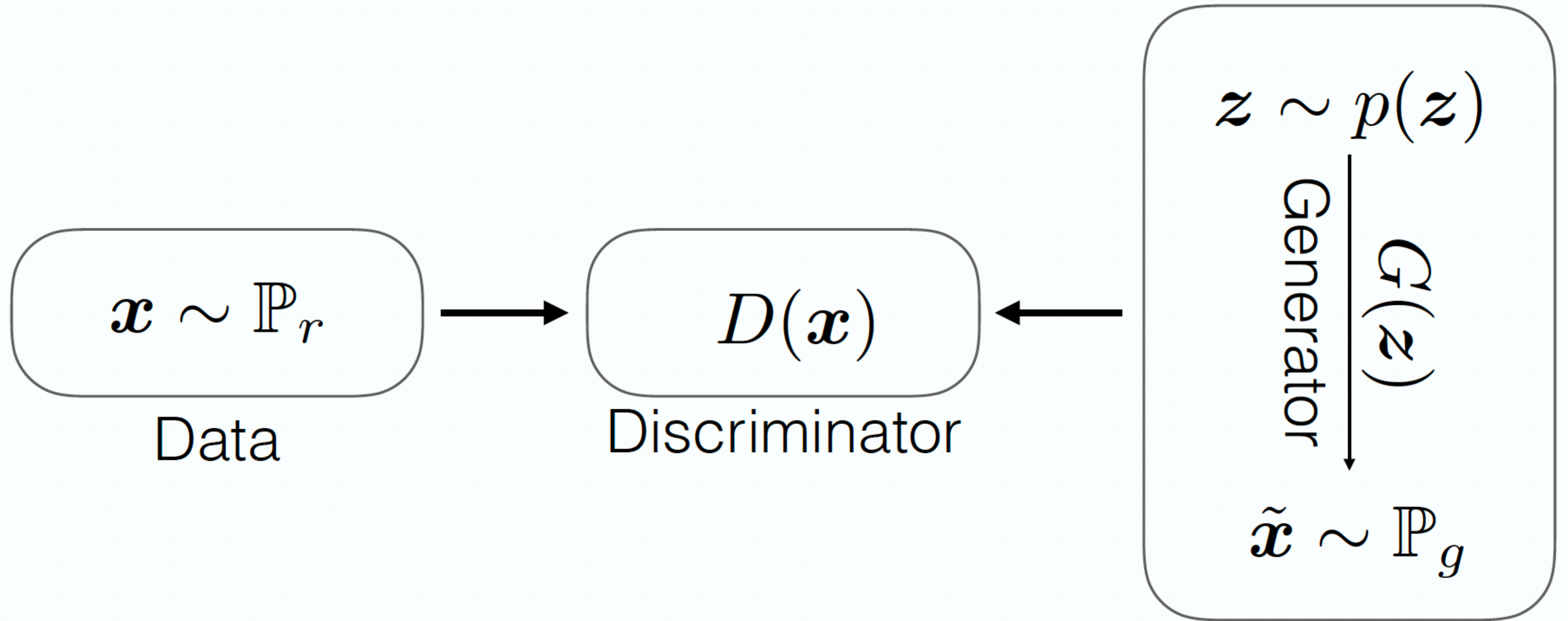
Generator

$$G(\mathbf{z})$$

$$\tilde{\mathbf{x}} \sim \mathbb{P}_g$$



# GANs - formalism



# GANs - objectives

GANs have two objectives (one for each player)

- The output of D is “1” for real and “0” for fake.

Objective of the  
discriminator

$$\max_D \mathbb{E}_{\boldsymbol{x} \sim \mathbb{P}_r} [\log(D(\boldsymbol{x}))] + \mathbb{E}_{\tilde{\boldsymbol{x}} \sim \mathbb{P}_g} [\log(1 - D(\tilde{\boldsymbol{x})))] .$$

Objective of the  
generator

$$\max_G \mathbb{E}_{\tilde{\boldsymbol{x}} \sim \mathbb{P}_g} [\log(D(\tilde{\boldsymbol{x})))] .$$



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where:

$\mathbb{P}_r$  Is the distribution that “generates” the real data



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$\mathbb{P}_g$  Is the distribution generating data

$\tilde{\mathbf{x}} = G(\mathbf{z}), \quad \mathbf{z} \sim p(\mathbf{z})$  Where  $\mathbf{z}$  follows a normal (e.g.)

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The game is framed as a two players game between the discriminator and the generator

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# Training GANs in practice alternates between training **D** and **G**

---

**Algorithm 1** Minibatch stochastic gradient descent training of generative adversarial nets. The number of steps to apply to the discriminator,  $k$ , is a hyperparameter. We used  $k = 1$ , the least expensive option, in our experiments.

---

**for** number of training iterations **do**

**for**  $k$  steps **do**

- Sample minibatch of  $m$  noise samples  $\{z^{(1)}, \dots, z^{(m)}\}$  from noise prior  $p_g(z)$ .
- Sample minibatch of  $m$  examples  $\{x^{(1)}, \dots, x^{(m)}\}$  from data generating distribution  $p_{\text{data}}(x)$ .
- Update the discriminator by ascending its stochastic gradient:

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^m \left[ \log D(x^{(i)}) + \log \left( 1 - D(G(z^{(i)})) \right) \right].$$

**end for**

- Sample minibatch of  $m$  noise samples  $\{z^{(1)}, \dots, z^{(m)}\}$  from noise prior  $p_g(z)$ .
- Update the generator by descending its stochastic gradient:

$$\nabla_{\theta_g} \frac{1}{m} \sum_{i=1}^m \log \left( 1 - D(G(z^{(i)})) \right).$$

**end for**

The gradient-based updates can use any standard gradient-based learning rule. We used momentum in our experiments.

---



# GANs - Varia

Monet ↔ Photos



Monet → photo

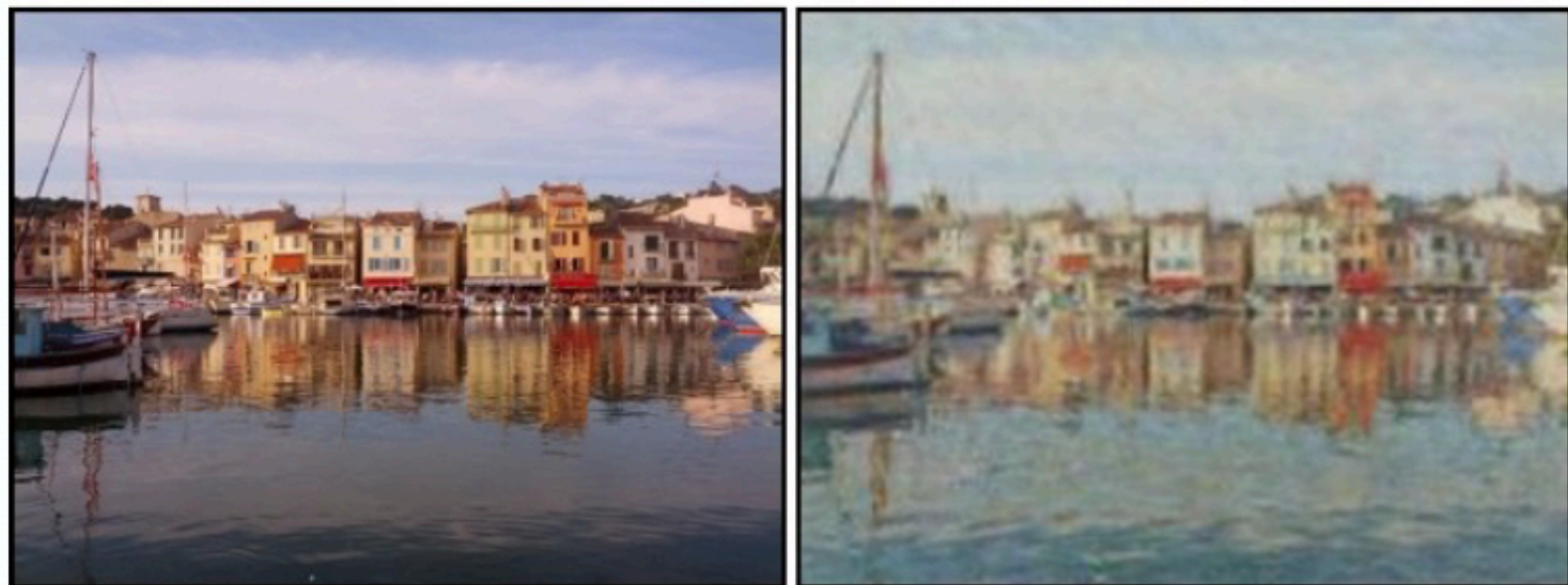


photo → Monet

Zebras ↔ Horses



zebra → horse



horse → zebra

Summer ↔ Winter



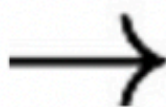
summer → winter



winter → summer



Photograph



Monet



Van Gogh



Cezanne



Ukiyo-e



# DALL-E 2

(As an example of using a  
diffusion model)



# DALL-E



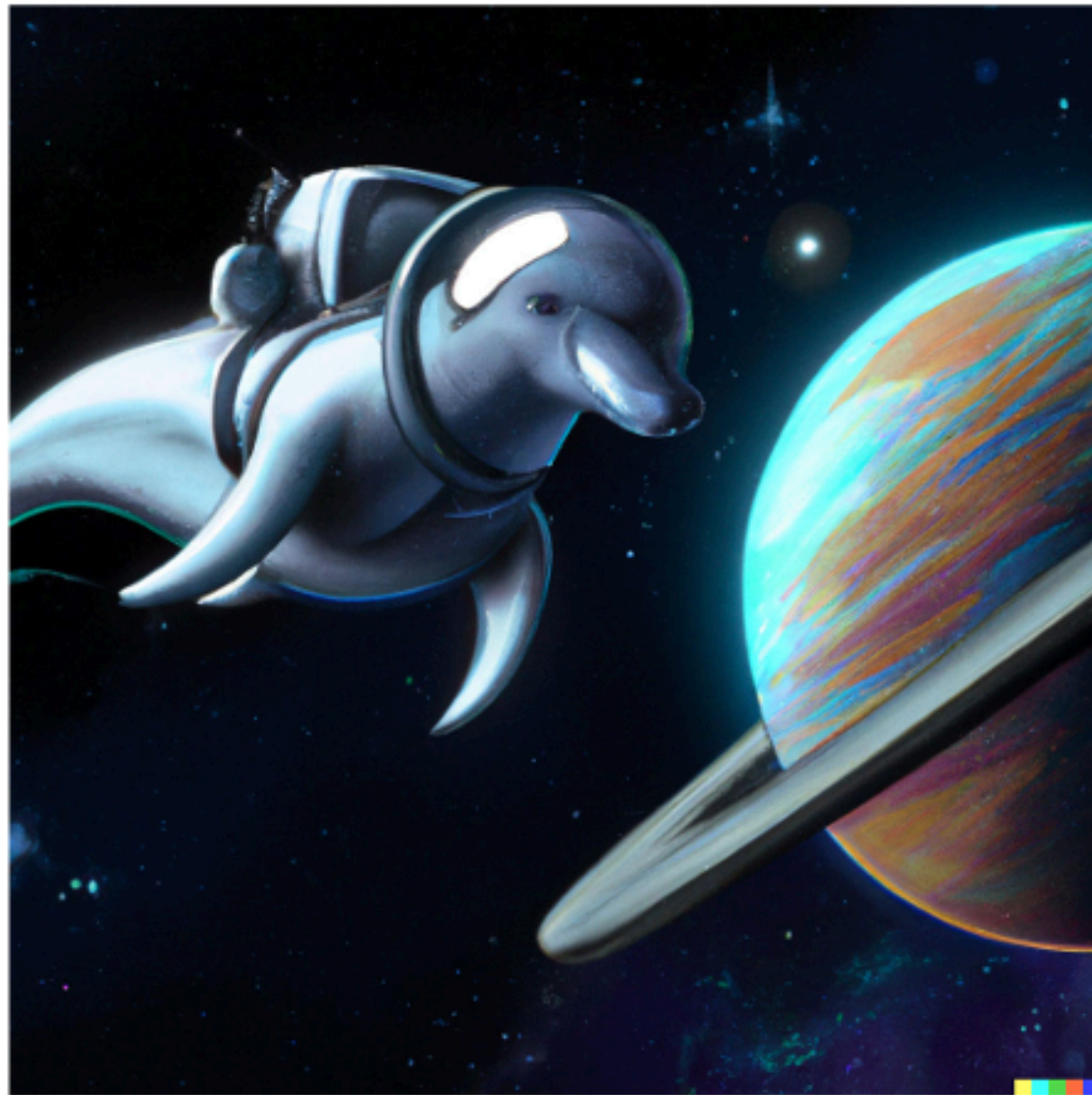
an espresso machine that makes coffee from human souls, artstation



panda mad scientist mixing sparkling chemicals, artstation



a corgi's head depicted as an explosion of a nebula



a dolphin in an astronaut suit on saturn, artstation



a propaganda poster depicting a cat dressed as french emperor napoleon holding a piece of cheese



a teddy bear on a skateboard in times square



# DALL-E - Overview

“a corgi  
playing a  
flame  
throwing  
trumpet”



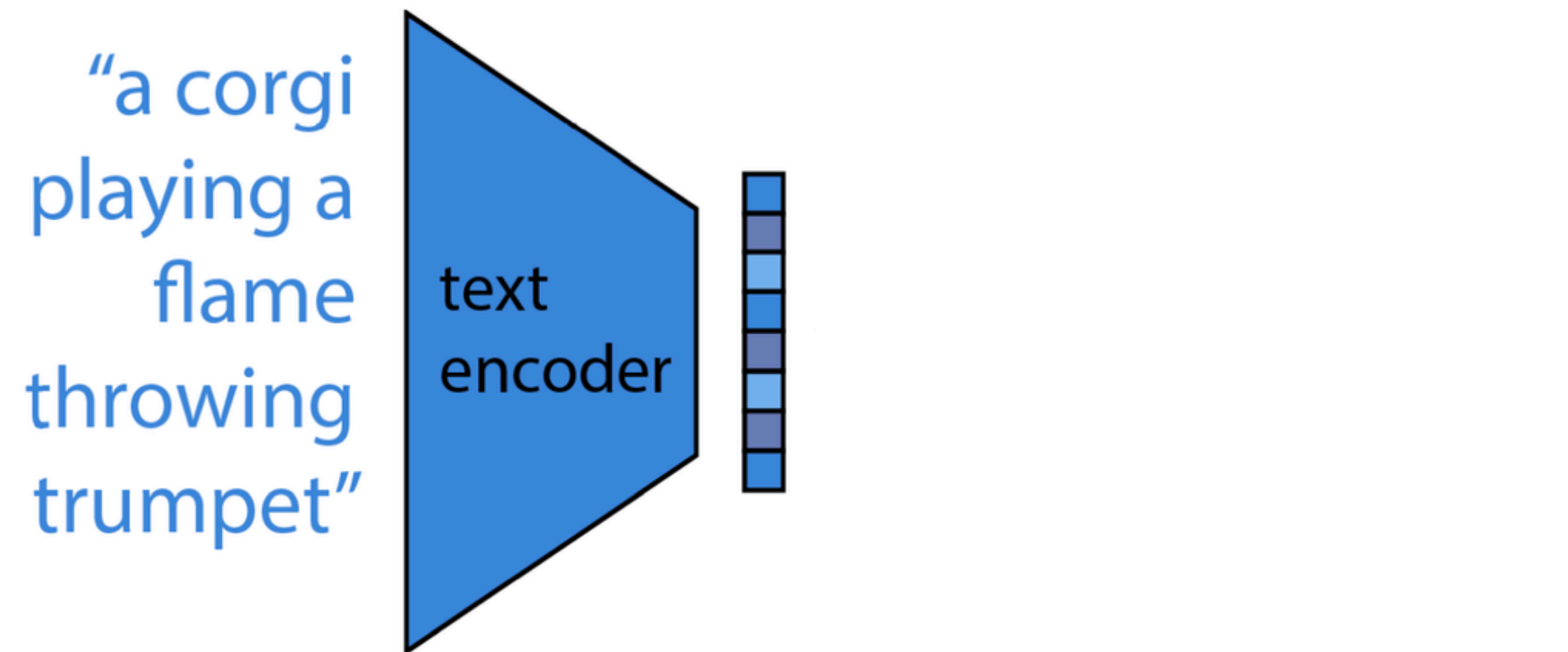
# DALL-E - Overview

“a corgi  
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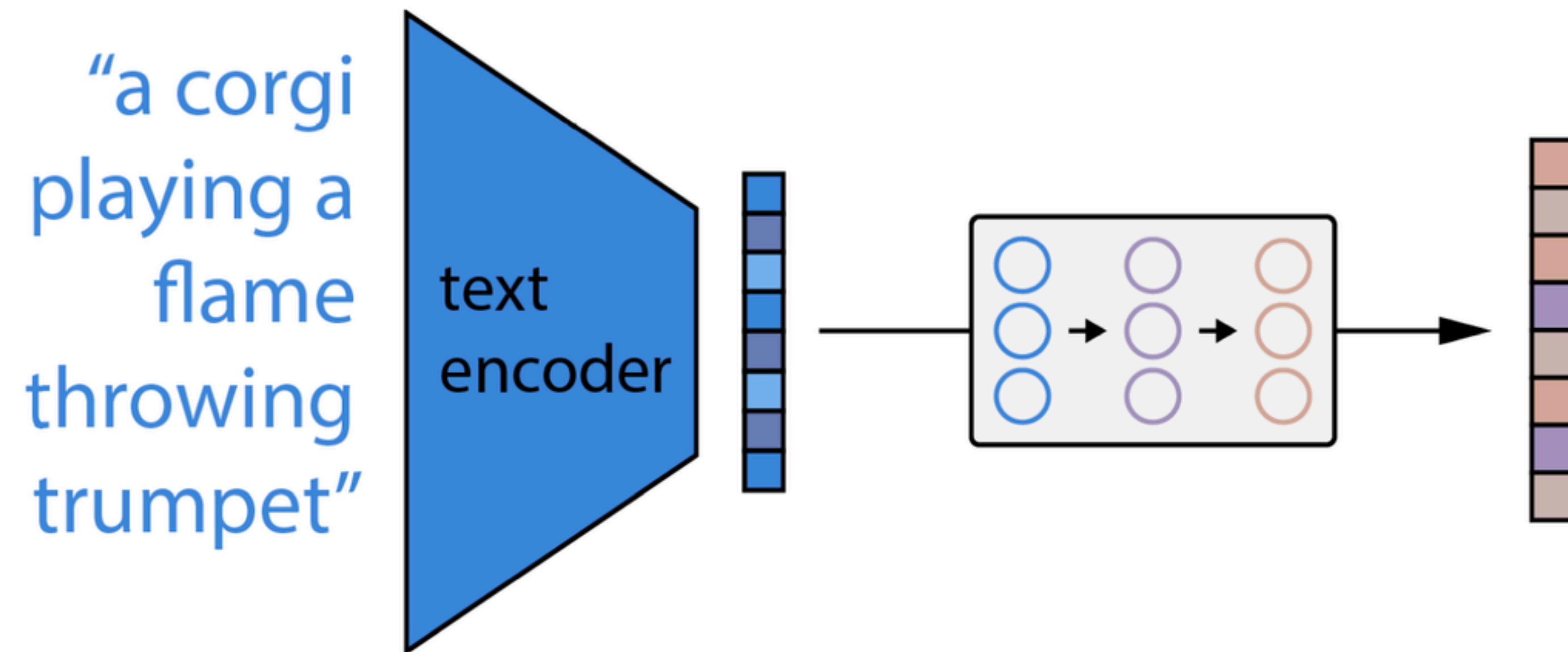
1. Input, a sentence (*prompt*) of the image we want to create

# DALL-E - Overview



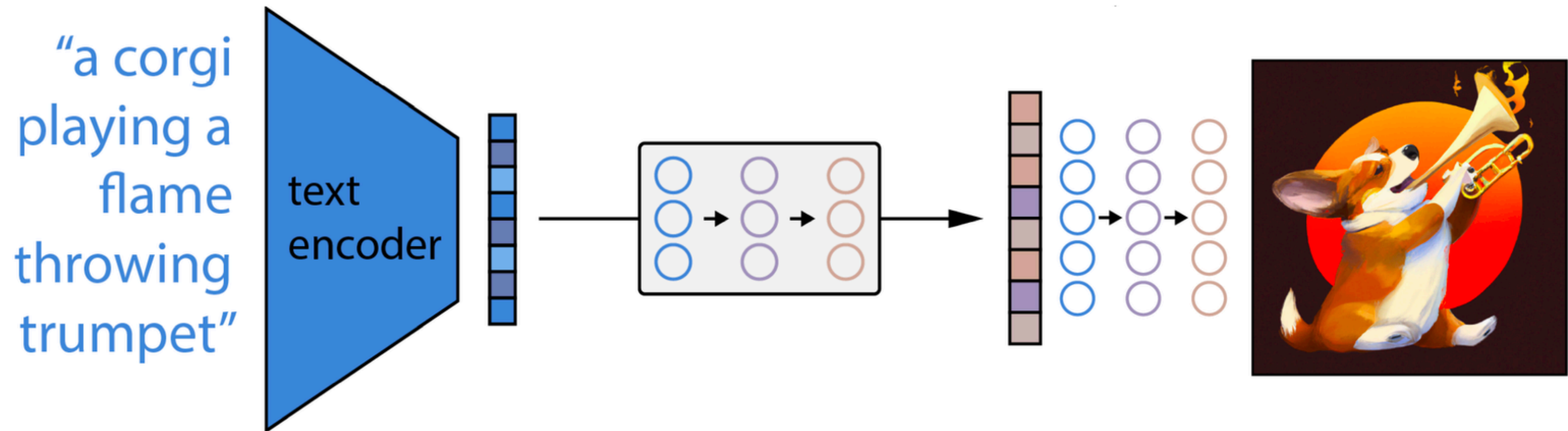
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# DALL-E - Overview



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3. Transform this representation in an image representation (image space) — not shown

