CSC C63 Midterm Exam

Feb 29, 2016

1 hour and 50 minutes

NAME:

Calculators are not permitted (nor would they be useful).

This is a closed book exam.

Ask an invigilator if there is anything that you do not understand completely.

1	/9	4	/12
2	/12	5	/12
3	/12		
			/57

1. **(9 pts) Short Answers.**

   (a) **(3 pts)** Define the set P.

   (b) **(3 pts)** Define an enumerator.

   (c) **(3 pts)** Explain why the set of all Turing machines is countable.

2. **(12 pts)** For each of the following languages, say whether it is decidable and whether it is recognizable. For example, for the language HALT, the correct answer would be "not decidable; recognizable".

   You do not need to prove your answer.

   In each language, P is a Turing machine and k, x, y are positive integers.

   (a) $\{x, y | y = x^2\}$

   (b) $\{< P >, x | P(x) \text{ returns } x^2\}$

   (c) $\{< P > | P(x) \text{ returns } x^2 \text{ for every } x\}$

   (d) $\{< P >, k | P(x) \text{ returns } x^2 \text{ for fewer than } k \text{ values of } x\}$

3. **(12 pts)** In the following language, P is a Turing Machine that takes positive integers as input; k, x, y are positive integers.

$A = \{(< P >, x, k)$: there are at least $2k$ integers y such that
$P(x)$ and $P(y)$ both halt and return the same output $\}$

Prove that A is recognizable.

4. **(12 pts)** Recall that an independent set in a graph is a subset S of the vertices such that no two vertices in S are joined by an edge.

   Consider the following problems.

   WEIGHTED-INDEPENDENT-SET
   Input: A connected graph G, with an integer weight $w_v \geq 0$ on each vertex v.
   Output: The total weight of an independent set in G with the greatest total weight.

   DECISION-WEIGHTED-INDEPENDENT-SET
   Input: A connected graph G, with an integer weight $w_v \geq 0$ on each vertex v, and an integer T.
   Question: Is there an independent set in G of total weight at least T?

   We will abbreviate the names of these two problems: WIS and DWIS.

   Prove that if there is a polytime algorithm for DWIS then there is a polytime algorithm for WIS.

5. **(12 pts)** A is a subset of the positive integers. Prove that A is decidable iff there is an enumerator which can list A *in increasing order and with no repeats.*

   Eg. if A is the set of squares, then the enumerator must provide the listing

   $$A = 1, 4, 9, 16, 25, 36, ...$$

   In other words, i^2 is the ith number listed by the enumerator.