CSC304 Lecture 11

Mechanism Design w/ Money:
Revelation principle; First price, second price, and ascending auctions; Revenue equivalence
Recap: Bayesian Framework

All distributions known to all agents

Private value of $i$ only known to $i$

All strategies known to all agents

All distributions known to all agents
Recap: Bayesian Framework

• Strategy profile $\tilde{s} = (s_1, \ldots, s_n)$

  ➢ Interim utility of agent $i$ is

  \[
  E_{\{v_j \sim D_j\}_{j \neq i}} [u_i(s_1(v_1), \ldots, s_n(v_n))]
  \]

  where utility $u_i$ is “value derived – payment charged”

  ➢ $\tilde{s}$ is a Bayes-Nash equilibrium (BNE) if $s_i$ is the best strategy for agent $i$ given $\tilde{s}_{-i}$ (strategies of others)

    o NOTE: I don’t know what others’ values are. But I know they are rational players, so I can reason about what strategies they might use.
Recap: 1\textsuperscript{st} Price Auction

• Sealed-bid first price auction for a single item
  ➢ Each agent $i$ privately submits a bid $b_i$
  ➢ Agent $i^*$ with the highest bid wins the item, pays $b_{i^*}$

• Suppose there are two agents
  ➢ Common prior: each has valuation drawn from $U[0,1]$

• Claim: Both players using $s_i(v_i) = v_i/2$ is a BNE.
  ➢ Proof on the board.
Direct Revelation Mechanisms & The Revelation Principle
Direct Revelation

• **Direct-revelation**: mechanisms that ask you to report your private values
  - Doesn’t mean agents will report their true values.
  - Makes sense to ask “Would they, in equilibrium?”

• **Non-direct-revelation**: different action space than type space
  - Suppose your value for an item is in [0,1], but the mechanism asks you to either dive left or dive right.
  - Strategy $s_i: [0,1] \rightarrow \{left, right\}$
  - Truthfulness doesn’t make much sense.
  - But we can still ask: What is the outcome in equilibrium?
BNIC Mechanisms

• A direct revelation mechanism is Bayes-Nash incentive compatible (BNIC) if all players playing $s_i(v_i) = v_i$ is a BNE.
  - I don’t know what other’s valuations are, only the distributions they’re drawn from.
  - But as long as they report their true values, in expectation I would like to report my true value.

• Compare to strategyproofness
  - I know what others’ values are, and for every possible values they can have, I want to report my true values.
Revelation Principle

• Outcome = (allocation, payments)

• **Strategyproof version** [Gibbard, ‘73]
  - If a mechanism implements an outcome in dominant strategies, there’s a direct revelation strategyproof mechanism implementing the same outcome.

• **BNIC version** [Dasgupta et al. ‘79, Holmstrom ‘77, Myerson ‘79]
  - If a mechanism implements an outcome as BNE, there’s a direct revelation BNIC mechanism implementing the same outcome.
Revelation Principle

• Informal proof:

Player 1 : $v_1$

\[ \vdots \]

Player $n : v_n$

Strategy $s_1$

\[ \vdots \]

Strategy $s_n$

Original Mechanism

Outcome

New direct revelation truthful mechanism
Applying Revelation Principle

• We already saw...
  ➢ Sealed-bid 1\textsuperscript{st} price auction
  ➢ 2 agents with valuations drawn from $U[0,1]$
  ➢ Each player halving his value was a BNE
  ➢ Not naturally BNIC (players don’t report value)

• Q: What is the BNIC variant of sealed-bid 1\textsuperscript{st} price auction that we get using the revelation principle?

• Can also be used on non-direct-revelation mechs
Revenue of Auction Mechanisms & Revenue Equivalence
1\textsuperscript{st} Price Auction

• For \( n \) players with iid valuations from \( U[0,1] \), “shadowing” the bid by a factor of \( (n - 1)/n \) is a BNE

• \( E[\text{Revenue}] \) to the auctioneer?

  \[ E_{\{v_i \sim U[0,1]\}_{i=1}^n} \left( \frac{n-1}{n} \right) \cdot \max_i v_i = \frac{n-1}{n+1} \]  
  (Exercise!)

• Interestingly, this is equal to \( E[\text{Revenue}] \) from 2\textsuperscript{nd} price auction

  \[ E_{\{v_i \sim U[0,1]\}_{i=1}^n} \left[ 2\text{nd highest } v_i \right] = \frac{n-1}{n+1} \]  
  (Exercise!)
Revenue Equivalence

• If two BNIC mechanisms A and B:
  1. Always produce the same allocation;
  2. Have the same expected payment to agent $i$ for some type $v_i^0$ (e.g., “zero value for all” → zero payment);
  3. Have agent valuations drawn from distributions with “path-connected support sets”;

• Then they:
  ø Charge the same expected payment to all agent types;
  ø Have the same expected total revenue.
Revenue Equivalence

• Informally...
  ➢ If two BNIC mechanisms always have the same allocation, then they have the same $E[payments]$ and $E[revenue]$.
  ➢ Very powerful as it applies to any pair of BNIC mechanism

• 1\textsuperscript{st} price (BNIC variant) and 2\textsuperscript{nd} price auctions
  ➢ Have the same allocation:
    Item always goes to the agent with the highest valuation
  ➢ Thus, also have the same revenue
Non-Direct-Revelation Auctions

• ascending auction (a.k.a. English auction)
  ➢ all agents + auctioneer meet in a room.
  ➢ auctioneer starts the price at 0.
  ➢ all agents want the item, and have their hands raised.
  ➢ auctioneer raise the price continuously.
  ➢ agents drop out when price > value for them

• descending auction (a.k.a. Dutch auction)
  ➢ start price at a very high value.
  ➢ keep decreasing the price until some agent agrees to buy.