# DALL-E - Overview

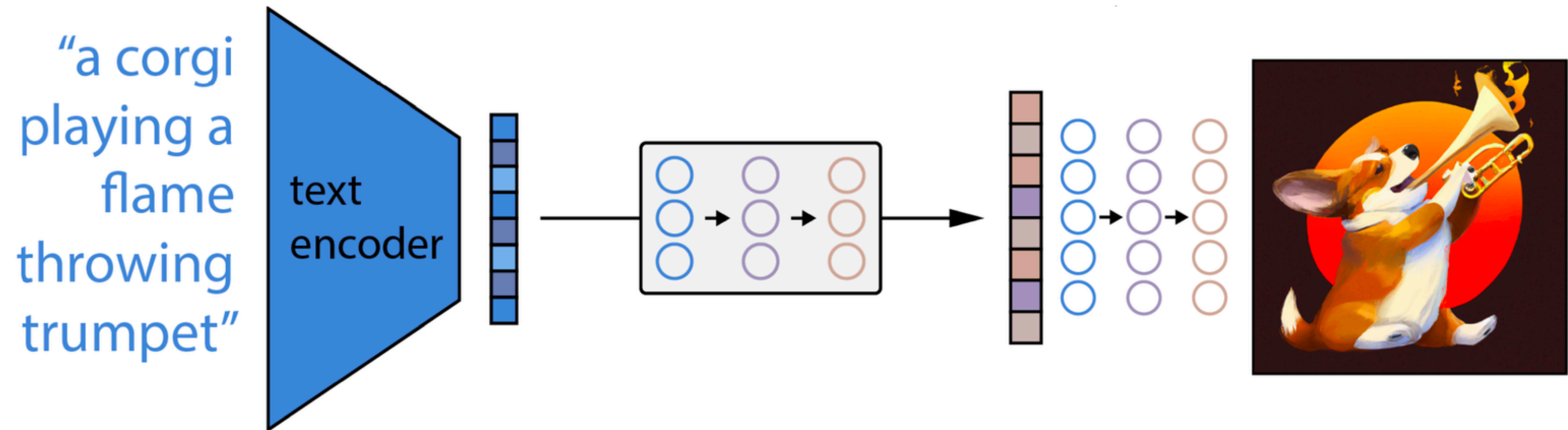


1. Input, a sentence (*prompt*) of the image we want to create
2. This prompt is encoded in a (latent) representation
3. Transform this representation in an image representation (image space) — not shown
4. Decode the image representation into an actual image



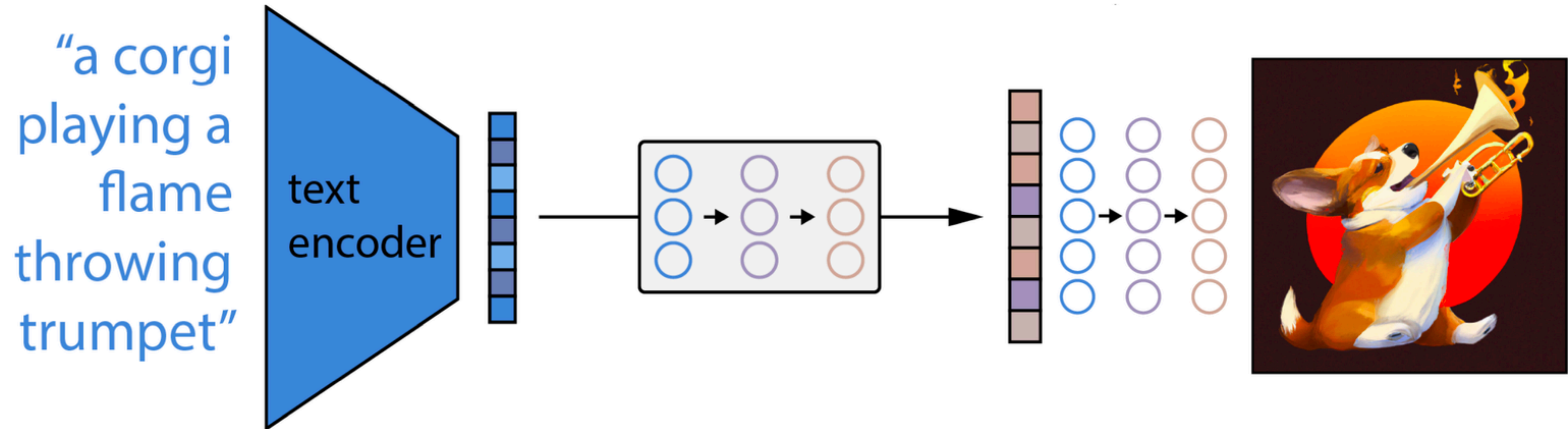
# DALL-E - Overview

Each step uses specific techniques:



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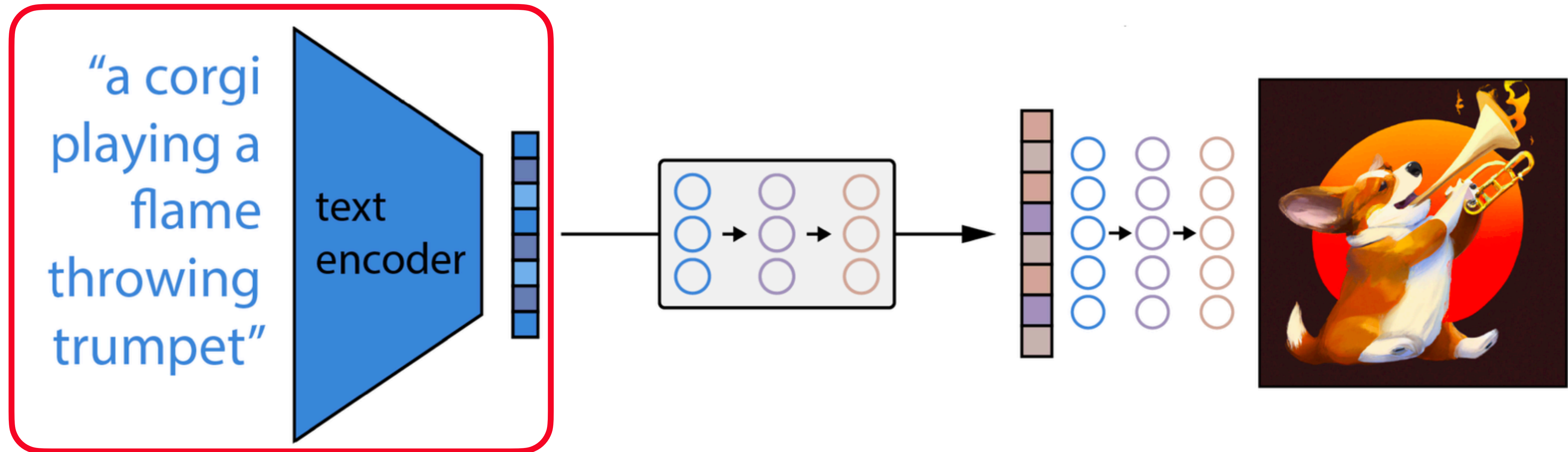


1. Contrastive Language-Image Pre-training (CLIP)



# DALL-E - Overview

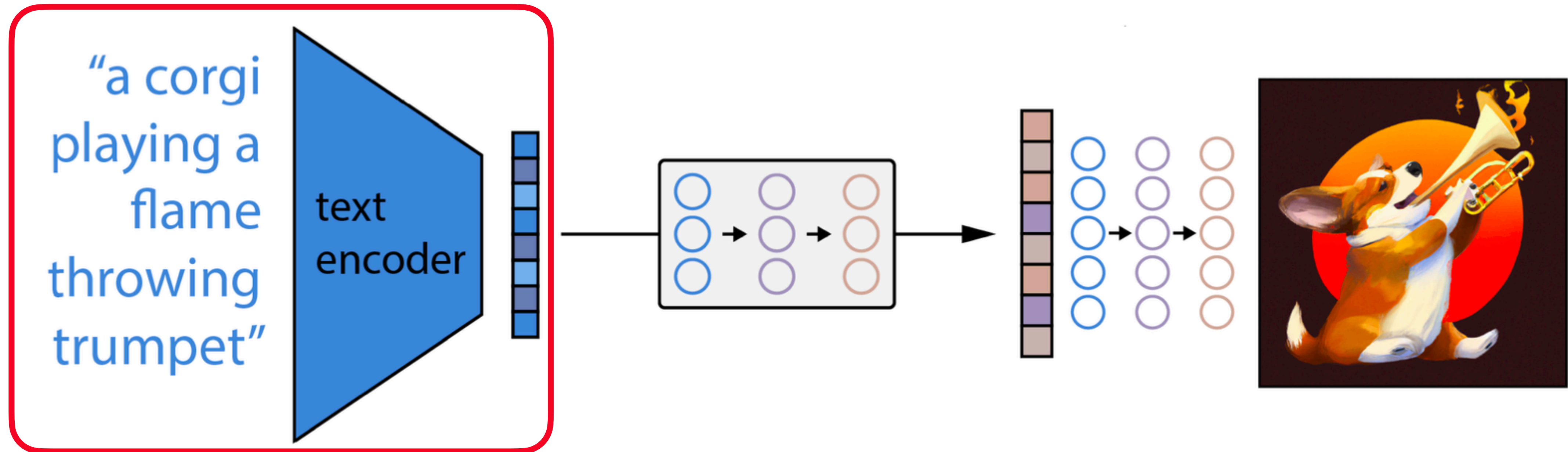
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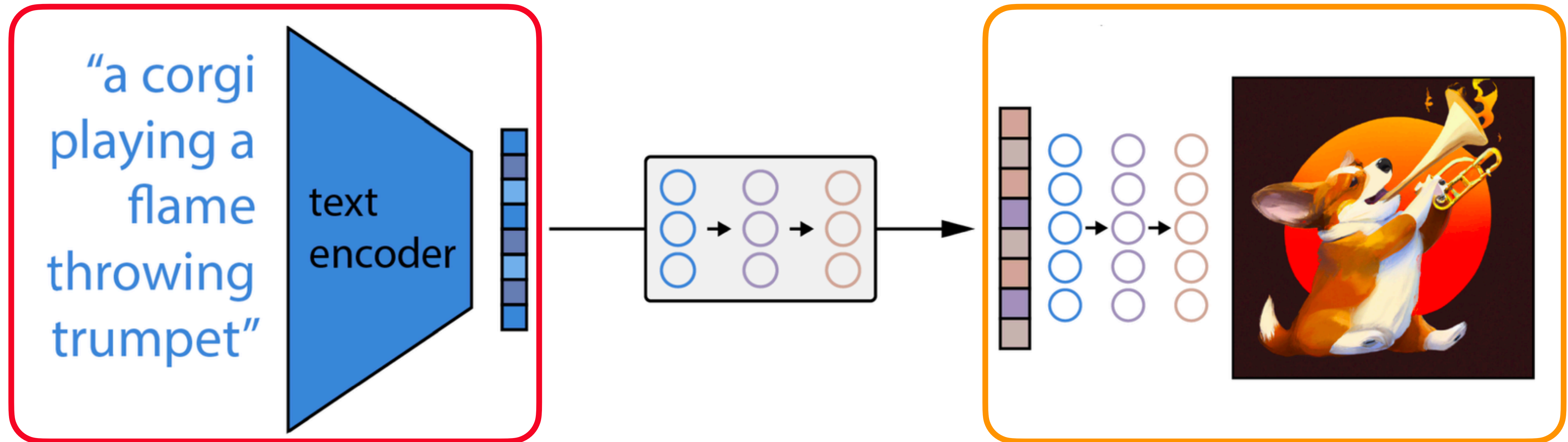


1. Contrastive Language-Image Pre-training (CLIP)

2. Generation of an image using a diffusion model

# DALL-E - Overview

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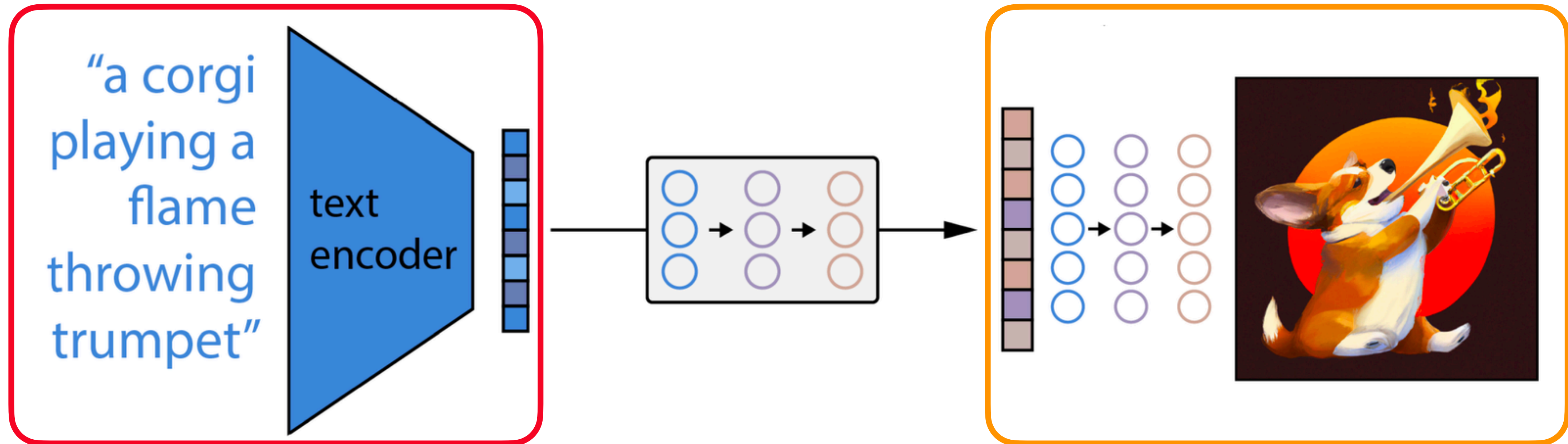
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# DALL-E - Overview

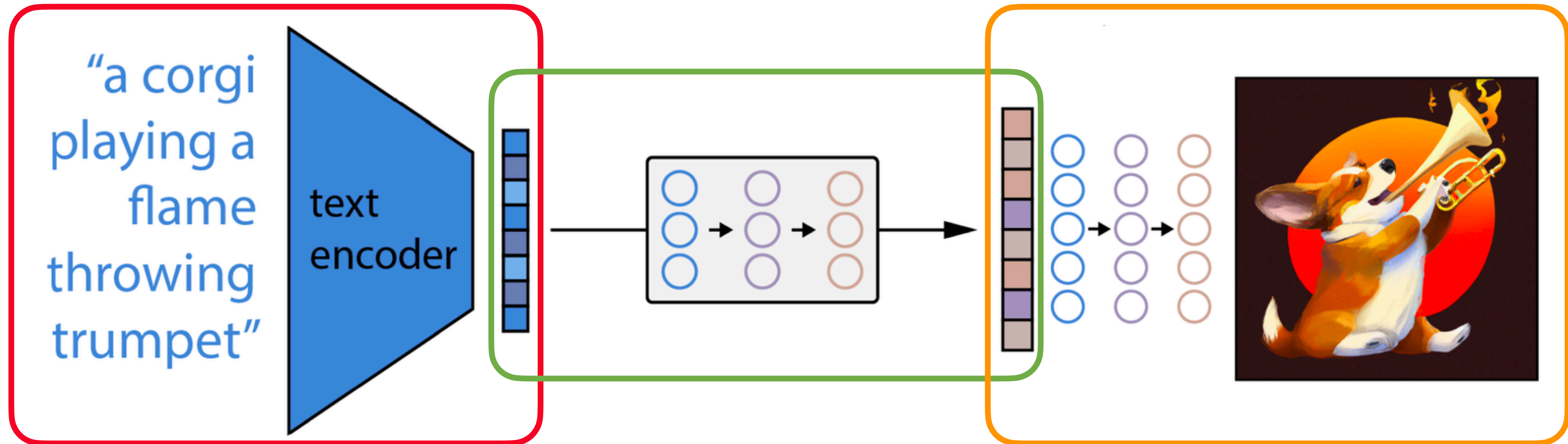
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1. Contrastive Language-Image Pre-training (CLIP)
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3. Learn the latent representations of text and images

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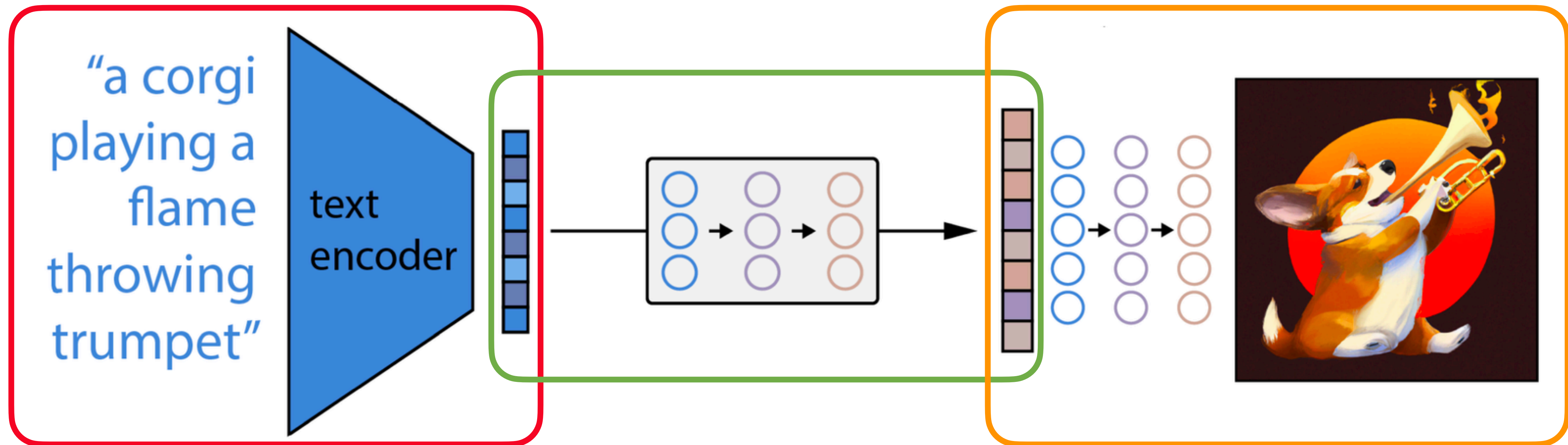


1. Contrastive Language-Image Pre-training (CLIP)
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3. Learn the latent representations of text and images



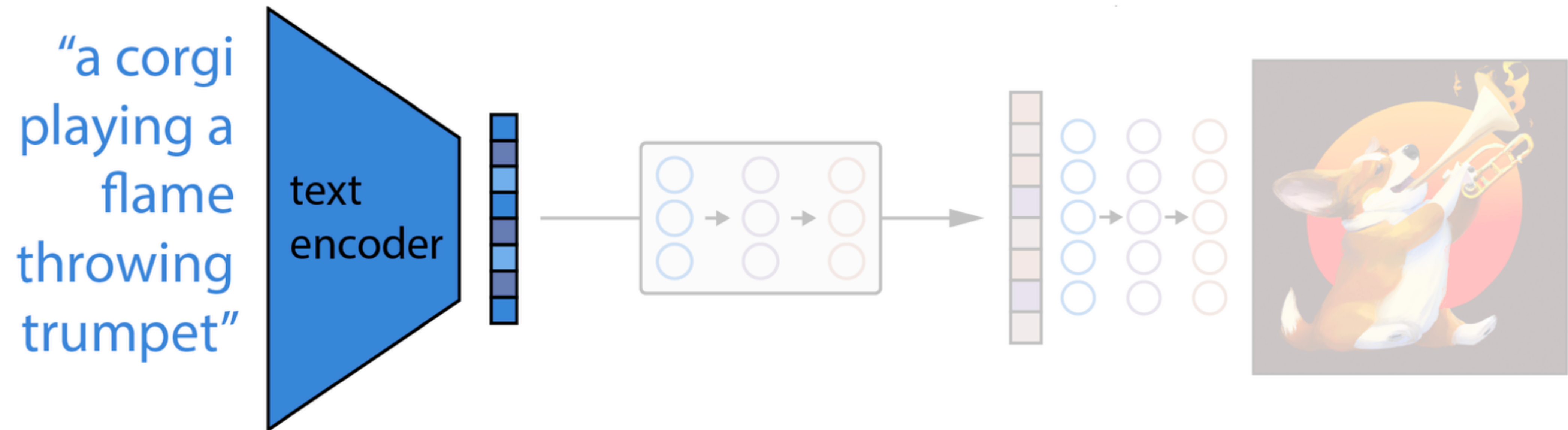
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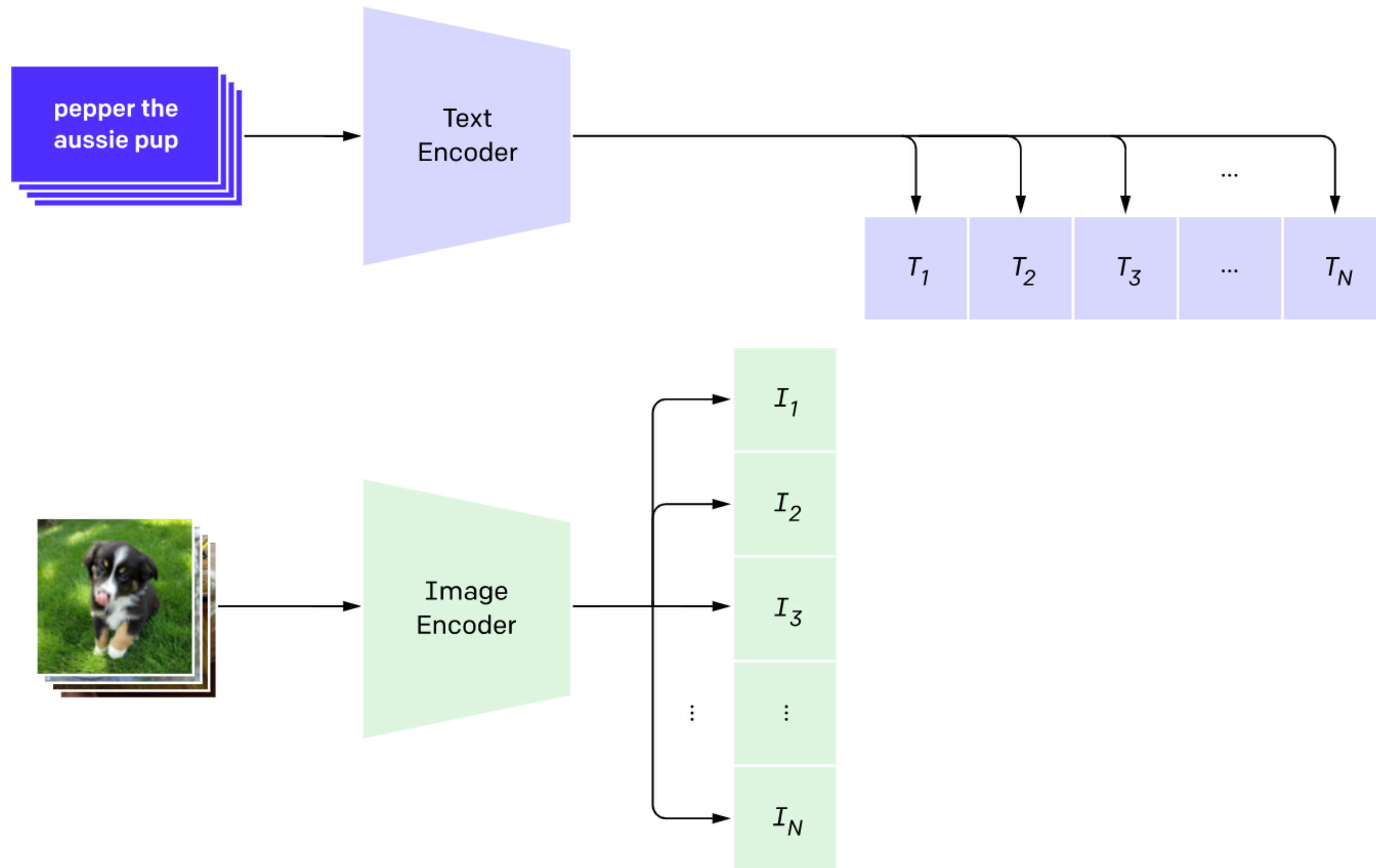


1. Contrastive Language-Image Pre-training (CLIP)
2. Generation of an image using a diffusion model
3. Learn the latent representations of text and images
4. Wrap-it up!

