CSC304 Lecture 2

Game Theory
(Basic Concepts)

Game Theory

- How do rational, self-interested agents act?
- Each agent has a set of possible actions
- Rules of the game:
  - Rewards for the agents as a function of the actions taken by different agents
- We focus on noncooperative games
  - No external force or agencies enforcing coalitions

Normal Form Games

- A set of players $\mathrm{N} = \{1, \dots, n\}$
- A set of actions S
  - Action of player $i \rightarrow s_i$
  - Action profile $\vec{s} = (s_1, \dots, s_n)$

- For each player i, utility function $u_i : S^n \rightarrow \mathbb{R}$
  - Given action profile $\vec{s} = (s_1, \dots, s_n)$, each player i gets reward $u_i(s_1, \dots, s_n)$

Normal Form Games

Recall: Prisoner's dilemma $S = \{\text{Silent}, \text{Betray}\}$

John's Actions / Sam's Actions	Stay Silent	Betray
Stay Silent	(-1 , -1)	(-3 , 0)
Betray	(0 , -3)	(-2 , -2)

$\boxed{u_{Sam}(Betray, Silent)}$ $\boxed{u_{John}(Betray, Silent)}$

s_{Sam} s_{John}

Player Strategies

- Pure strategy
  - Choose an action to play
  - E.g., “Betray”
  - For our purposes, simply an action.
    - In repeated or multi-move games (like Chess), need to choose an action to play at every step of the game based on history.

- Mixed strategy
  - Choose a probability distribution over actions
  - Randomize over pure strategies
  - E.g., “Betray with probability 0.3, and stay silent with probability 0.7”

Domination among Strategies

- s_i dominates s_i' if player i is always "better off" playing s_i than s_i', regardless of the strategies of other players.

- Two variants: weak and strict domination
  - $u_i(s_i, \vec{s}_{-i}) \geq u_i(s_i', \vec{s}_{-i}), \forall \vec{s}_{-i}$ (needed for both)
  - Strict inequality for some $\vec{s}_{-i}$ $\leftarrow$ *s_i weakly dominates s_i'*
  - Strict inequality for all $\vec{s}_{-i}$ $\leftarrow$ *s_i strictly dominates s_i'*

Example

P1 \ P2	b_1	b_2
a_1	(2 , 3)	(4 , 1)
a_2	(2 , 5)	(6 , 3)
a_3	(3 , 1)	(5 , 2)

- P1
  - a_1 vs a_2 ?
  - a_1 vs a_3 ?
  - a_2 vs a_3 ?
- P2
  - b_1 vs b_2 ?

Dominant Strategies

- s_i is a strictly (weakly) dominant strategy for player i if it strictly (weakly) dominates every other strategy

- Strict dominance is a strong concept
  - A player who has a strictly dominant strategy has no reason *not* to play it
  - If every player has a strictly dominant strategy, such strategies will very likely dictate the outcome of the game

Example

P1 \ P2	b_1	b_2
a_1	**(2 , 3)**	**(4 , 1)**
a_2	**(2 , 5)**	**(6 , 3)**
a_3	**(3 , 1)**	**(5 , 2)**

- Does either player have a dominant strategy?

Example

P2 / P1	b_1	b_2	b_3
a_1	(2 , 3)	(4 , 1)	(2 , 3)
a_2	(2 , 5)	(6 , 3)	(3 , 5)
a_3	(3 , 1)	(5 , 2)	(4 , 3)

- How about now?

Example

P1 \ P2	b_1	b_2	b_3
a_1	(2 , 3)	(4 , 1)	(2 , 4)
a_2	(2 , 5)	(6 , 3)	(3 , 6)
a_3	(3 , 1)	(5 , 2)	(4 , 3)

- How about now?

Example: Prisoner’s Dilemma

• Recap:

John’s Actions / Sam’s Actions	Stay Silent	Betray
Stay Silent	**(-1 , -1)**	**(-3 , 0)**
Betray	**(0 , -3)**	**(-2 , -2)**

• Betraying is a strictly dominant strategy for each player

Iterated Elimination

- What if there are no dominant strategies?
  - No single strategy dominates every other strategy
  - But some strategies might still be dominated

- Assuming everyone knows everyone is rational…
  - Can remove their dominated strategies
  - Might reveal a newly dominant strategy

- Two variants depending on what we eliminate:
  - Only strictly dominated? Or also weakly dominated?

Iterated Elimination

- Toy example:
  - Microsoft vs Startup
  - Enter the market or stay out?

Microsoft \ Startup	Enter	Stay Out
Enter	(2 , -2)	(4 , 0)
Stay Out	(0 , 4)	(0 , 0)

- Q: Is there a dominant strategy for startup?
- Q: Do you see a rational outcome of the game?

Iterated Elimination

- More serious: "Guess 2/3 of average"
  - Each student guesses a real number between 0 and 100 (inclusive)
  - The student whose number is the closest to 2/3 of the average of all numbers wins!

- In-class poll!

- Recall: We have a unique optimal strategy only if everyone is rational, and everyone thinks everyone is rational, and so on.

Nash Equilibrium

- What if we don't find a unique outcome after iterated elimination of dominated strategies?

Students \ Professor	Attend	Be Absent
Attend	**(3 , 1)**	**(-1 , -3)**
Be Absent	**(-1 , -1)**	**(0 , 0)**

Nash Equilibrium

- Nash Equilibrium
  - A strategy profile $\vec{s}$ is in Nash equilibrium if s_i is the best action for player i given that other players are playing $\vec{s}_{-i}$

$$u_i(s_i, \vec{s}_{-i}) \geq u_i(s_i', \vec{s}_{-i}), \forall s_i'$$

No quantifier on $\vec{s}_{-i}$

  - Each player's strategy is only best *given* the strategies of others, and not *regardless*.

Recap: Prisoner's Dilemma

John's Actions / Sam's Actions	Stay Silent	Betray
Stay Silent	(-1 , -1)	(-3 , 0)
Betray	(0 , -3)	(-2 , -2)

- Nash equilibrium?

- Food for thought:
  - What is the relation between iterated elimination of weakly/strictly dominated strategies and Nash equilibria?

Recap: Microsoft vs Startup

Microsoft \ Startup	Enter	Stay Out
Enter	(2 , -2)	(4 , 0)
Stay Out	(0 , 4)	(0 , 0)

- Nash equilibrium?

Recap: Attend or Not

Students \ Professor	Attend	Be Absent
Attend	**(3 , 1)**	**(-1 , -3)**
Be Absent	**(-1 , -1)**	**(0 , 0)**

- Nash equilibrium?

Example: Stag Hunt

Hunter 1 / Hunter 2	Stag	Hare
Stag	(4 , 4)	(0 , 2)
Hare	(2 , 0)	(1 , 1)

- Game:
  - Each hunter decides to hunt stag or hare
  - Stag = 8 days of food, hare = 2 days of food
  - Catching stag requires both hunters, catching hare requires only one
  - If they catch one animal together, they share
- Nash equilibrium?