CSCC63 Assignment #2, 2018

Due: Fri Feb 16 at 11:00 PM

You may work with a partner. **You must write your own solution. You may not look at your partner's written solution.** You can only have one partner. Neither you nor your partner can discuss this assignment with anyone else other than the course instructor and the TA. If you work with a partner then you must name your partner on your assignment.

For problems 1 - 6, answer: (i) is the set decidable? (ii) is the set recognizable? You must prove your answers.

The variables P, Q represent algorithms (formally: Turing Machines with outputs) which have positive integers as input and output (formally: the input and output are binary strings which are interpreted as integers). The variable R represents an algorithm that has a pair of positive integers as input and a single positive integer as output. The variables x, y, k, ℓ represent positive integers.

(20 marks each)

1. $\{< P >, < Q >, k|$ there are at least k natural numbers x for which $P(x), Q(x)$ both halt$\}$.
2. $\{< P >, k|$ there are at most k natural numbers x for which $P(x)$ outputs 7 $\}$.
3. $\{< R > |$ for every input (x, y), $R(x, y)$ outputs $x + y + 25\}$.
4. $\{x, k|$ there are at least k Mersenne primes less than $2x\}$.
5. $\{< P >, \ell|$ there is some $x > y \geq \ell$ such that $P(x), P(y)$ both halt and their outputs sum to $x - y + 10\}$.
6. $\{< P >, < Q >, x|P(x)$ runs for at least as many steps as $Q(x)\}$.

   (If $P(x)$ loops forever, then we consider it to run for ∞ steps. So if $P(x), Q(x)$ both loop forever then (P, Q, x) belong to the set. But if $P(x)$ halts and $Q(x)$ loops forever, then (P, Q, x) do not belong to the set.)

7. (25 marks) When A, B are languages, $(AB)^*$ is defined to be the set of strings of the form $a_1 b_1 a_2 b_2 ... a_t b_t$ where each a_i is in A and each b_i is in B. For example, if $A = \{xxx, zyy\}, B = \{yx, xzz\}$ then $zyyyxxxxxzz \in (AB)^*$ as it can be split into the strings zyy, yx, xxx, xzz, but $zyyx \notin (AB)^*$ and $xxxyxxxxzz \notin (AB)^*$.

   (a) Suppose that A, B are decidable languages. Prove that $(AB)^*$ is decidable.

   (b) Suppose that A, B are recognizable languages. Prove that $(AB)^*$ is recognizable.