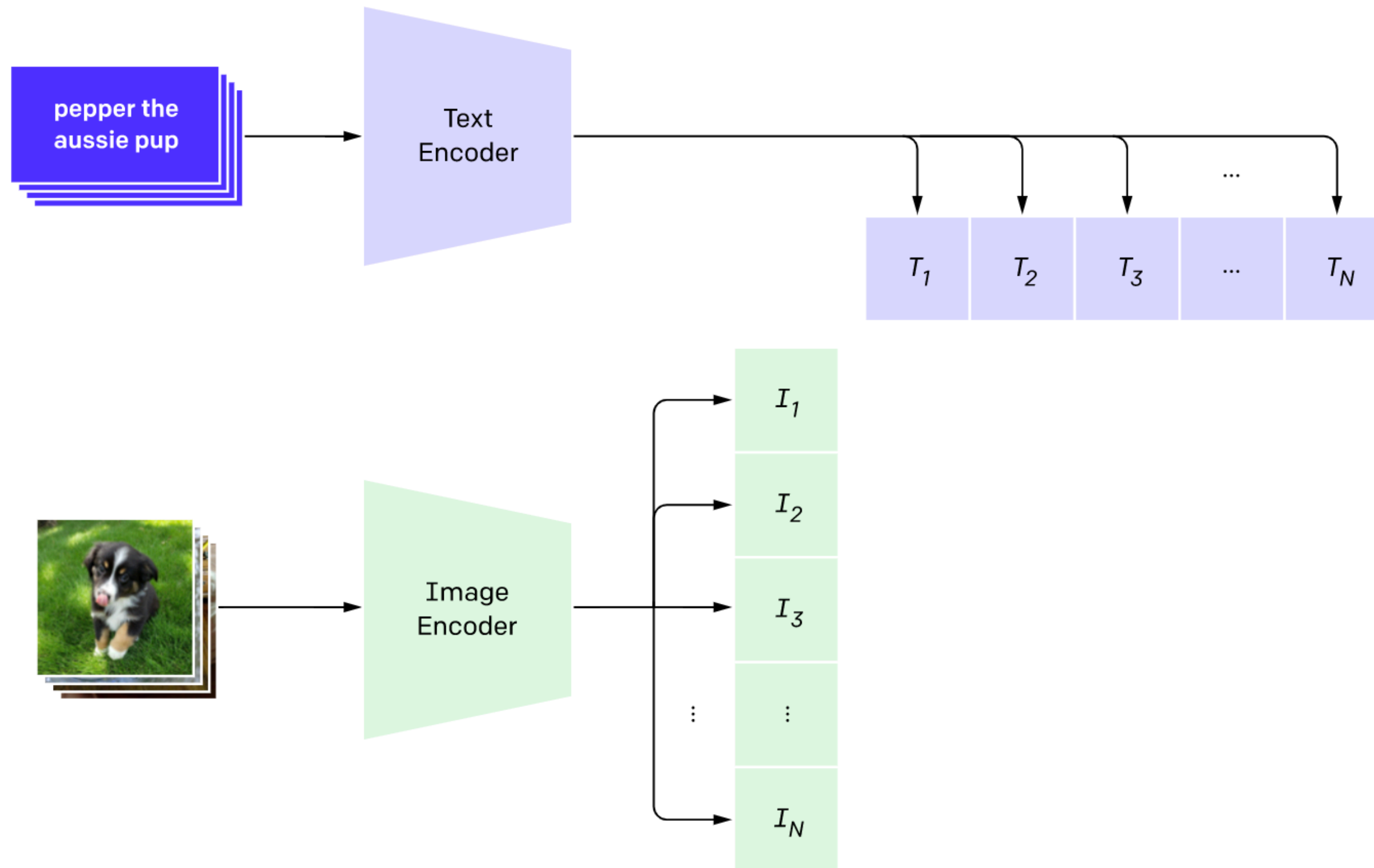
# DALL-E - CLIP



# DALL-E - CLIP

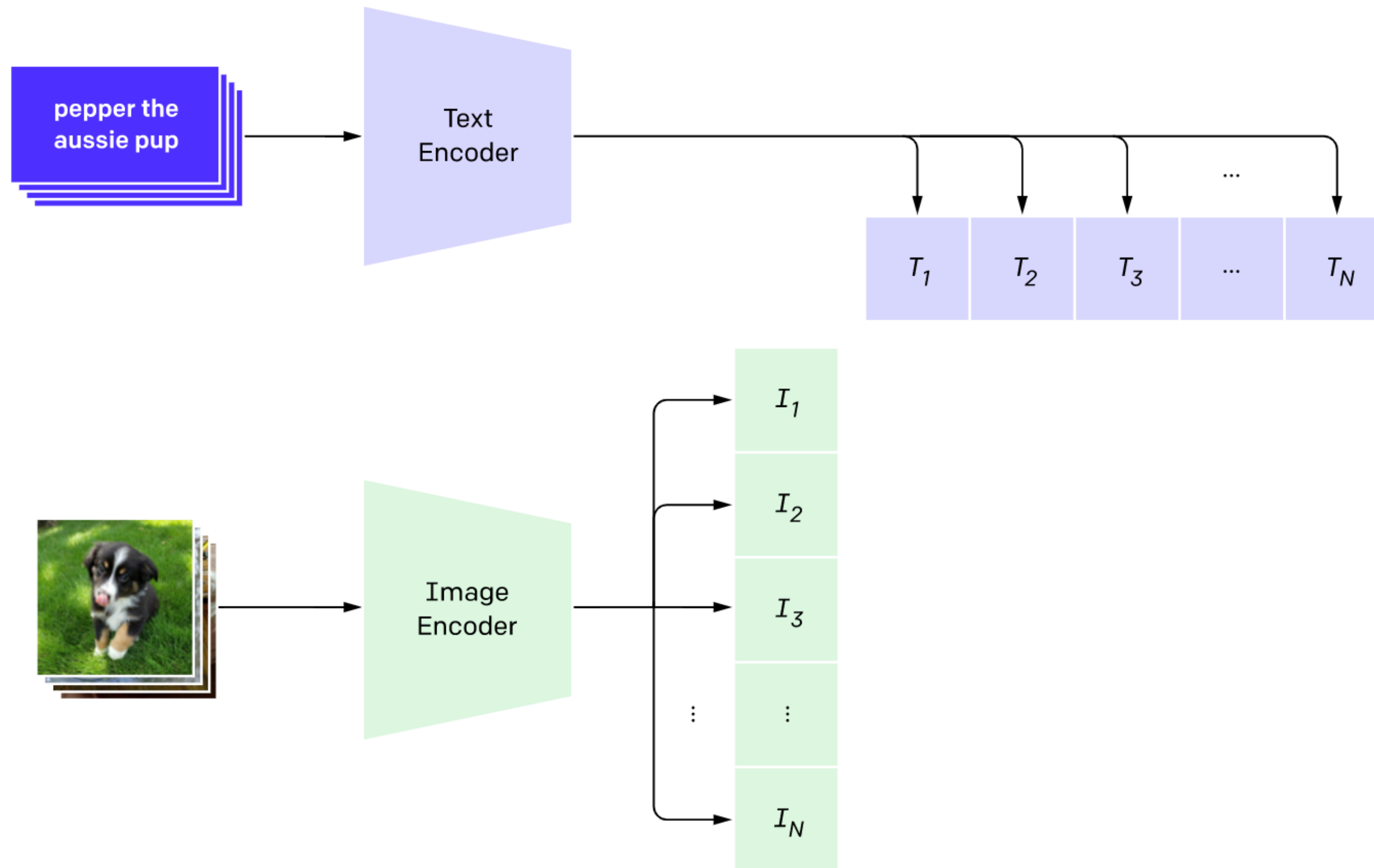


# DALL-E - CLIP



1. Learn latent representations of text  $T_i$  and images  $I_i$

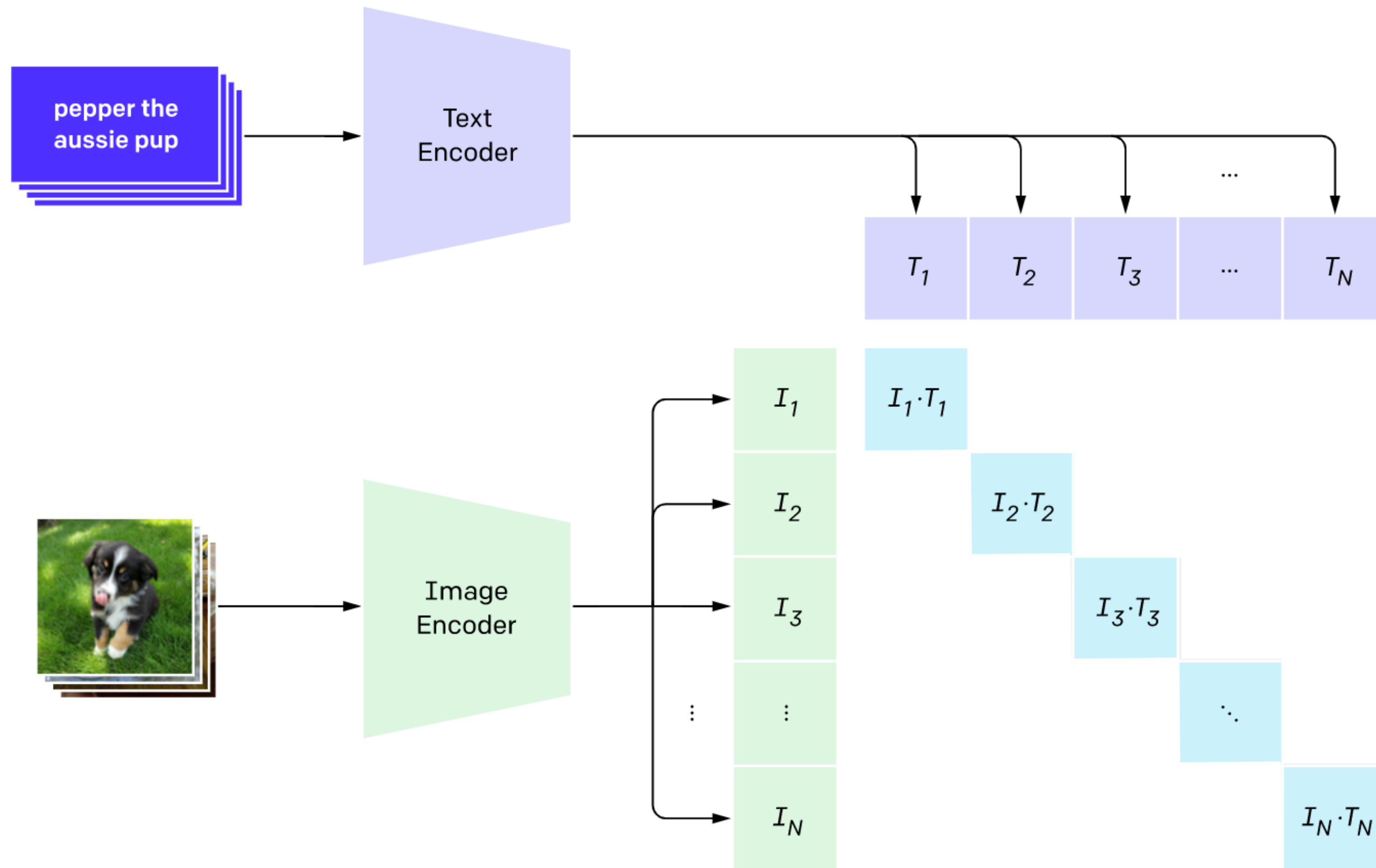
# DALL-E - CLIP



1. Learn latent representations of text  $T_i$  and images  $I_i$ 
  - From a batch of size N

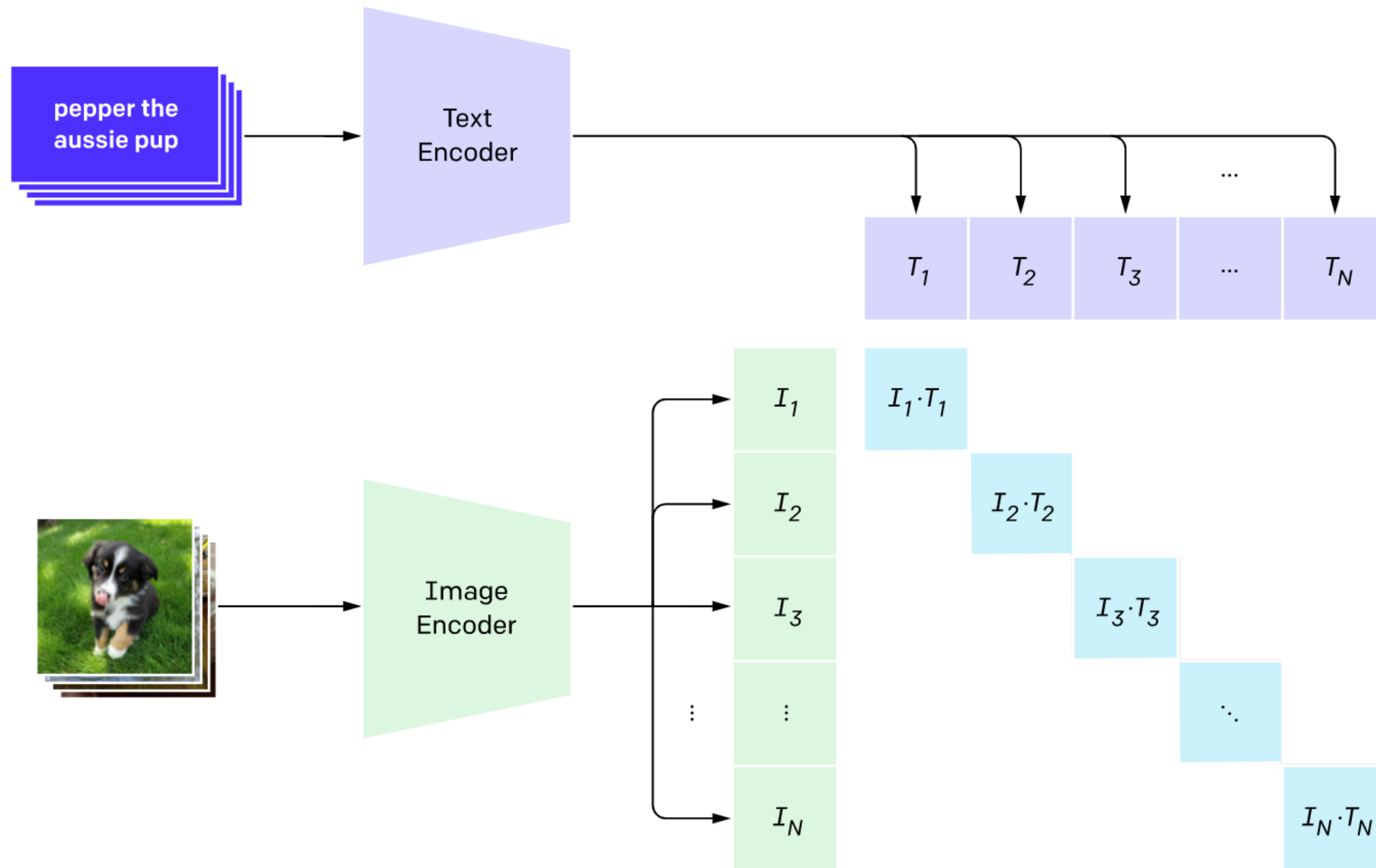


# DALL-E - CLIP



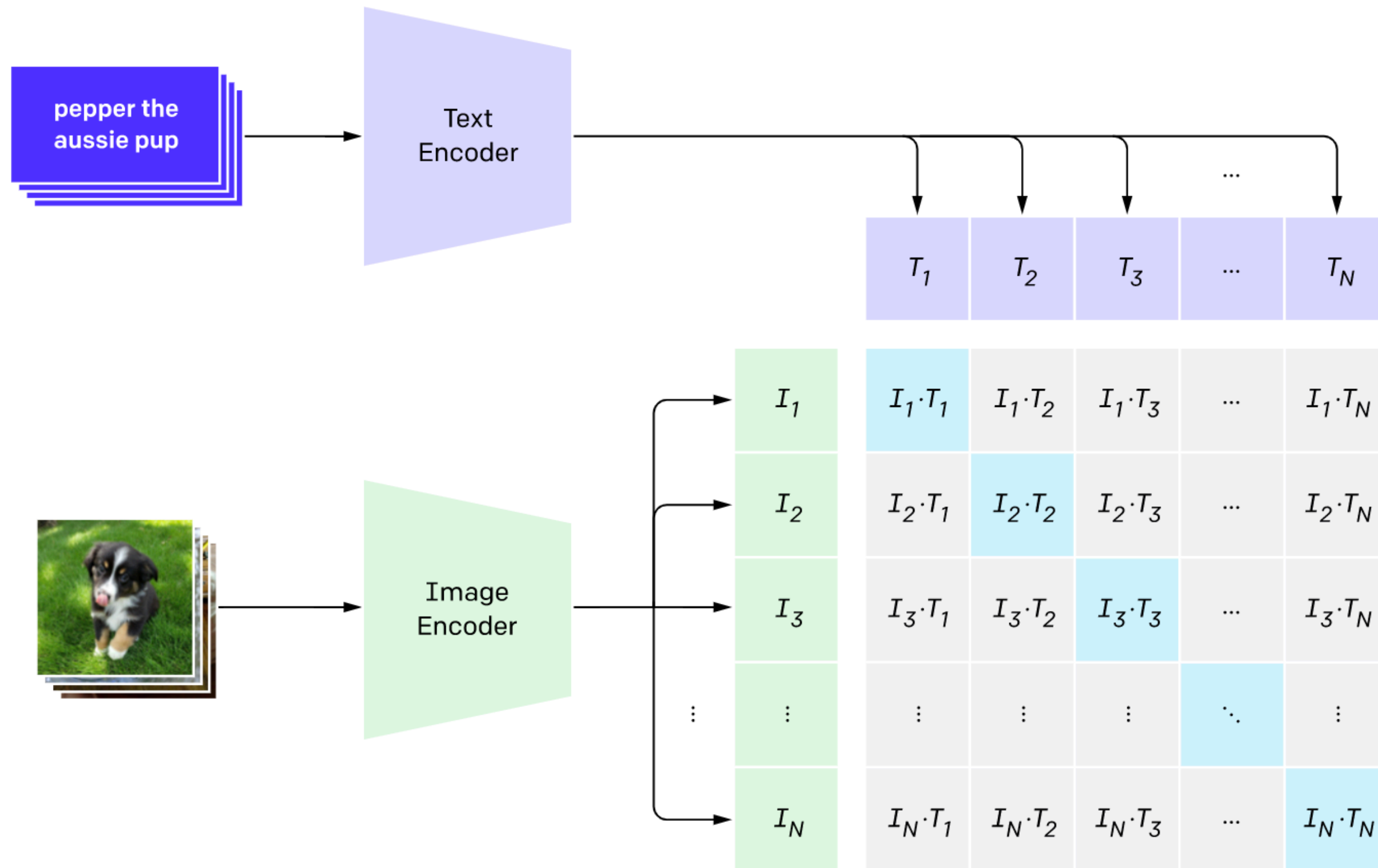
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# DALL-E - CLIP



