

Machine Learning and Data Mining

Lecture Notes

CSC C11/D11

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Conventions and Notation

Scalars are written with lower-case italics, e.g., x . Column-vectors are written in bold, lower-case: \mathbf{x} , and matrices are written in bold uppercase: \mathbf{B} .

The set of real numbers is represented by \mathbb{R} ; N -dimensional Euclidean space is written \mathbb{R}^N .

Aside:

Text in “aside” boxes provide extra background or information that you are not required to know for this course.

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