CSC 2410, Fall 2017 Assignment #1

Due: Monday Oct 9. Either submit by email before midnight, or slip it under my office door (SF3301B) before I arrive Tuesday.

You may consult the text. You may not consult any other materials. You may not search for help on the internet.

For the first three problems, you may consult with each other but you must each write your own solution. For each problem, list all students with whom you discussed the problem.

For the last three problems, **you may not consult with each other** - you can only consult with the instructor.

Of course, each problem requires a well-written proof. Proofs that are unneccessarily lengthy might not get full marks, even if they are correct. And they might not be read thoroughly by the grader.

Numbered exercises are all taken from the 2nd edition of West:

Problem 1: (30 pts) Suppose that you have a graph G with non-negative weights on its edges, and that you have already found a Minimum Spanning Tree T of G. Someone updates G by adding a new vertex to it, along with non-negative weighted edges from that vertex to some of the original vertices. Give an algorithm to find a Minimum Spanning Tree of the updated graph which is asymptotically faster than just applying Prim's algorithm to the new graph.

Anything asymptotically faster than the worst-case running time of Prim's algorithm can get partial marks. For full marks, your algorithm should run in $O(n)$ time, where n is the number of vertices.

(Of course, you must prove that your algorithm works.)

If you aren't familiar with the Minimum Spanning Tree problem, read Section 2.3 of West.

Problem 2: (25 pts) Let T be any tree, and let ϕ be any automorphism of T. Prove that ϕ must fix at least one vertex or edge. In other words, prove that there is a vertex v such that $\phi(v) = v$ or there is an edge uv such that $\phi(u) = v, \phi(v) = u$.

Problem 3: (15 pts) G is said to be *claw-free* if it has no induced subgraph that is isomorphic to $K_{1,3}$. See definitions 1.1.27, 1.1.35, 1.3.22, 1.2.12. Prove that if G is a claw-free graph with an even number of vertices, then G has a perfect matching.

Note: Exercise 3.3.23 describes one approach to proving this. Alternatively, you can use Tutte's Theorem.

(15 pts) 5.2.25 (a,b,c) See Definition 1.1.27. Note that $K_{2,2}$ is the 4-cycle.

(25 pts) 3.1.37 (see definitions 1.2.17 and 3.1.1)

(25 pts) 3.3.29 (see definition 3.3.11)

Note: There is a typo in part (c). It should begin with "Conclude that $d_1 \geq ... \geq d_n \geq 0$ are the . . . "