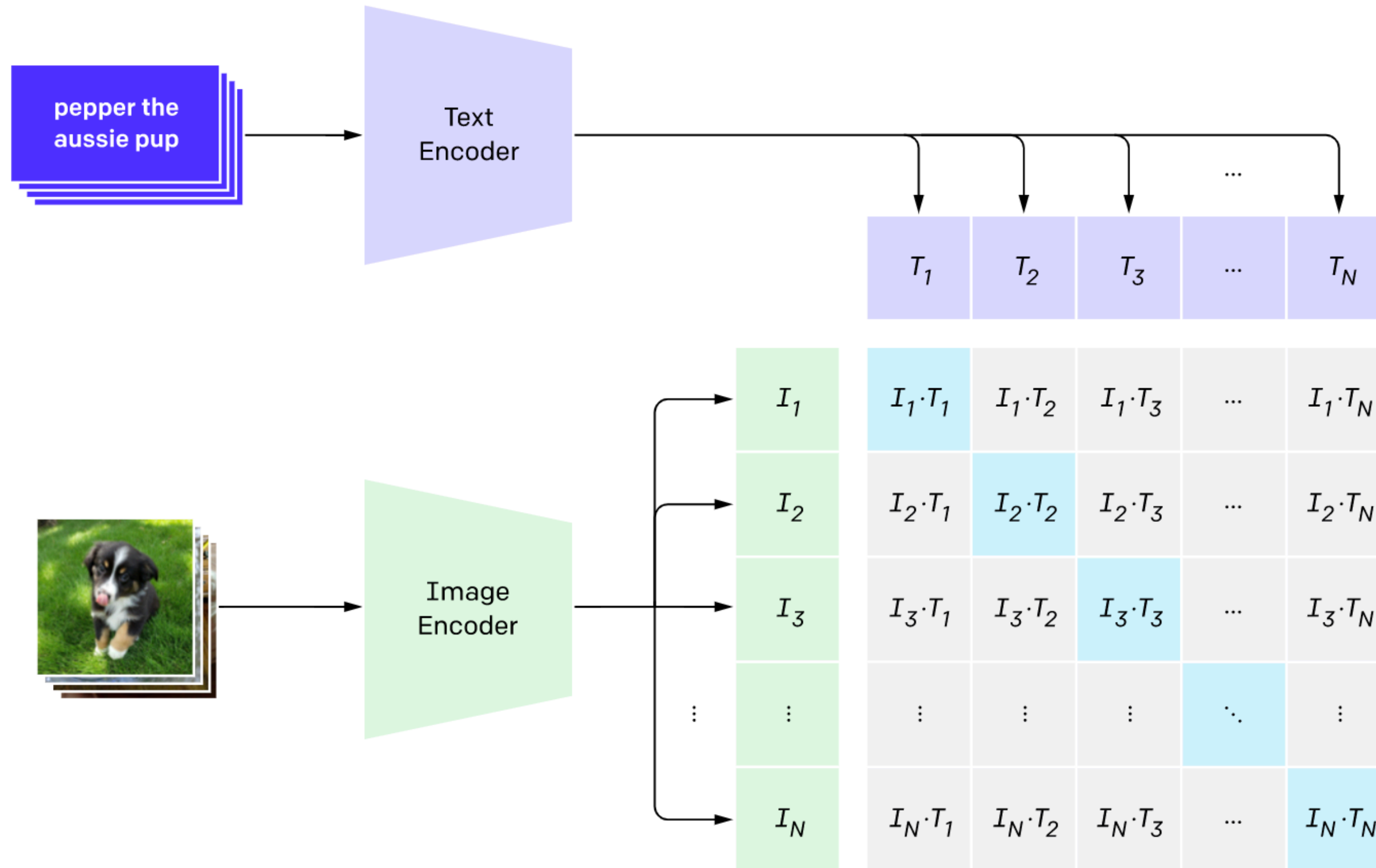
1. Learn latent representations of text  $T_i$  and images  $I_i$ 
  - From a batch of size N
2. Similarity measure

# DALL-E - CLIP



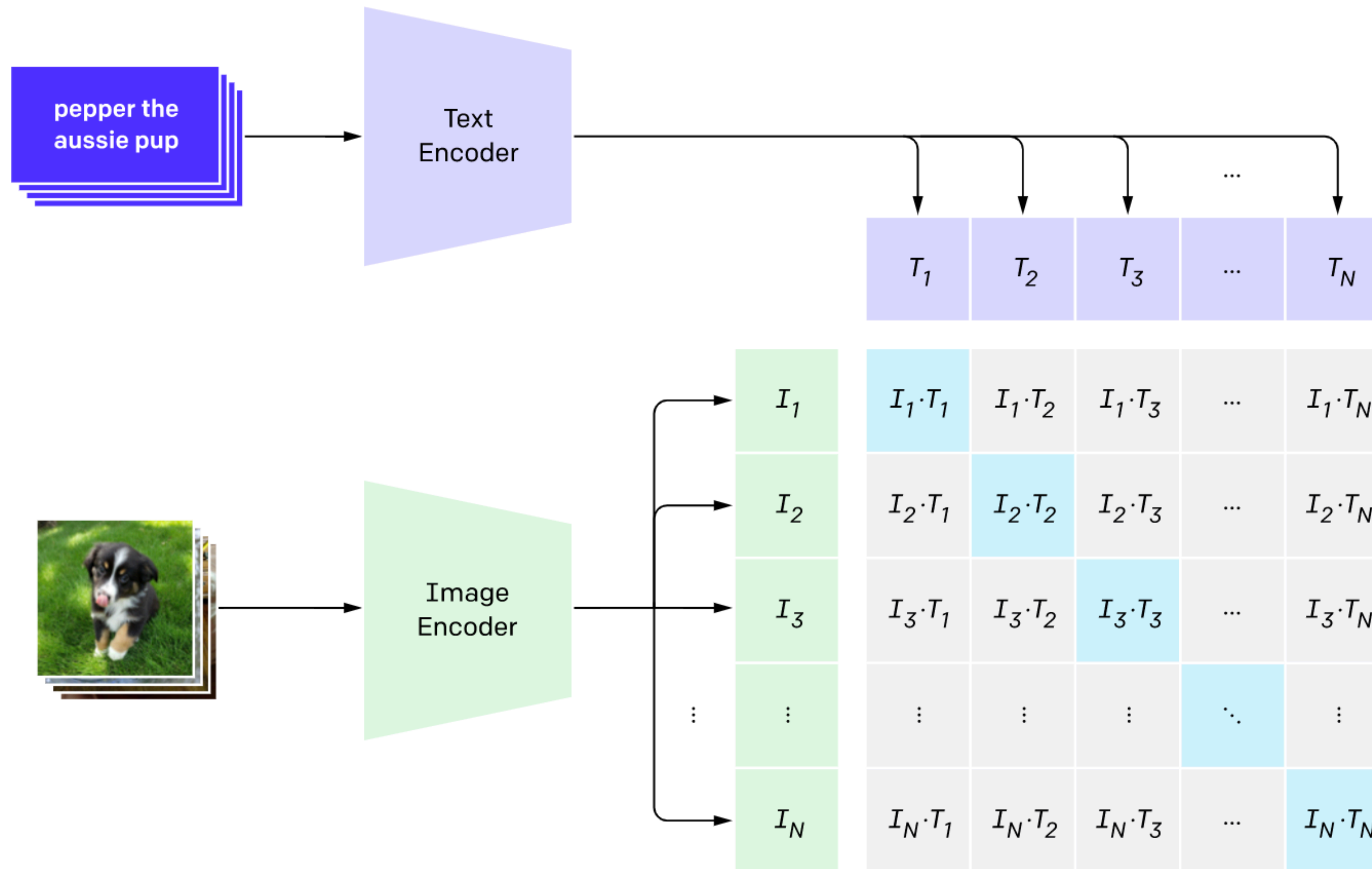
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# DALL-E - CLIP



1. Learn latent representations of text  $T_i$  and images  $I_i$ 
  - From a batch of size N
2. Similarity measure
  - $\max T_i I_i$

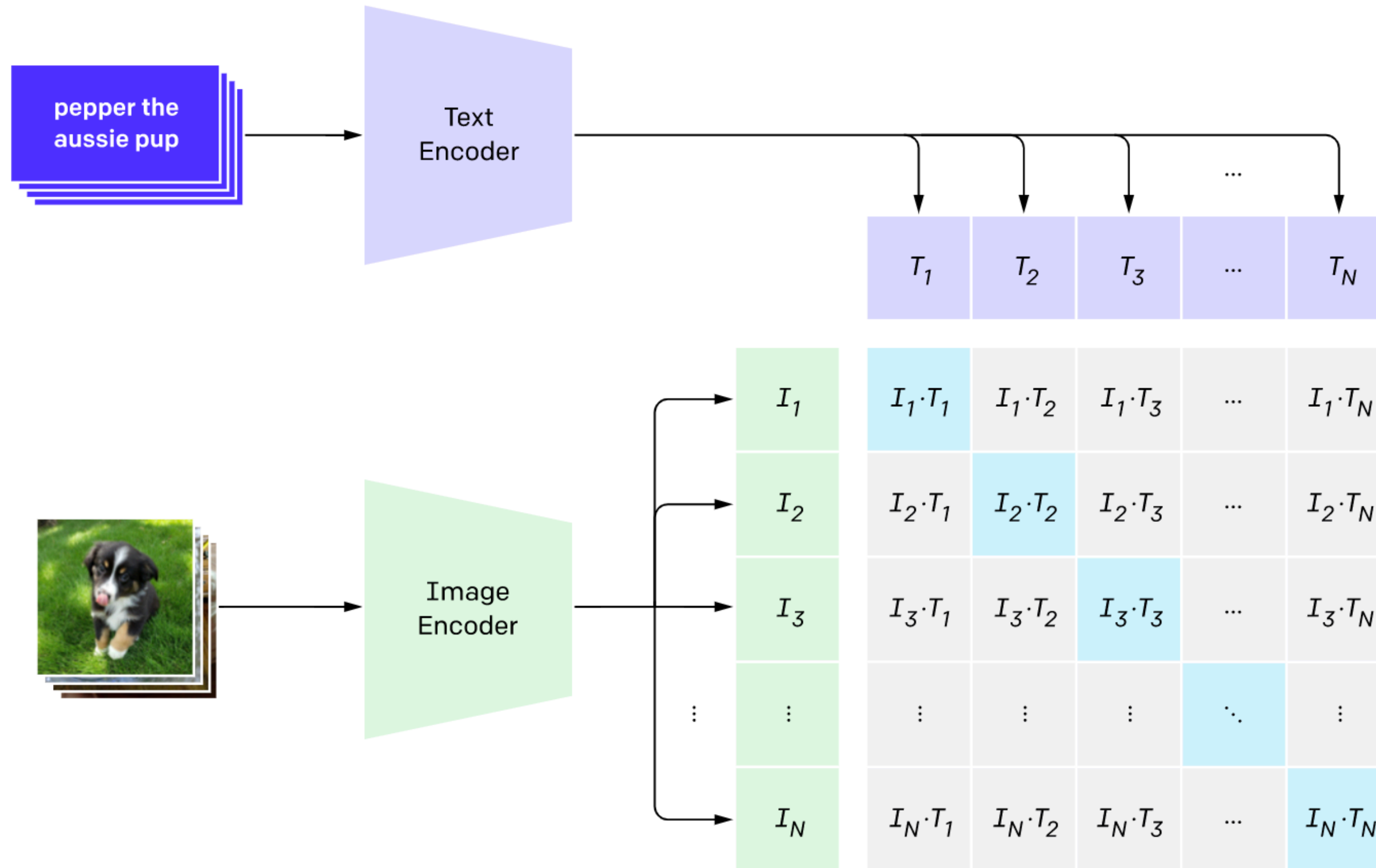
# DALL-E - CLIP



1. Learn latent representations of text  $T_i$  and images  $I_i$ 
  - From a batch of size N
2. Similarity measure
  - $\max T_i I_i$
3. Dissimilarity measure

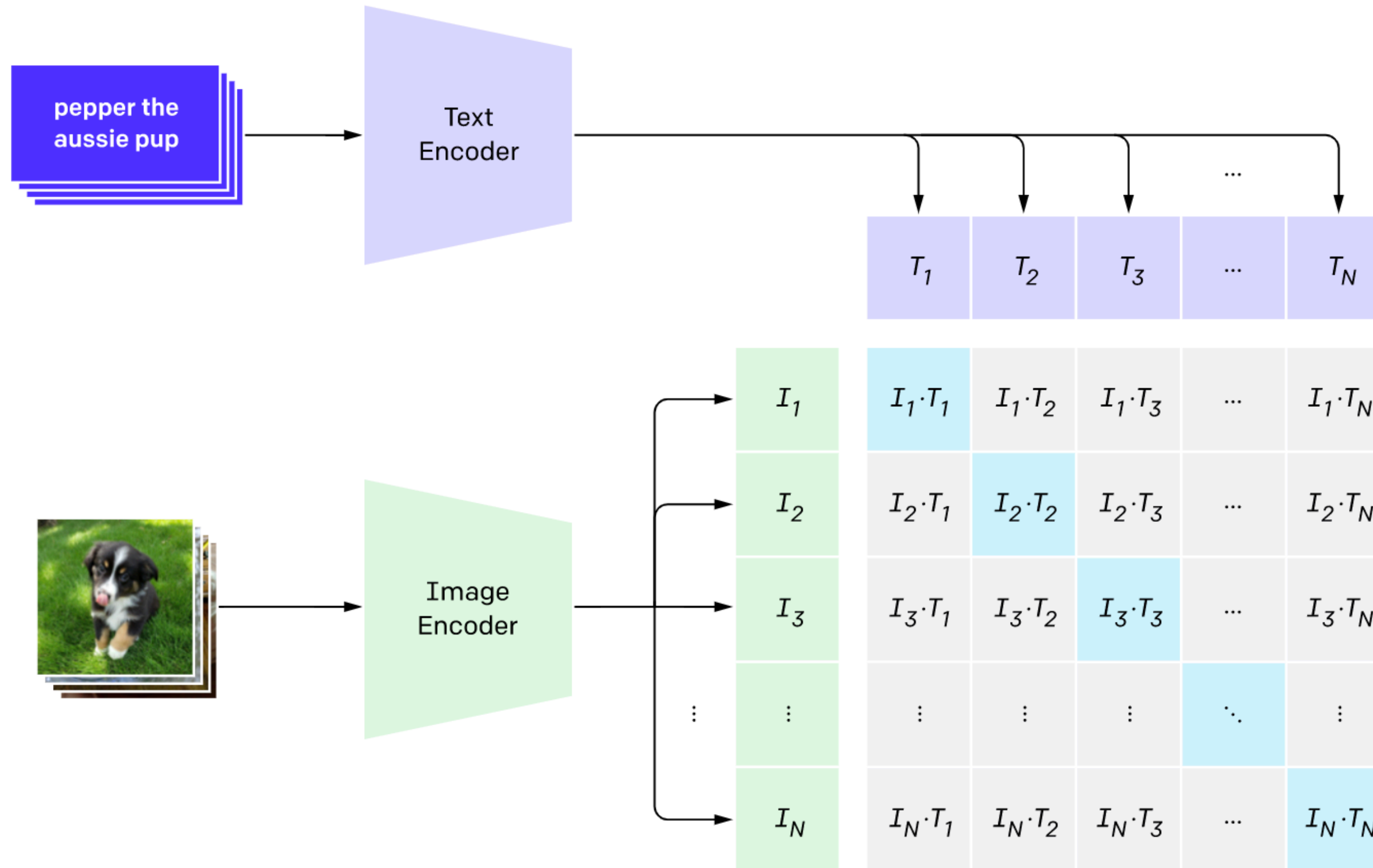


# DALL-E - CLIP



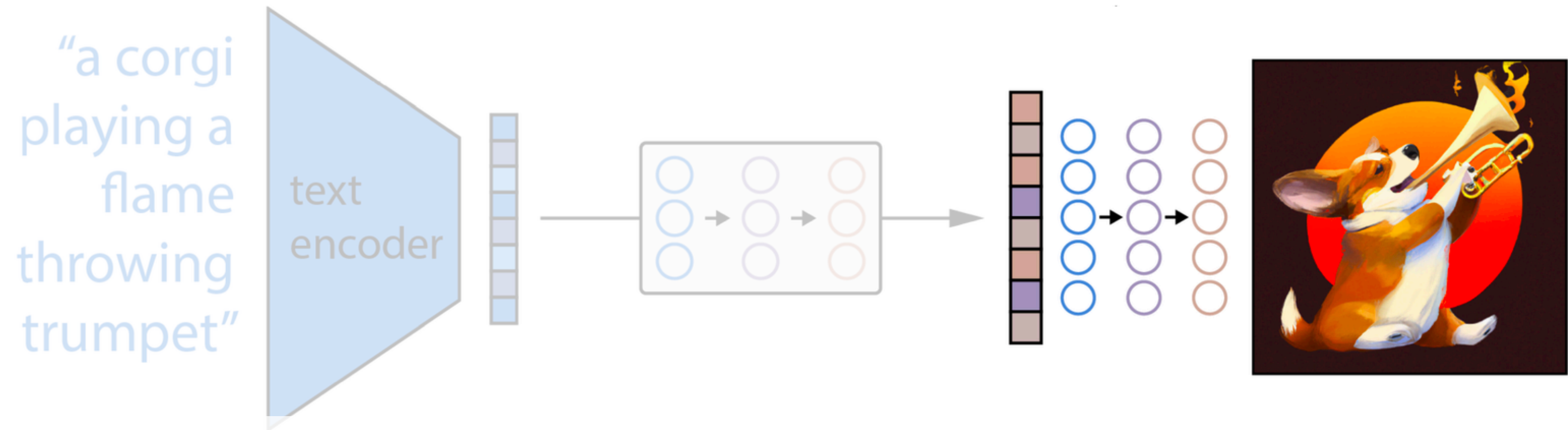
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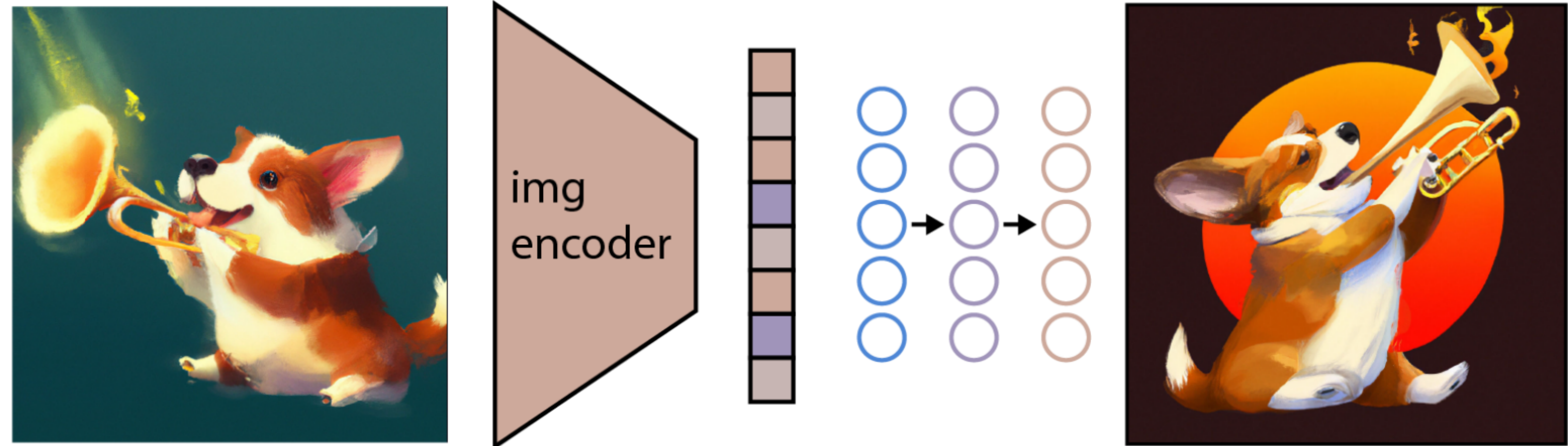


1. Learn latent representations of text  $T_i$  and images  $I_i$ 
  - From a batch of size N
2. Similarity measure
  - $\max T_i I_i$
3. Dissimilarity measure
  - $\min T_i I_j \quad \forall j \neq i$
4. Maximize the similarities and minimize the dissimilarities

# DALL-E - Diffusion Models

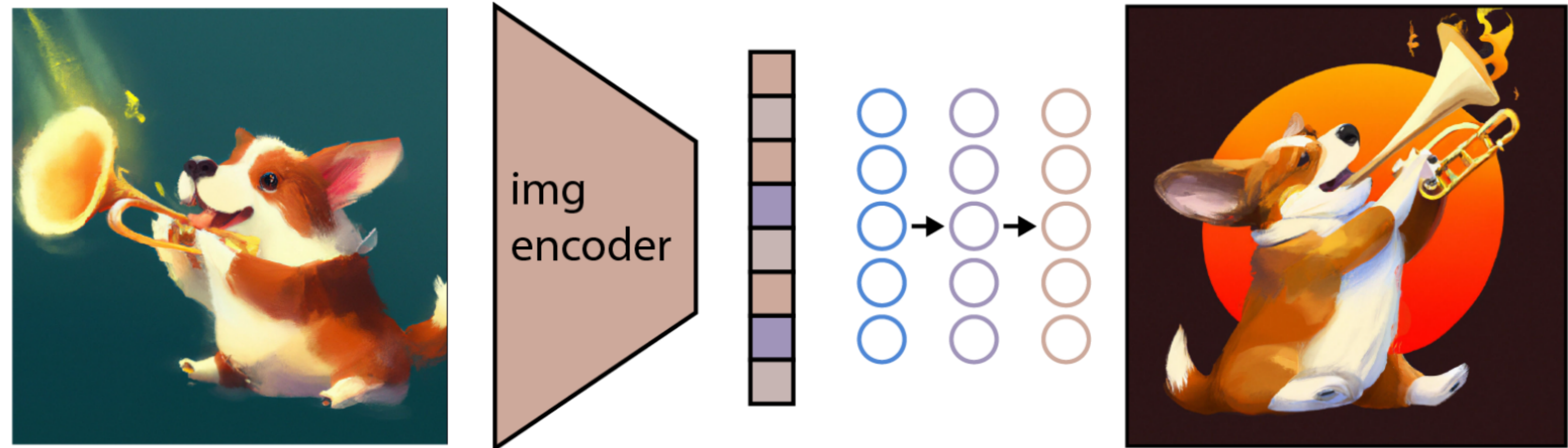


# DALL-E - Diffusion Models





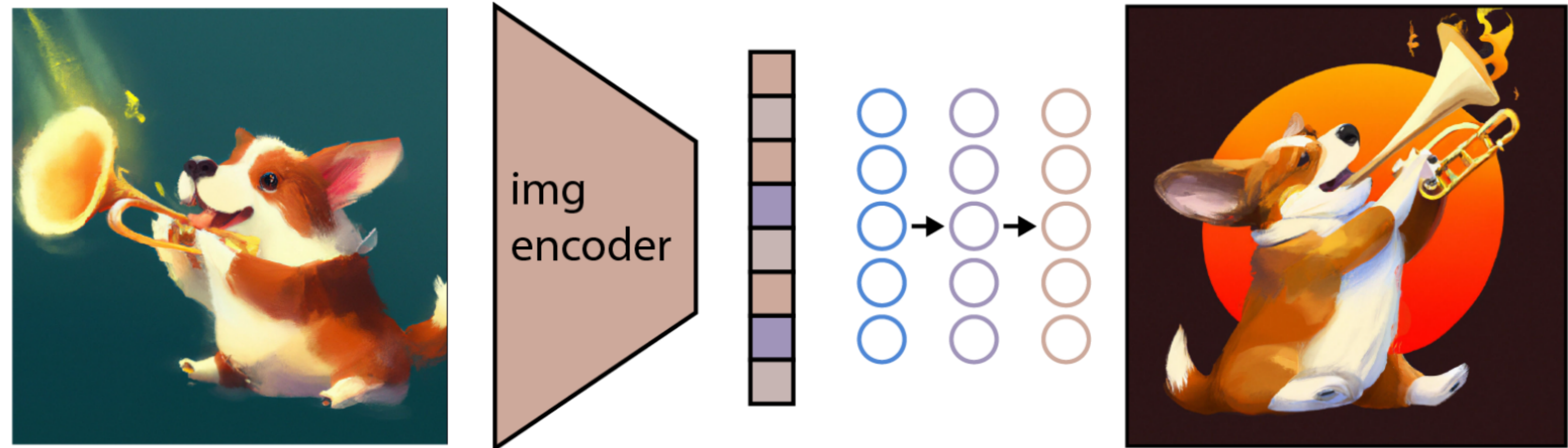
# DALL-E - Diffusion Models



1. Contrastive Language-Image Pre-training (CLIP)



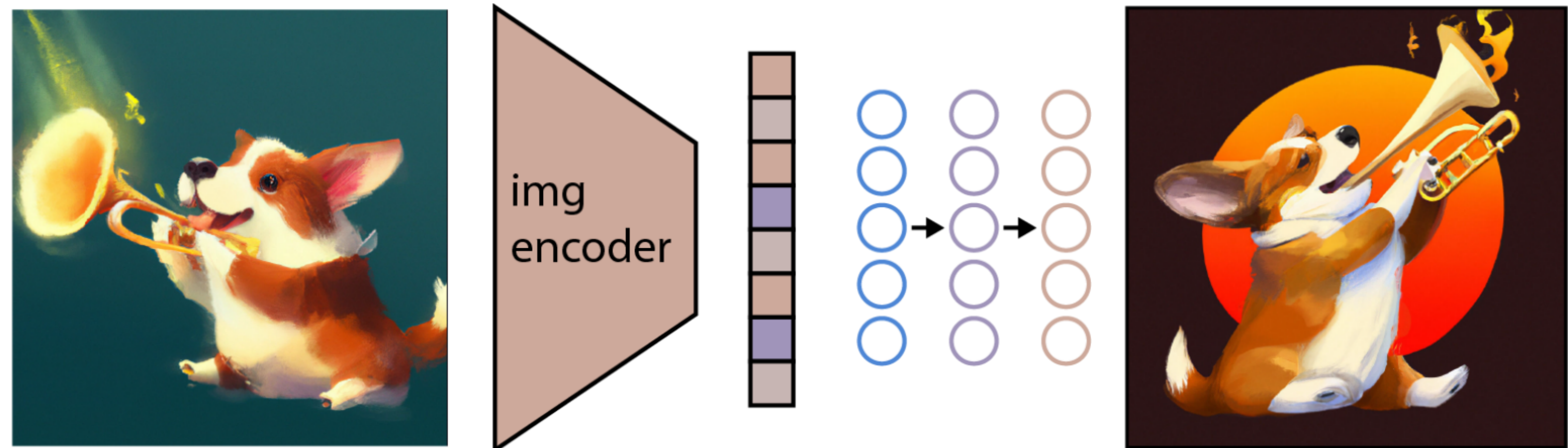
# DALL-E - Diffusion Models



1. Contrastive Language-Image Pre-training (CLIP)

2. Generation of an image using a diffusion model

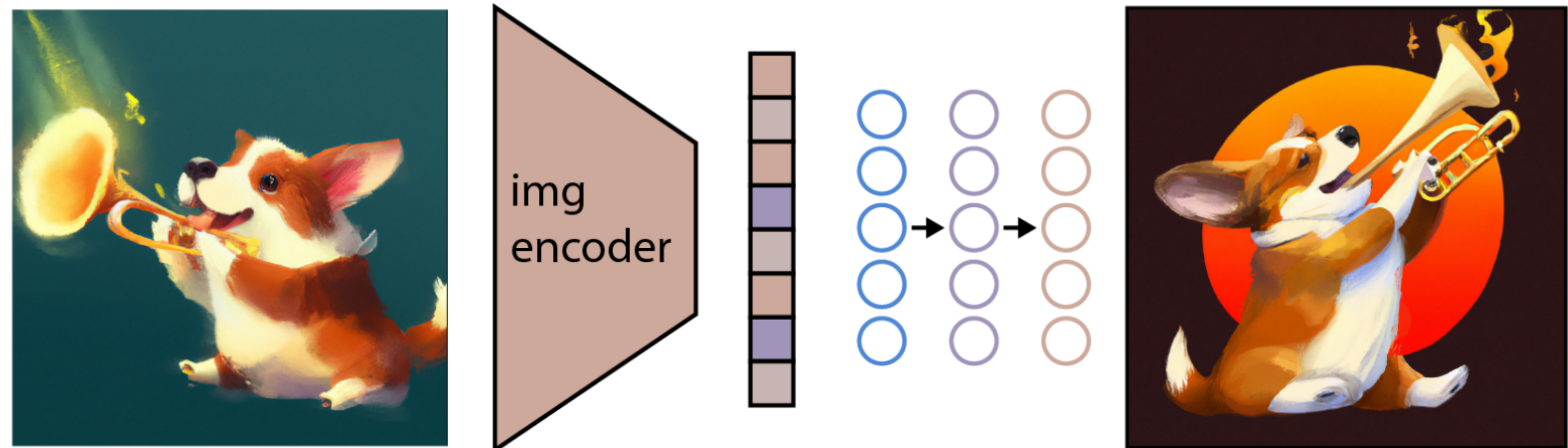
# DALL-E - Diffusion Models



1. Contrastive Language-Image Pre-training (CLIP)
2. Generation of an image using a diffusion model
3. Learn the latent representations of text and images



# DALL-E - Diffusion Models

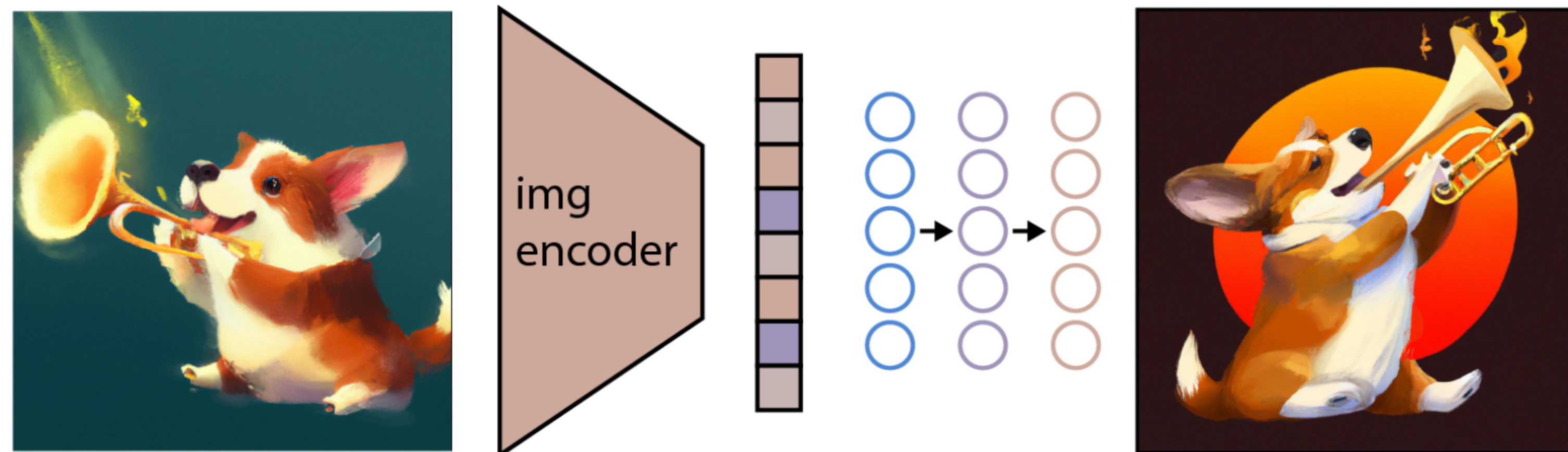


1. Contrastive Language-Image Pre-training (CLIP)
2. Generation of an image using a diffusion model
3. Learn the latent representations of text and images
4. Wrap-it up!

# DALL-E - Diffusion Model

Idea:

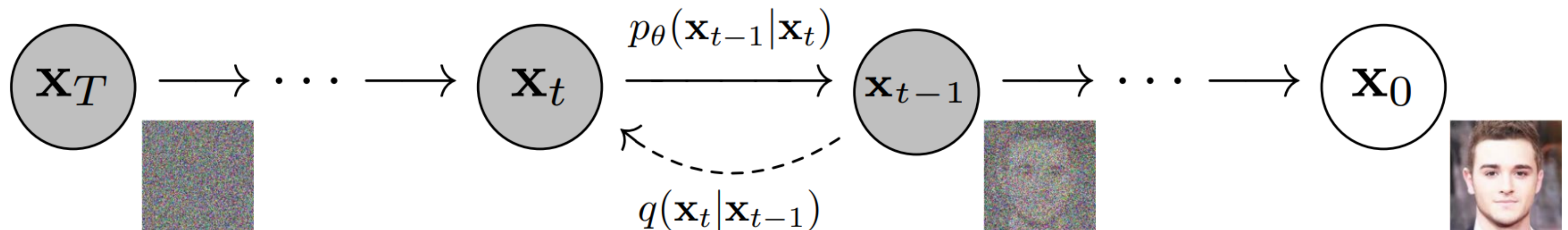
- From an image (left), generate other similar ones
- This is where diffusion models are used



# Diffusion Models

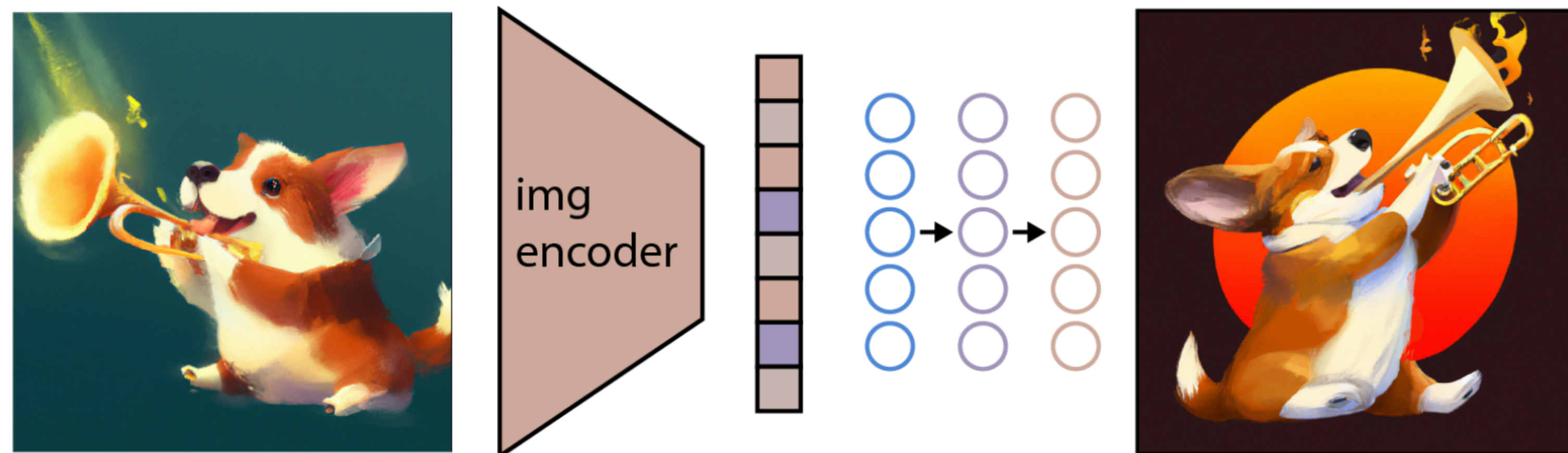
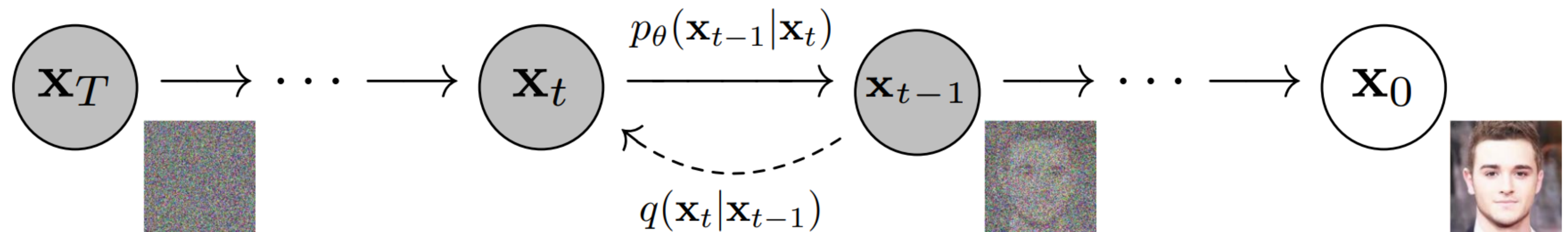
Idea:

- Add noise incrementally to an image until it is pure white noise
- Danoise the image to obtain the original image
- If we know the noise mechanism, starting from white noise, we can then generate an image



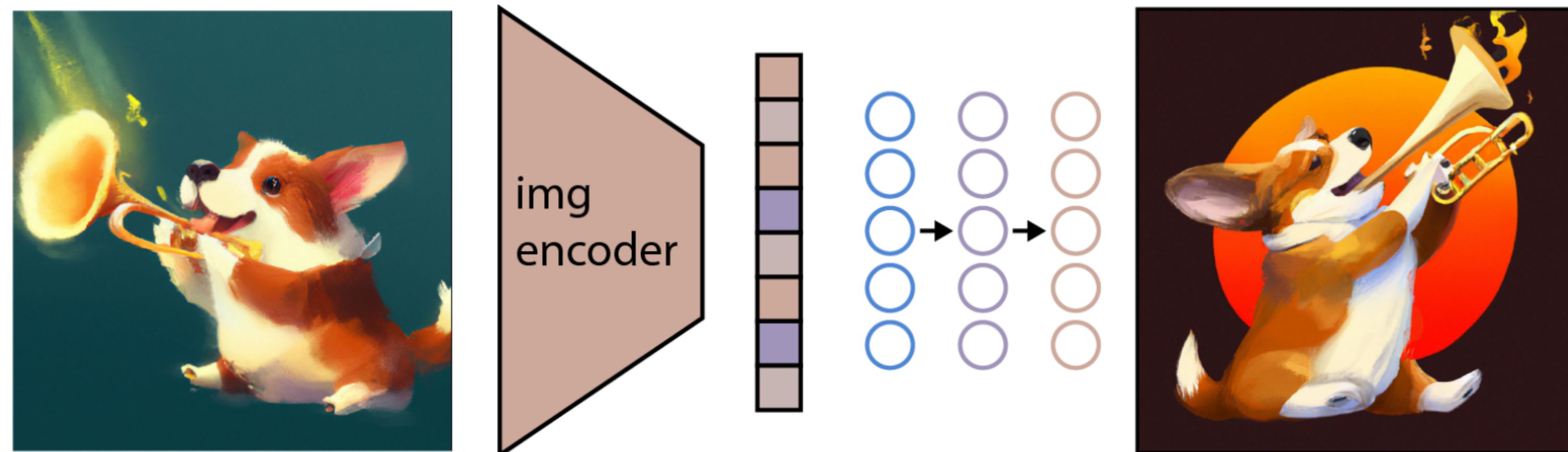
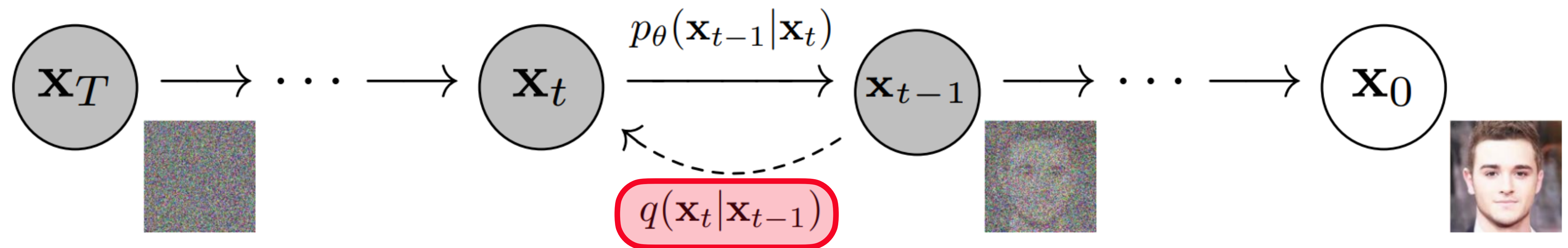


# DALL-E - Diffusion Model



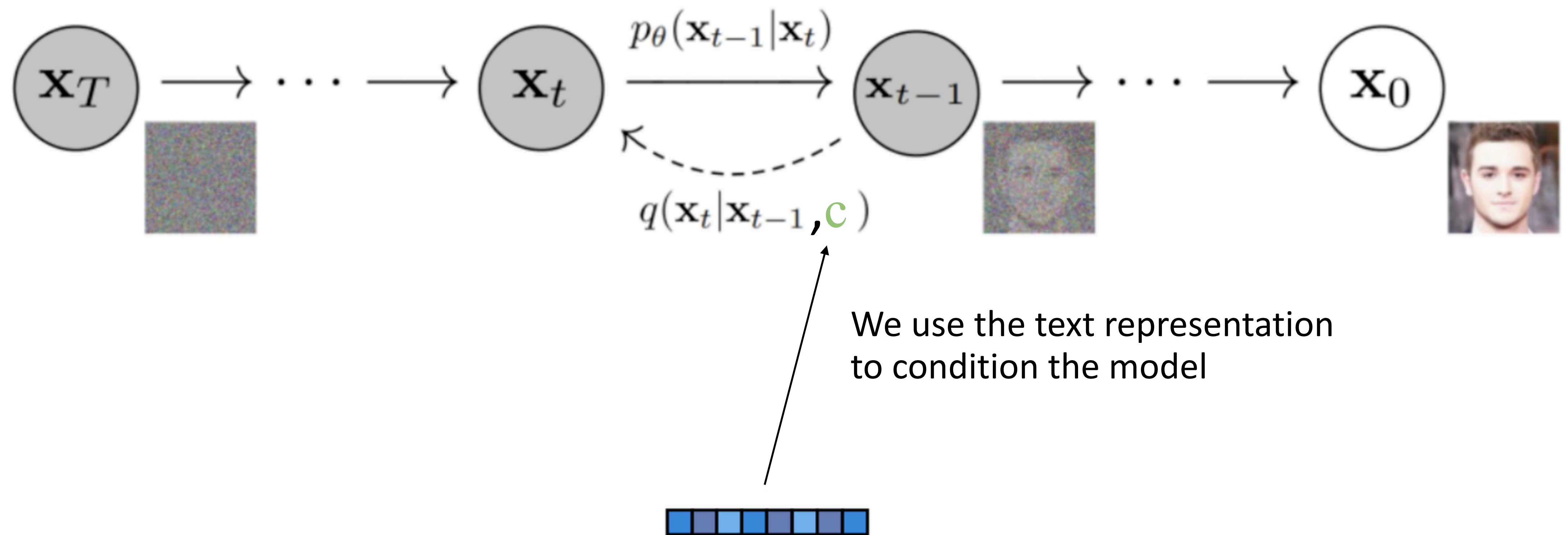
# DALL-E - Diffusion Model

For text-to-image generation, we add information from the text  
During the diffusion process

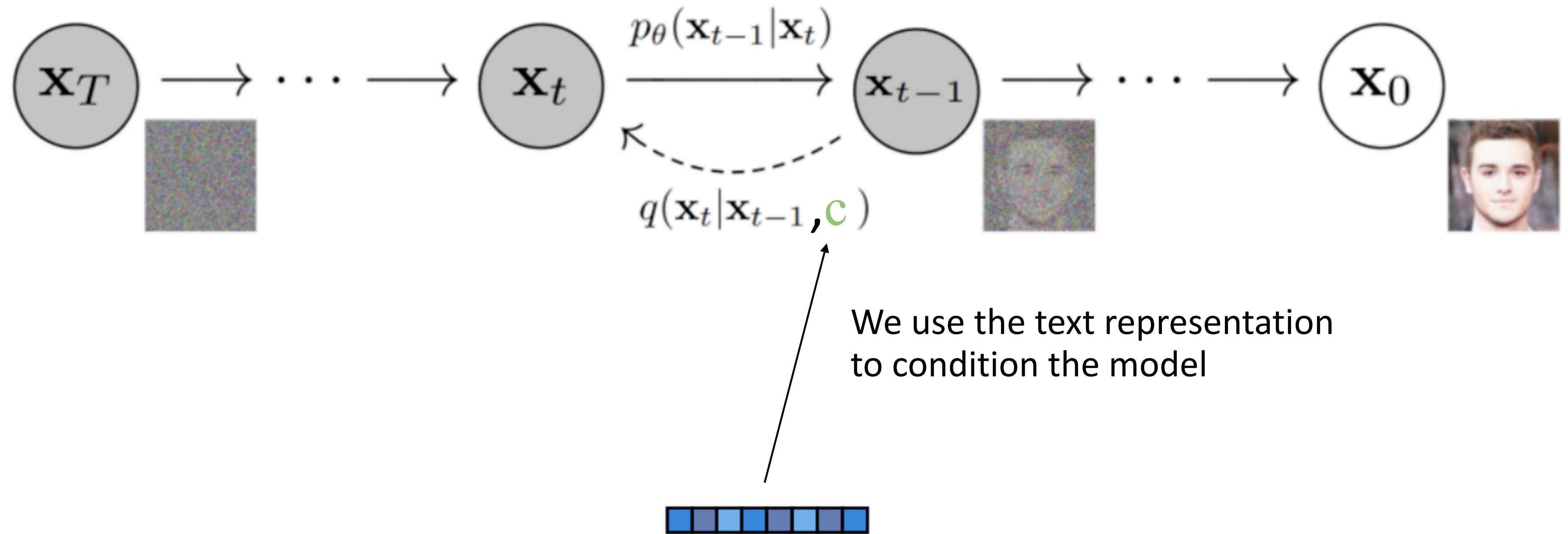




# DALL-E

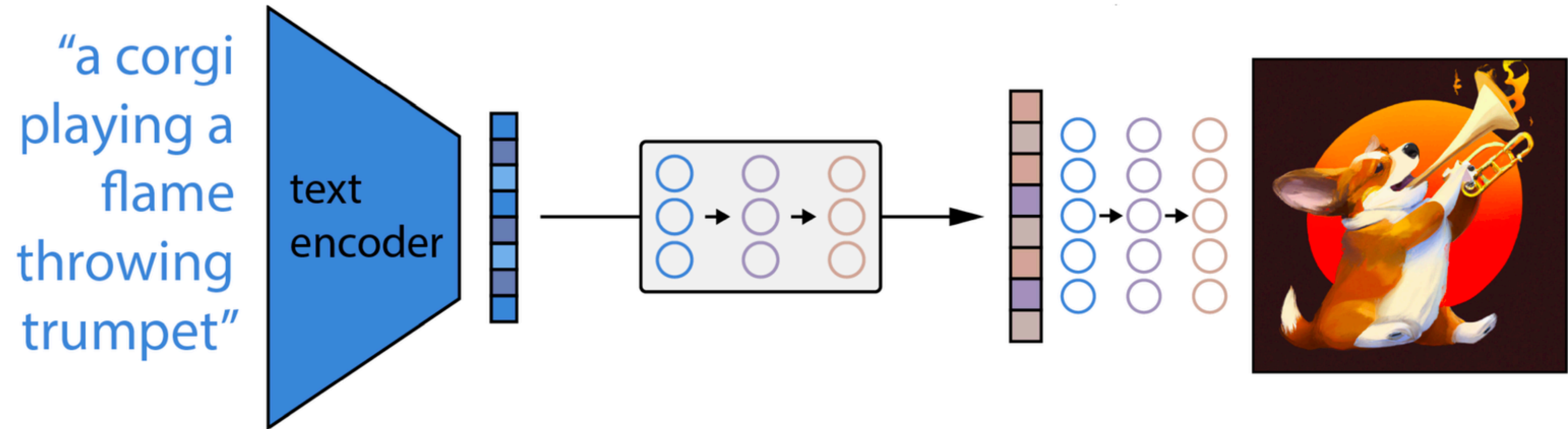


# DALL-E



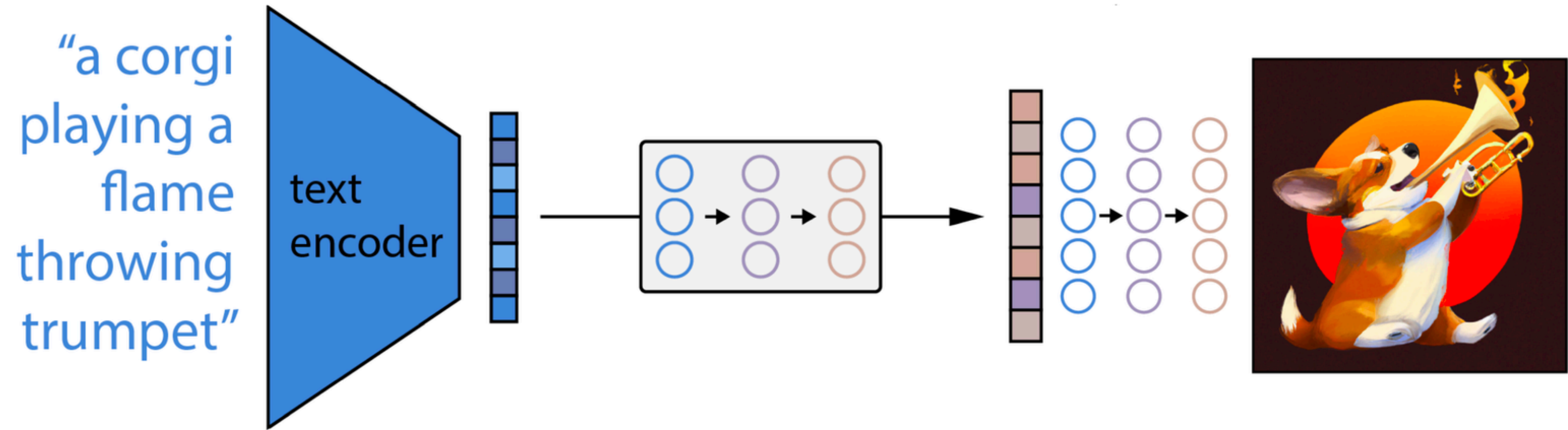
“Face face of a man with red hair”

# DALL-E - Wrap-it up!



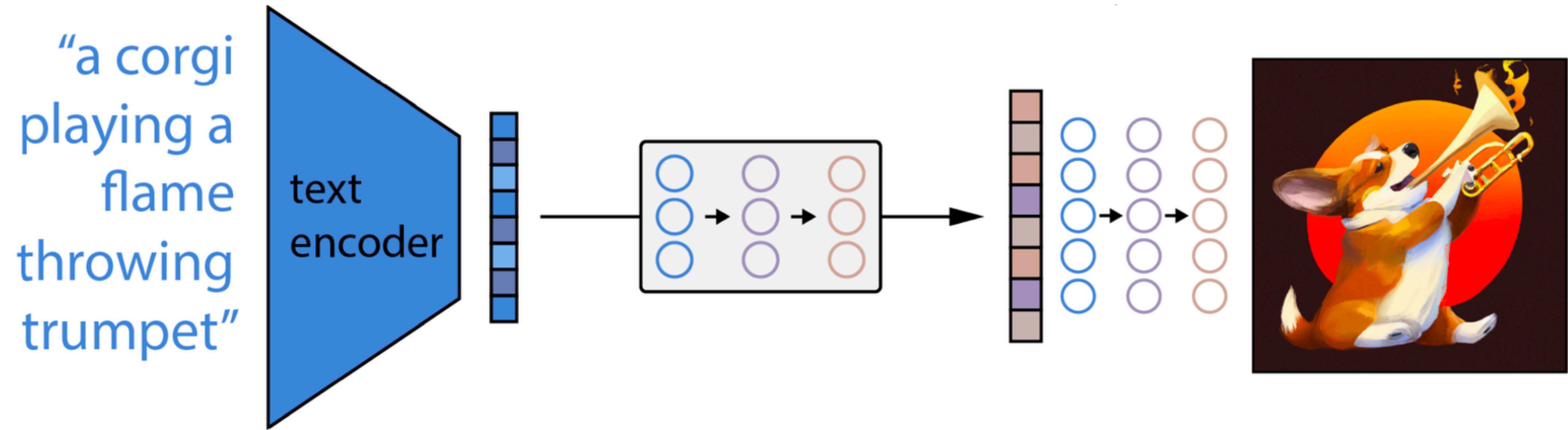


# DALL-E - Wrap-it up!



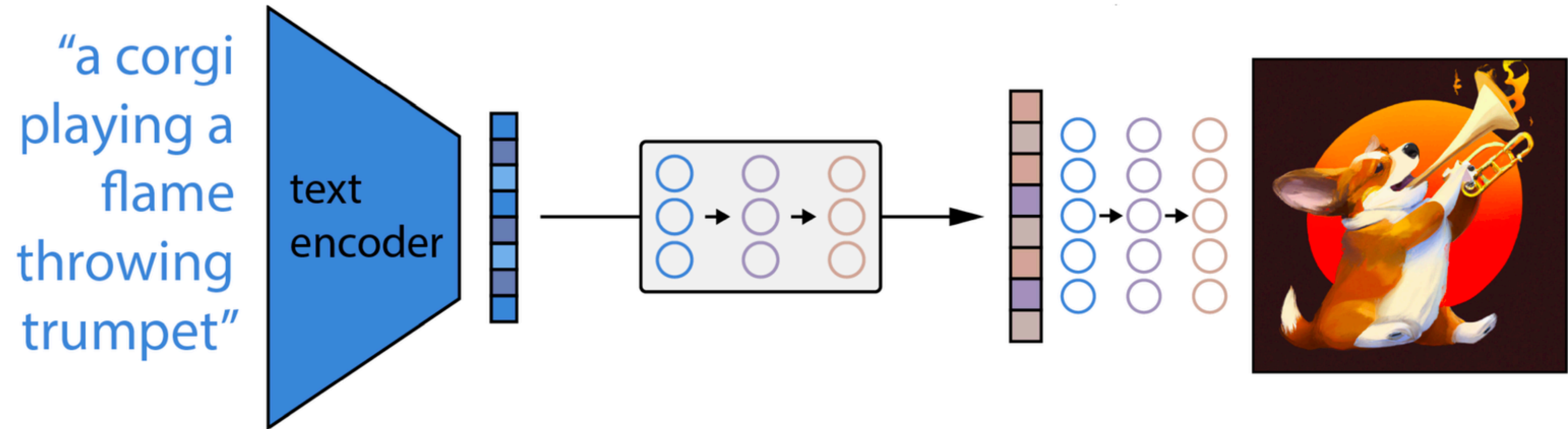
1. Contrastive Language-Image Pre-training (CLIP)

# DALL-E - Wrap-it up!



1. Contrastive Language-Image Pre-training (CLIP)
2. Generation of an image using a diffusion model

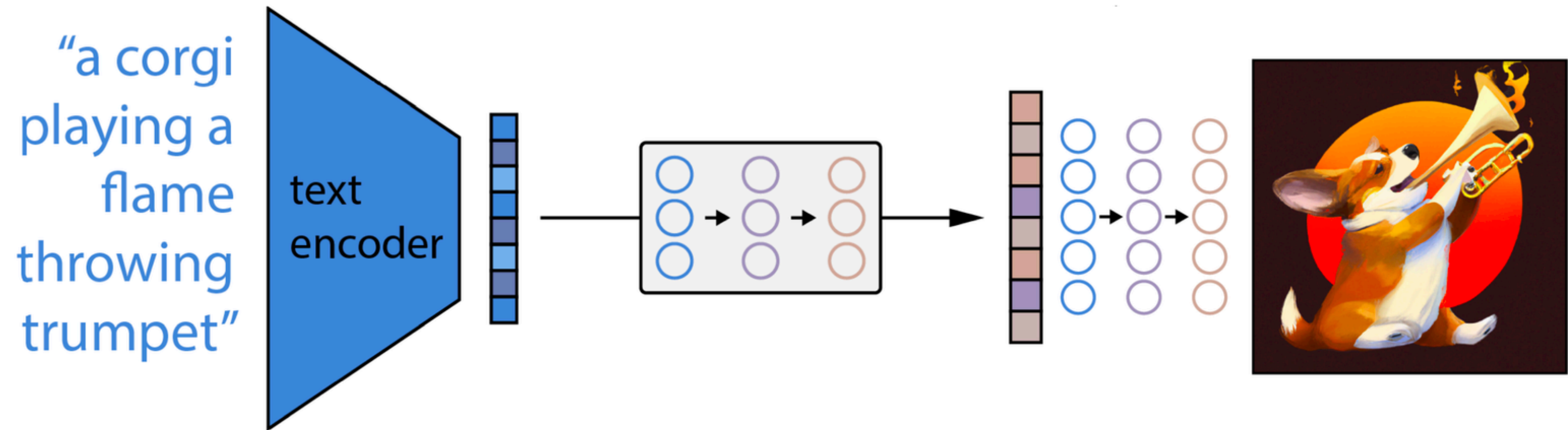
# DALL-E - Wrap-it up!



1. Contrastive Language-Image Pre-training (CLIP)
2. Generation of an image using a diffusion model
3. Learn the latent representations of text and images

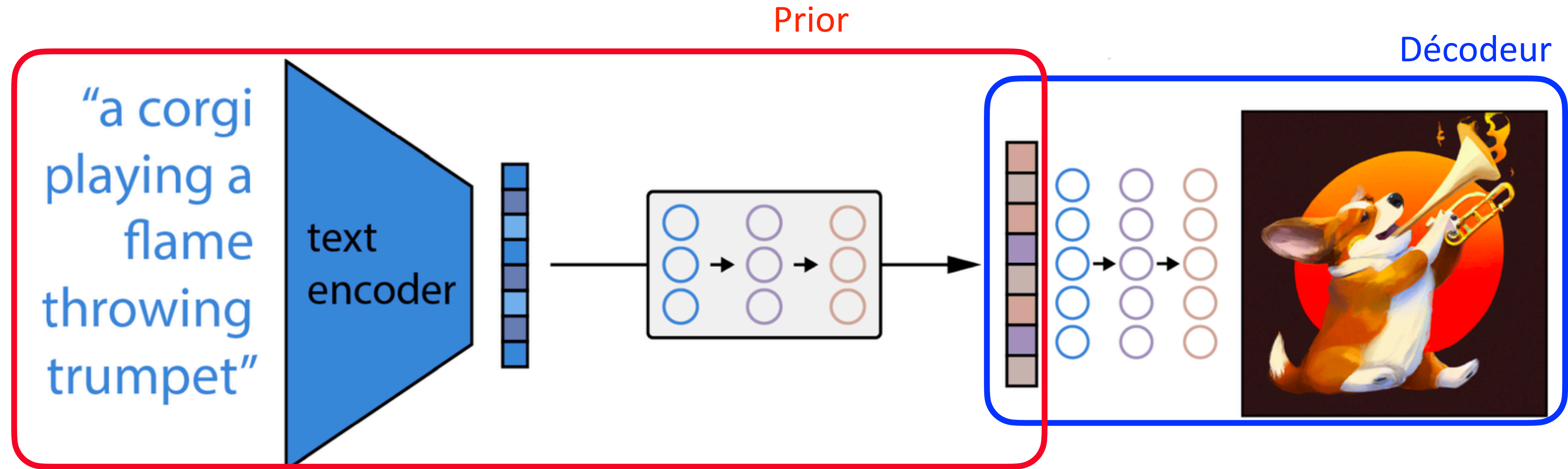


# DALL-E - Wrap-it up!



1. Contrastive Language-Image Pre-training (CLIP)
2. Generation of an image using a diffusion model
3. Learn the latent representations of text and images
4. Wrap-it up!

# DALL-E - Wrap-it up!



Idea:

- Given  $(x, y)$  a tuple of an image  $x$  and text  $y$ .
- Given the representation of an image  $z$ .
- The distribution of the image given the text is:

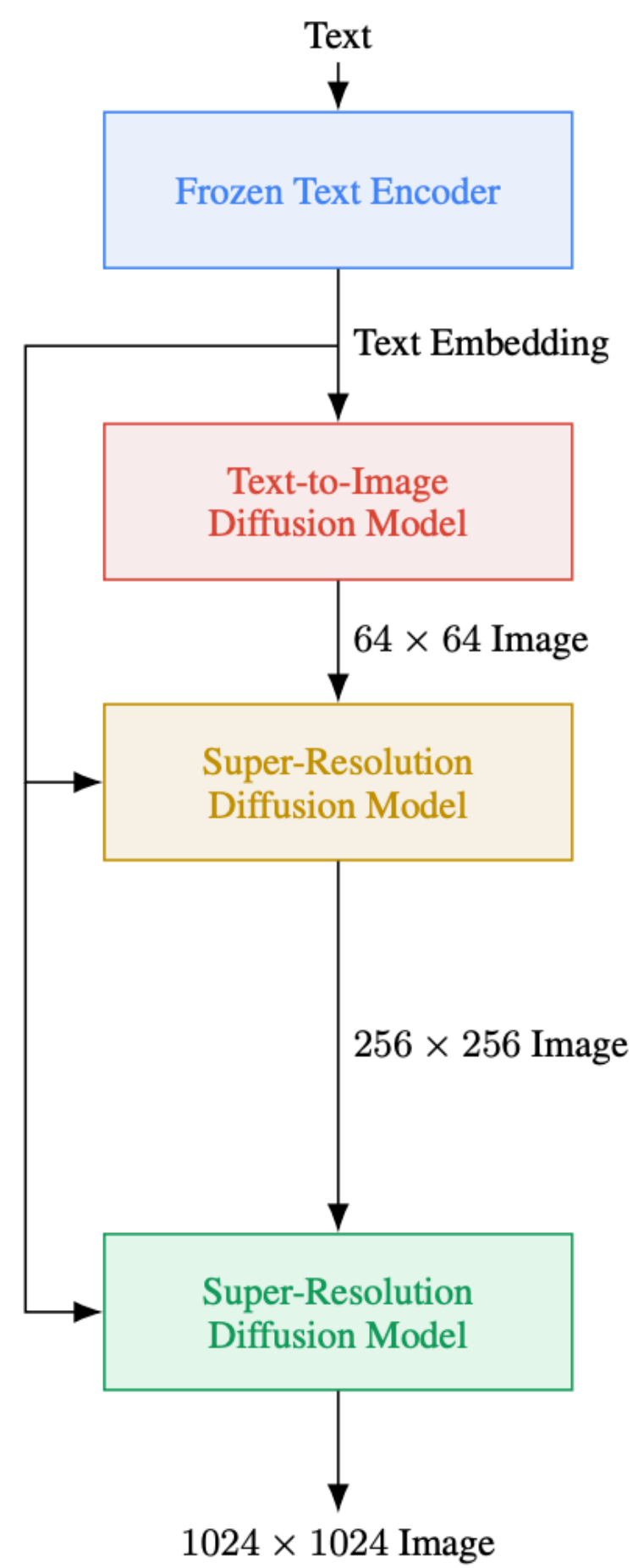
$$P(x|y) = P(x, z_i|y) = P(x|z_i, y)P(z_i|y)$$



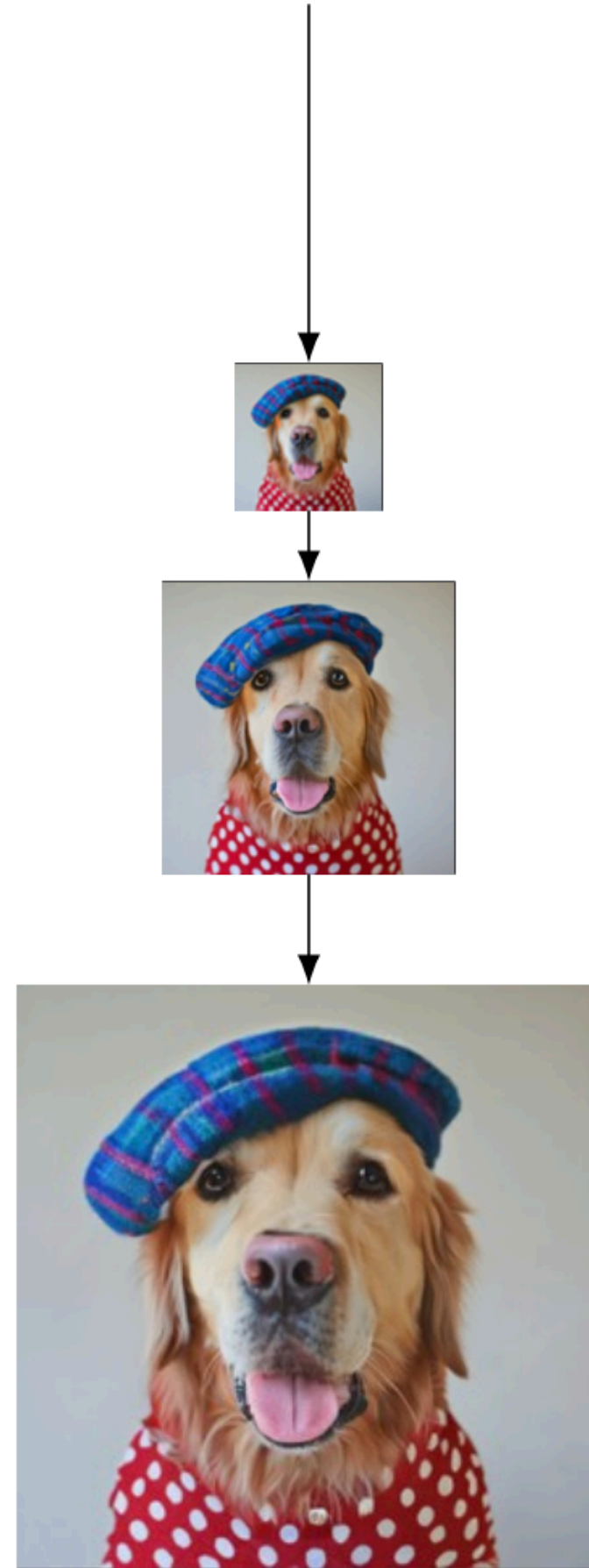
# Imagen



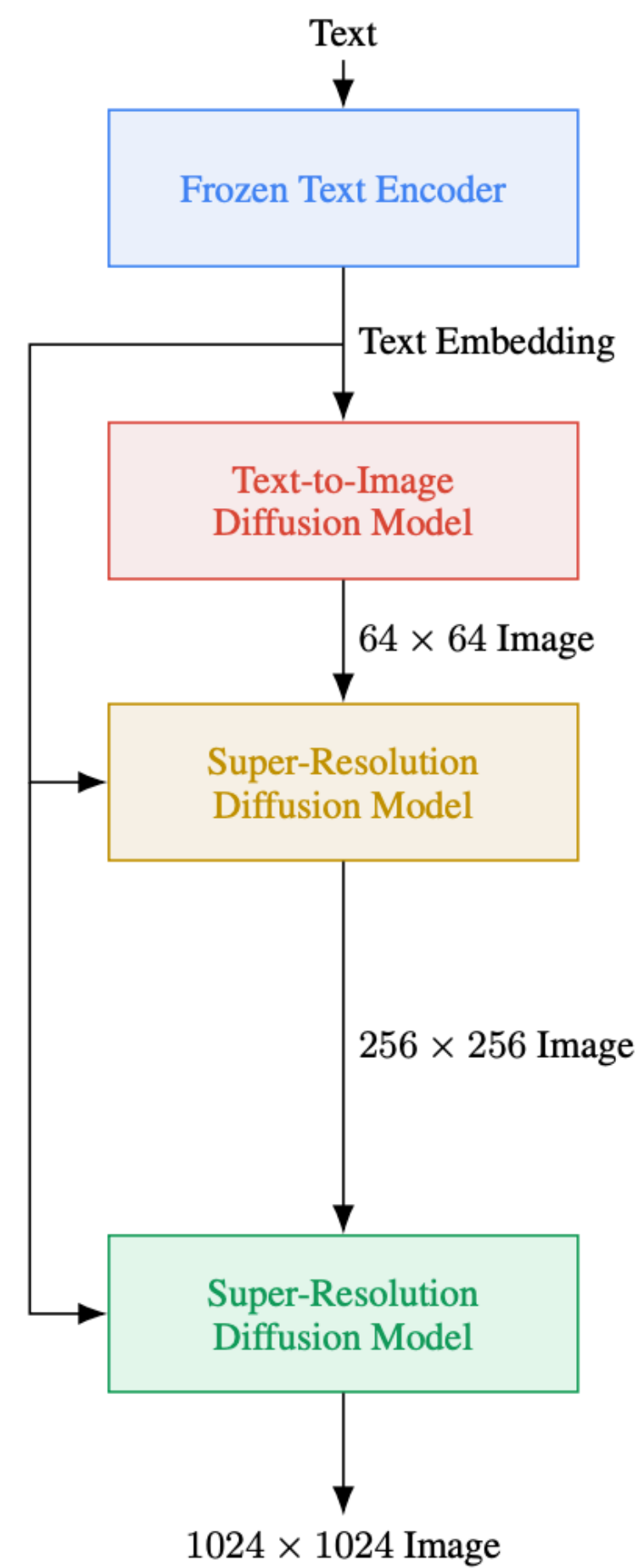
# Architecture



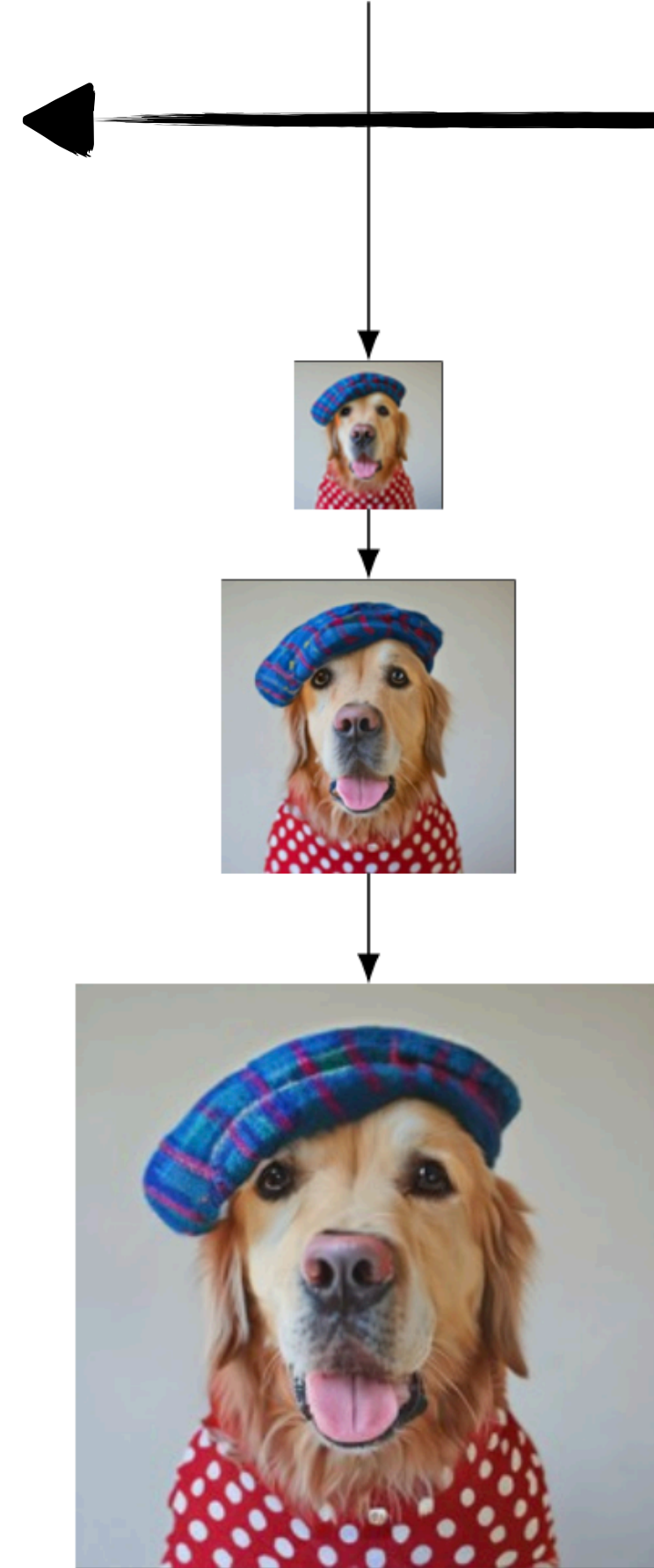
“A Golden Retriever dog wearing a blue checkered beret and red dotted turtleneck.”



# Architecture

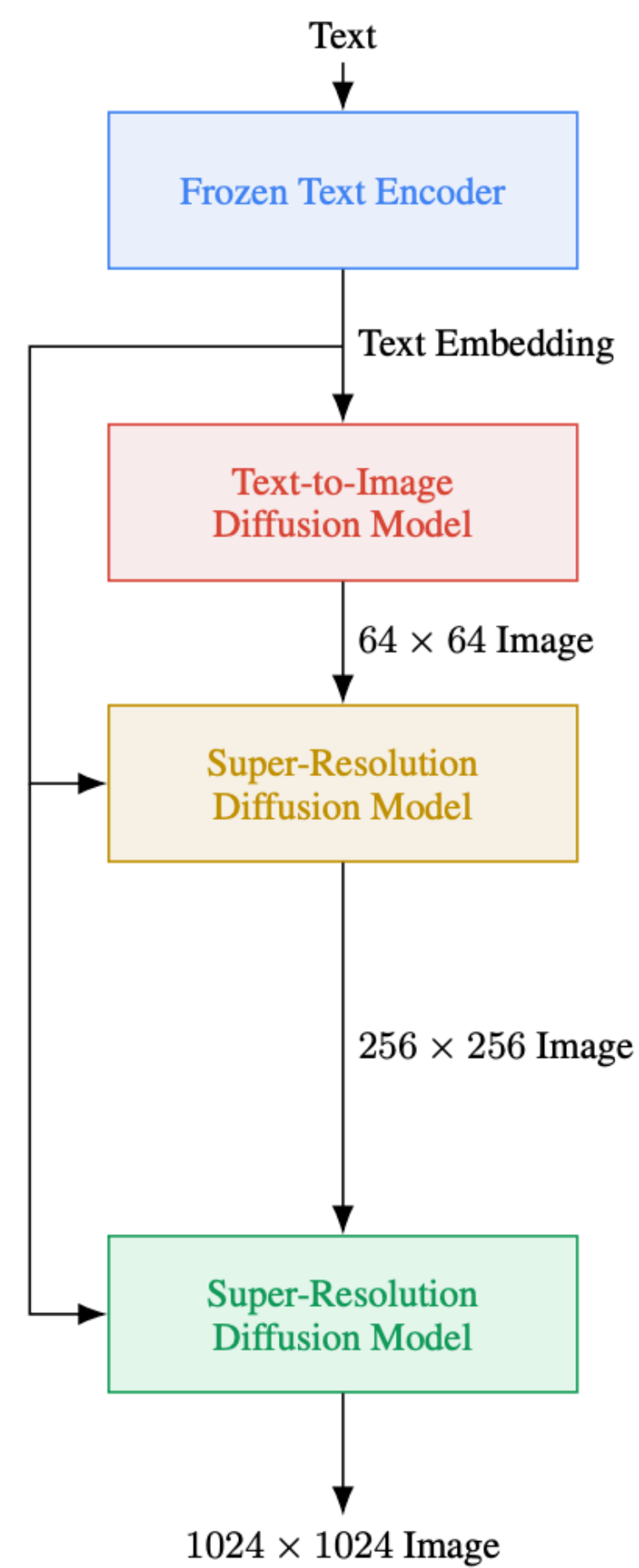


“A Golden Retriever dog wearing a blue checkered beret and red dotted turtleneck.”

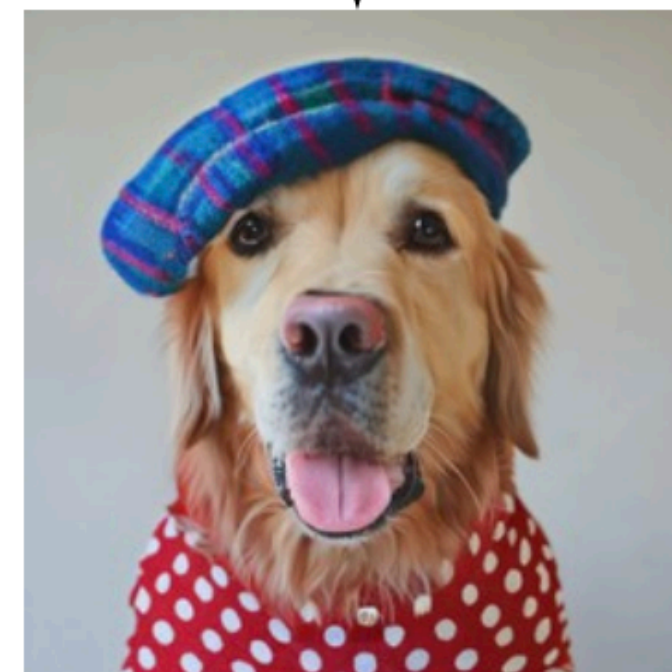
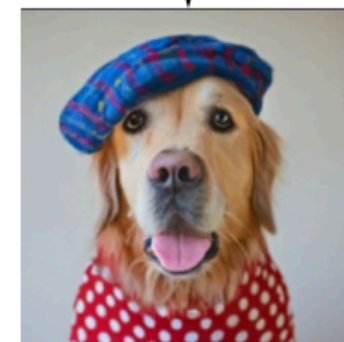
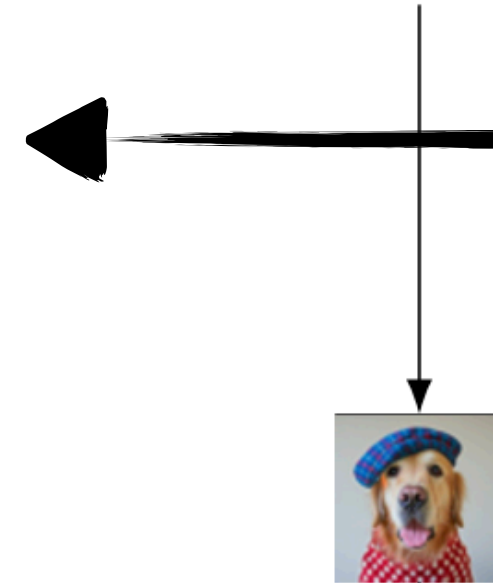


- Uses a fixed and pertained encoder

# Architecture



“A Golden Retriever dog wearing a blue checkered beret and red dotted turtleneck.”

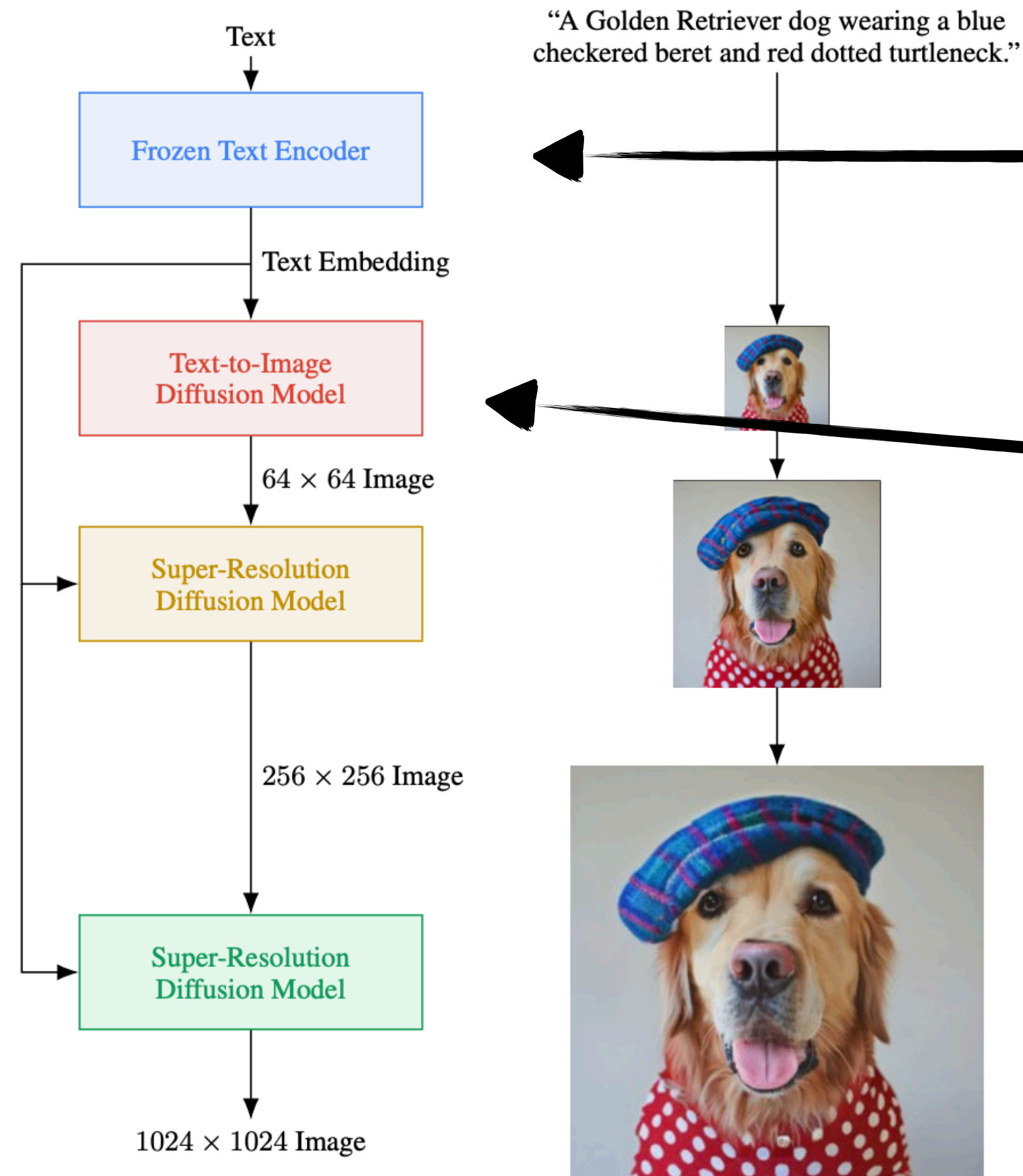


- Uses a fixed and pertained encoder

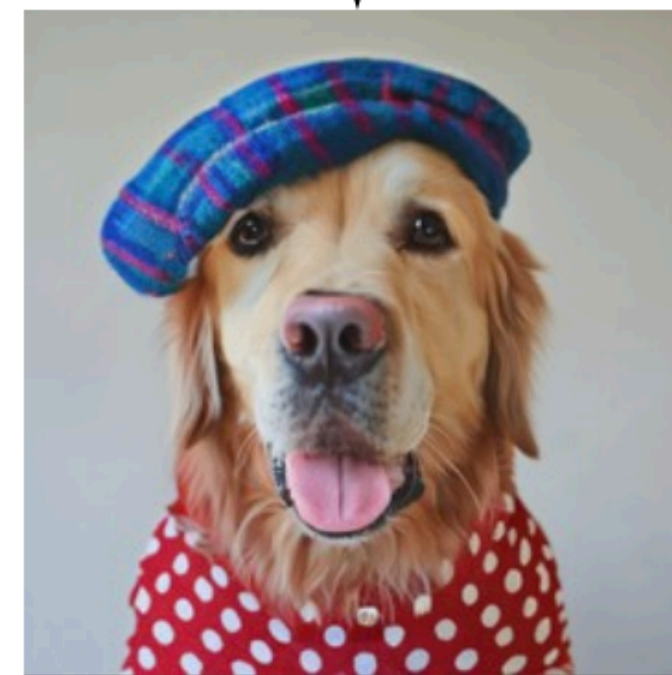
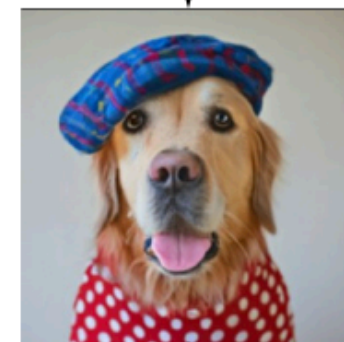
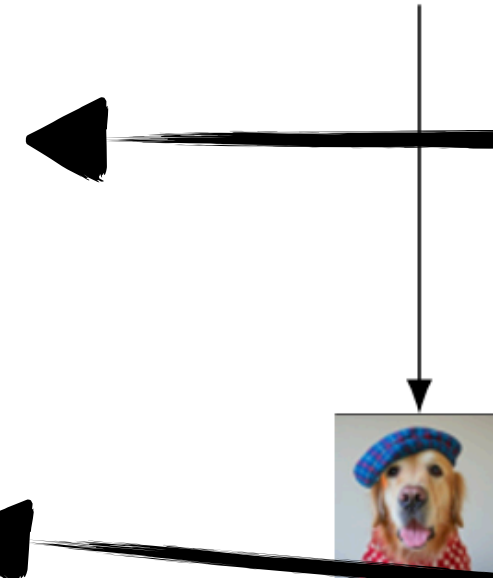
For example a transformer learned from a large-scale text dataset to predict the next word.  
**No image!**



# Architecture



“A Golden Retriever dog wearing a blue checkered beret and red dotted turtleneck.”

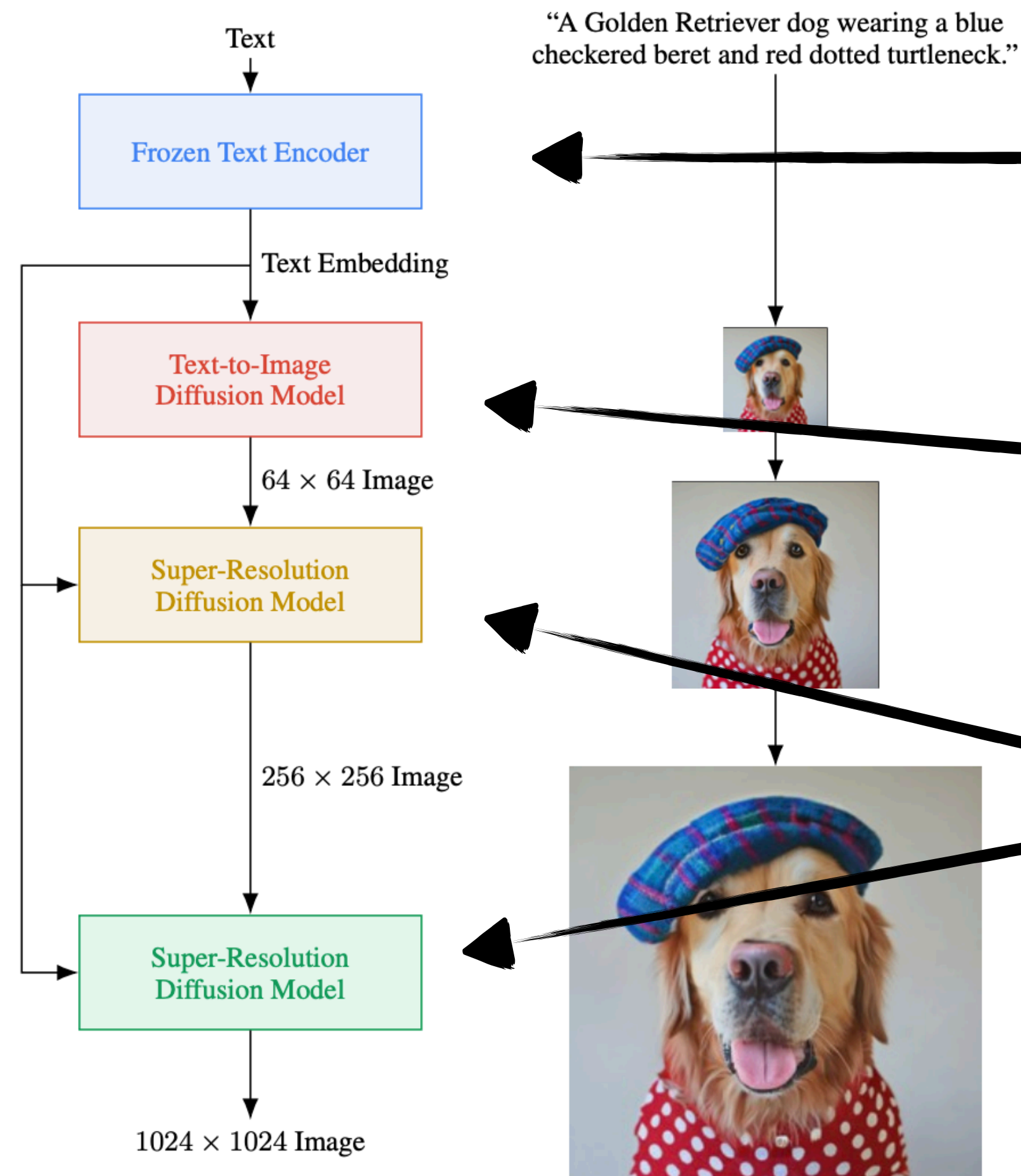


- Uses a fixed and pertained encoder

For example a transformer learned from a large-scale text dataset to predict the next word.  
**No image!**

- *Followed by a diffusion model to obtain a first image*

# Architecture



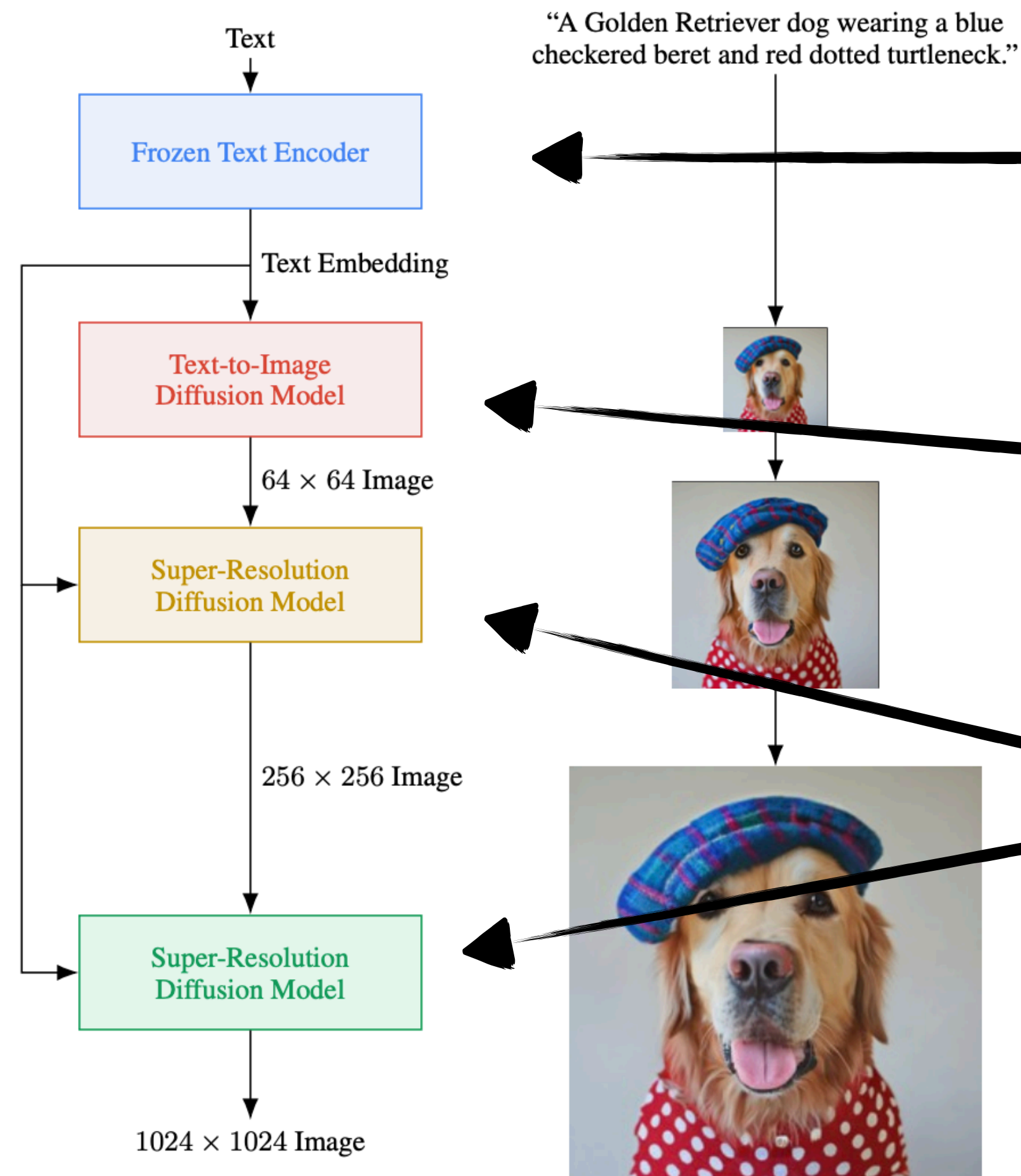
“A Golden Retriever dog wearing a blue checkered beret and red dotted turtleneck.”

- Uses a fixed and pertained encoder

For example a transformer learned from a large-scale text dataset to predict the next word.  
**No image!**

- *Followed by a diffusion model to obtain a first image*
- Followed by a few other diffusion models to obtain images of higher and higher resolution

# Architecture



“A Golden Retriever dog wearing a blue checkered beret and red dotted turtleneck.”

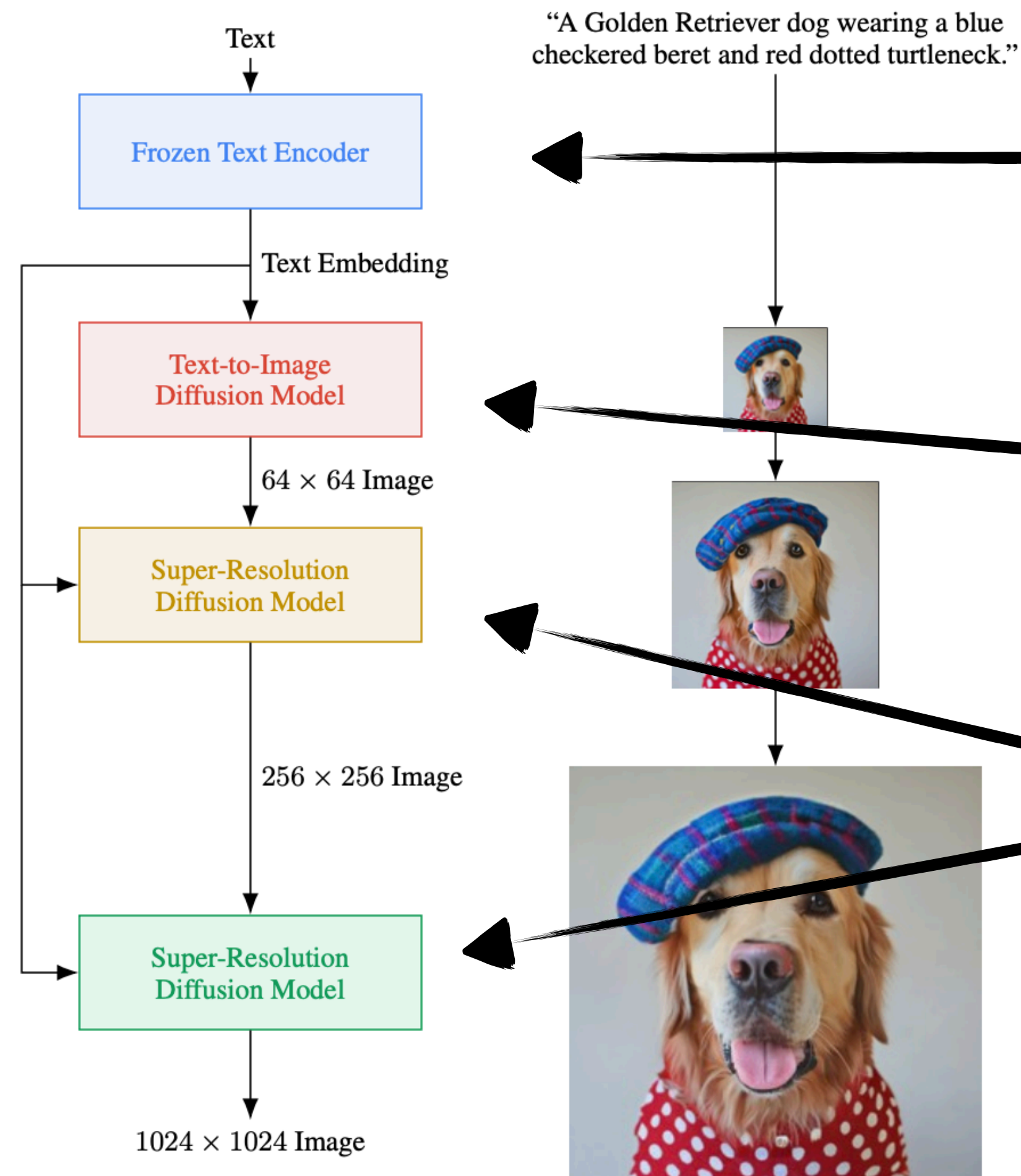
- Uses a fixed and pertained encoder

For example a transformer learned from a large-scale text dataset to predict the next word.  
**No image!**

- *Followed by a diffusion model to obtain a first image*
- Followed by a few other diffusion models to obtain images of higher and higher resolution
- The diffusion models use an attention mechanism on the text representation



# Architecture



“A Golden Retriever dog wearing a blue checkered beret and red dotted turtleneck.”

- Uses a fixed and pertained encoder

For example a transformer learned from a large-scale text dataset to predict the next word.  
**No image!**

- *Followed by a diffusion model to obtain a first image*
- Followed by a few other diffusion models to obtain images of higher and higher resolution
- The diffusion models use an attention mechanism on the text representation
- The diffusion moles are parametrized using a U-Net



# « Classifier-free » guidance

- The diffusion model is trained using two objectives
  1. Generate images from the text
  2. (Also) Generate images
    - This allows to obtain high-quality images (1) that are diversified (2)
- Imagen proposes a method to ensure pixels don't saturate during diffusion (somewhat similar problem to clipping in RNNs)

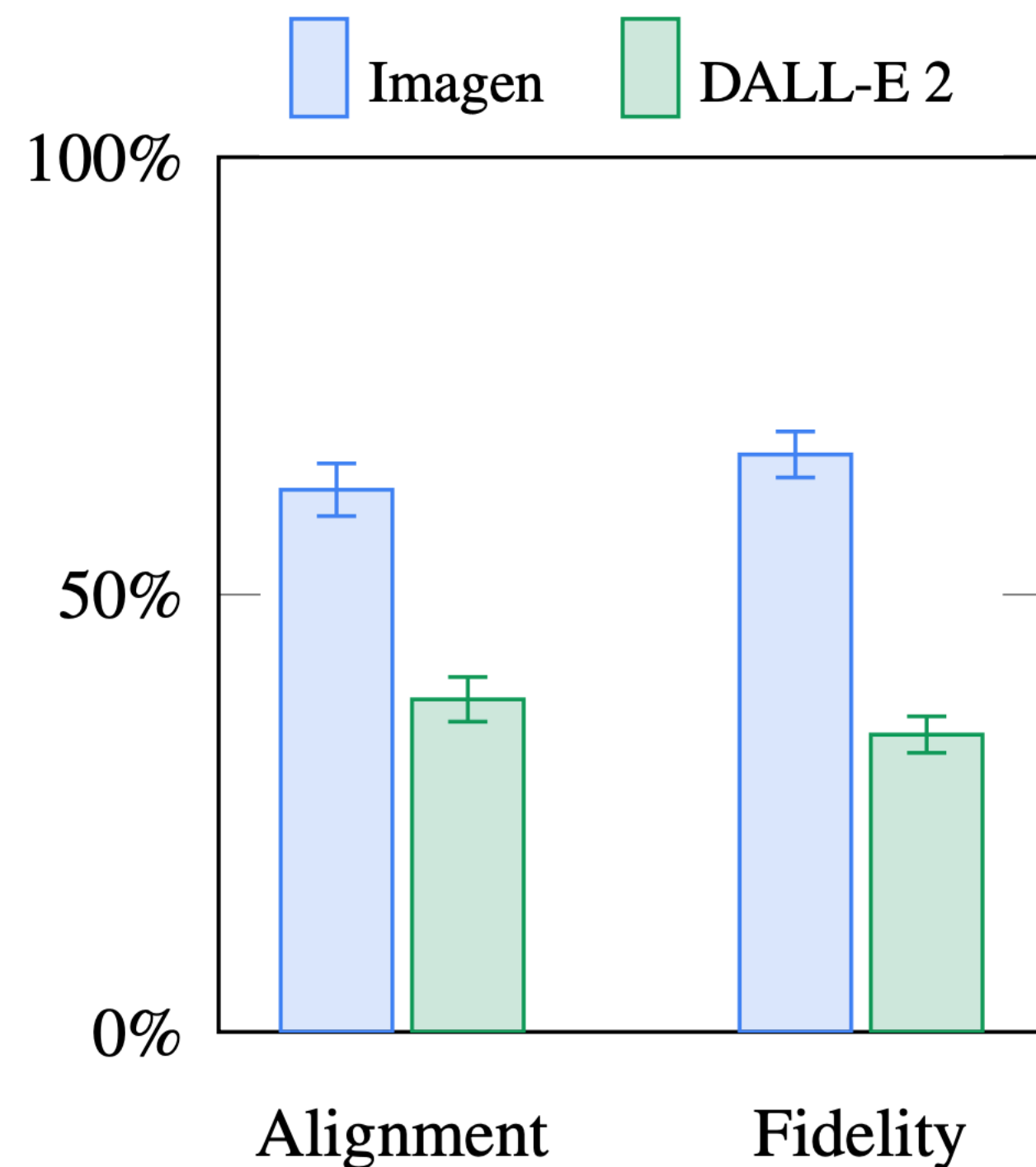
Avant Imagen



Imagen

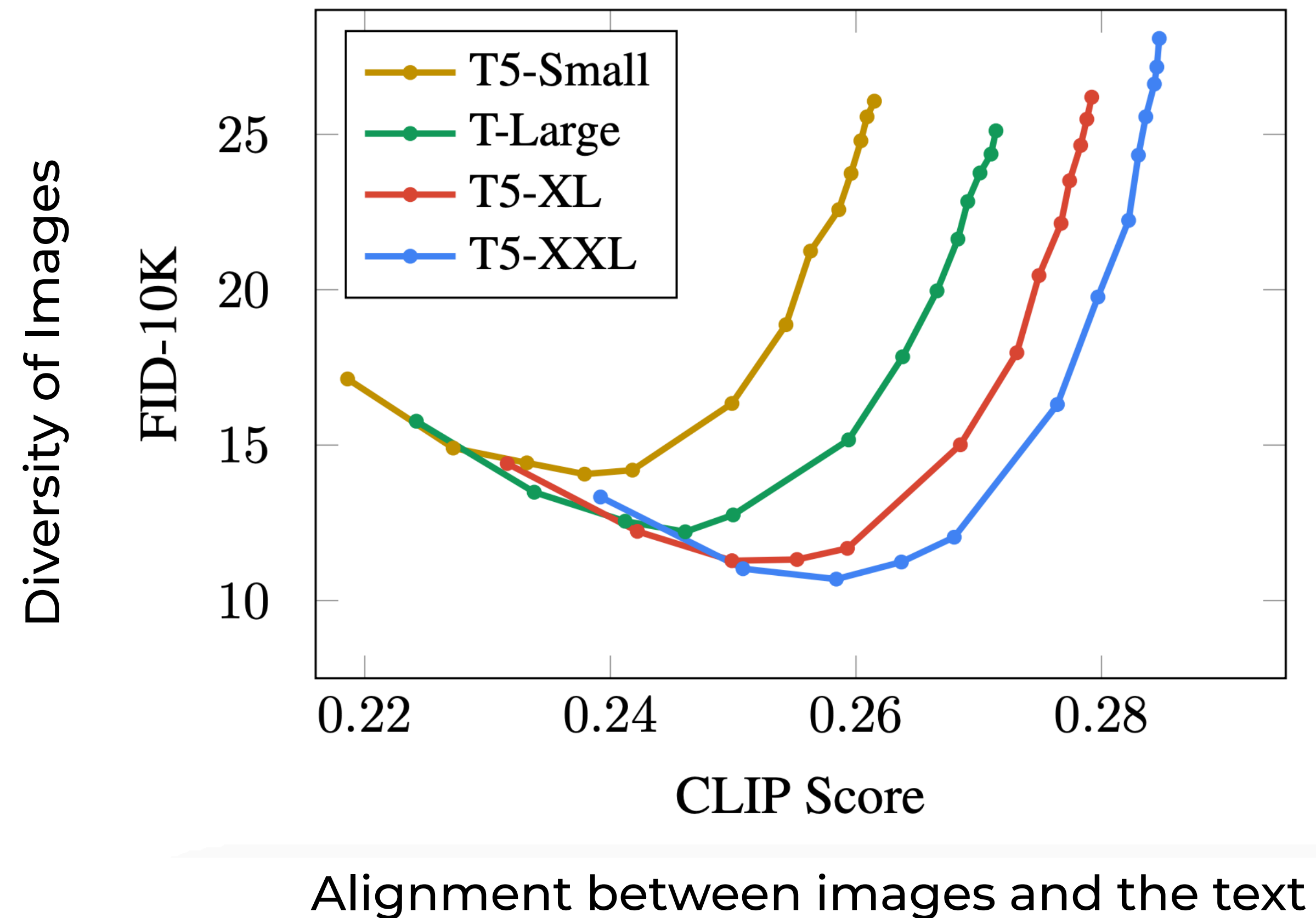


# Compared to Dall-E 2



- Better empirical performance (according to a human study)
- “Alignment” -> “Does the caption accurately describe the above image”
- “Fidelity” -> “Which image is more photorealistic”
- Users a simpler architecture (no CLIP)

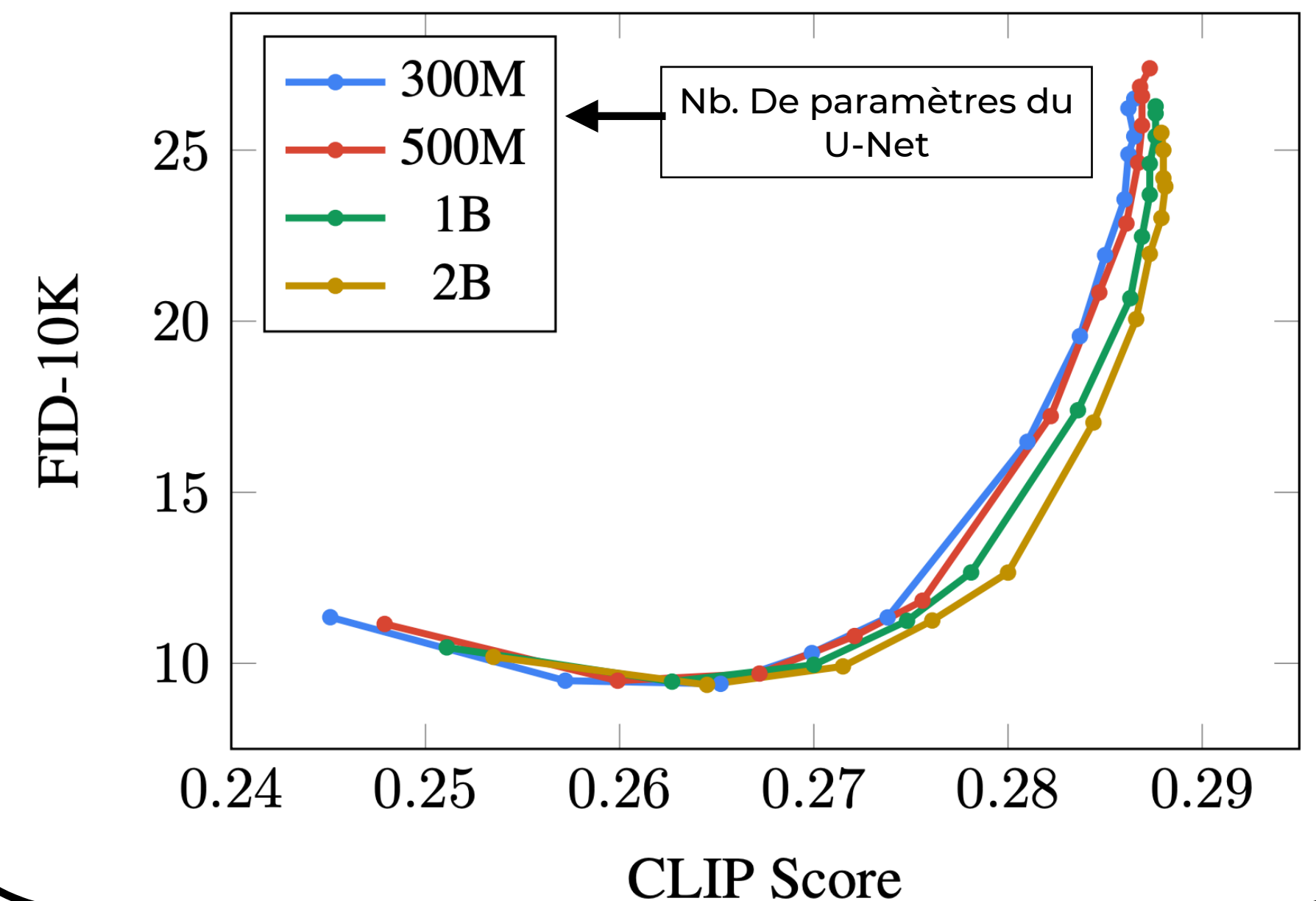
# The size of the text encoder is an important hyper-ammeter



T5 XXL (2.6B)

- Trans. Encodeur-Décodeur
- Uses only the encoder

Comparatively, the size of the image generation model is less important





# Still far from perfect...



A pear cut into seven pieces arranged in a ring.



One cat and two dogs sitting on the grass.

- E.g., operations that require counting and logic remain difficult