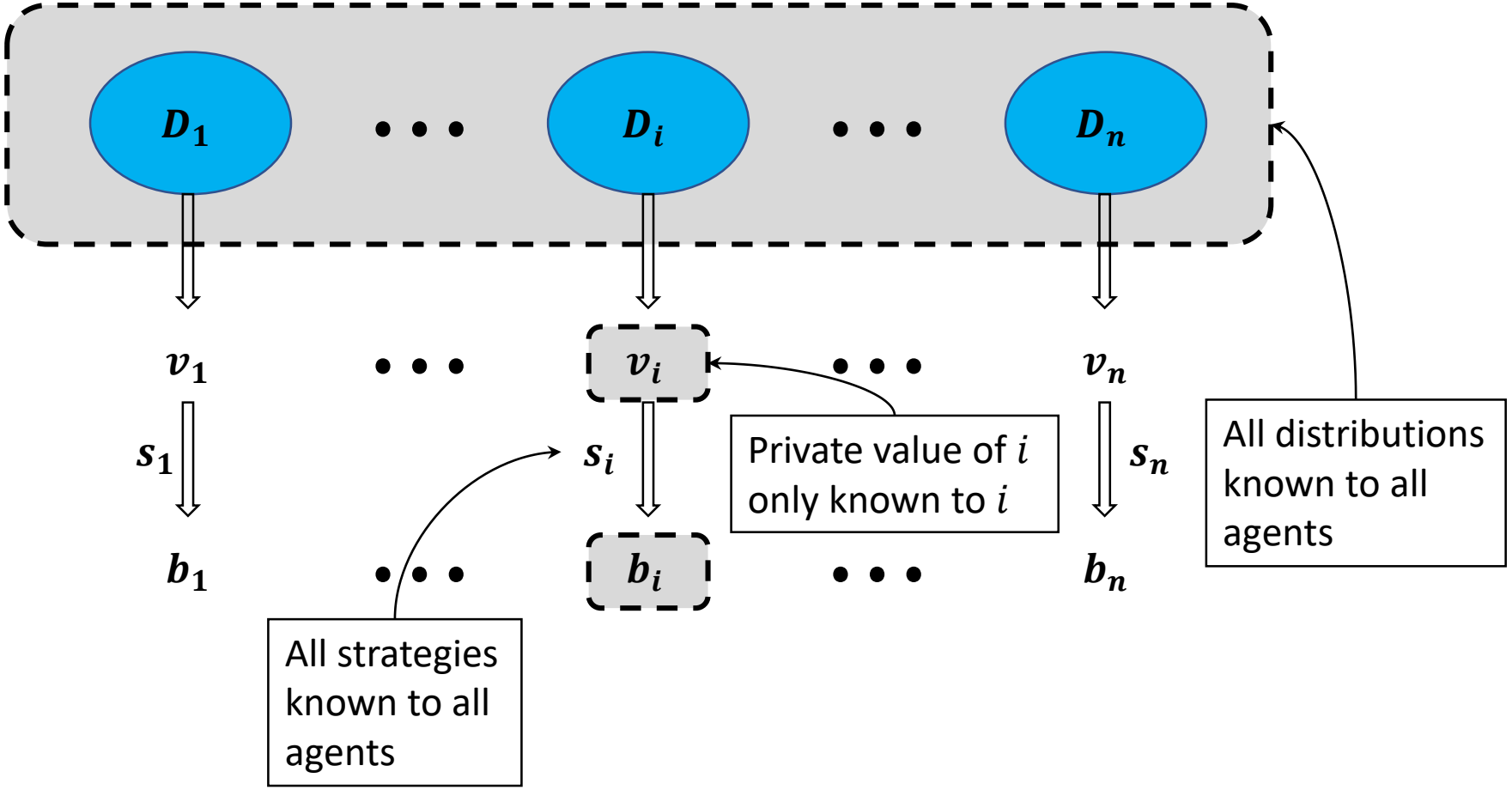
CSC304 Lecture 11

Mechanism Design w/ Money:
Revelation principle; First price, second price, and ascending auctions; Revenue equivalence

Recap: Bayesian Framework



Recap: Bayesian Framework

- Strategy profile $\vec{s} = (s_1, \dots, s_n)$
  - Interim utility of agent i is

$$E_{\{v_j \sim D_j\}_{j \neq i}}\left[u_i\left(s_1(v_1), \dots, s_n(v_n)\right)\right]$$

where utility u_i is "value derived – payment charged"

  - $\vec{s}$ is a Bayes-Nash equilibrium (BNE) if s_i is the best strategy for agent i given $\vec{s}_{-i}$ (strategies of others)
    - NOTE: I don't know what others' values are. But I know they are rational players, so I can reason about what strategies they might use.

Recap: 1st Price Auction

- Sealed-bid first price auction for a single item
  - Each agent i privately submits a bid b_i
  - Agent i^* with the highest bid wins the item, pays b_{i^*}

- Suppose there are two agents
  - Common prior: each has valuation drawn from $U[0,1]$

- Claim: Both players using $s_i(v_i) = v_i/2$ is a BNE.
  - Proof on the board.

Direct Revelation Mechanisms
&
The Revelation Principle

Direct Revelation

- Direct-revelation: mechanisms that ask you to report your private values
  - Doesn't mean agents will report their true values.
  - Makes sense to ask "Would they, in equilibrium?"

- Non-direct-revelation: different action space than type space
  - Suppose your value for an item is in [0,1], but the mechanism asks you to either dive left or dive right.
  - Strategy $s_i: [0,1] \rightarrow \{left, right\}$
  - Truthfulness doesn't make much sense.
  - But we can still ask: What is the outcome in equilibrium?

BNIC Mechanisms

- A direct revelation mechanism is Bayes-Nash incentive compatible (BNIC) if all players playing $s_i(v_i) = v_i$ is a BNE.
  - I don't know what other's valuations are, only the distributions they're drawn from.
  - But as long as they report their true values, in expectation I would like to report my true value.

- Compare to strategyproofness
  - I know what others' values are, and for every possible values they can have, I want to report my true values.

Revelation Principle

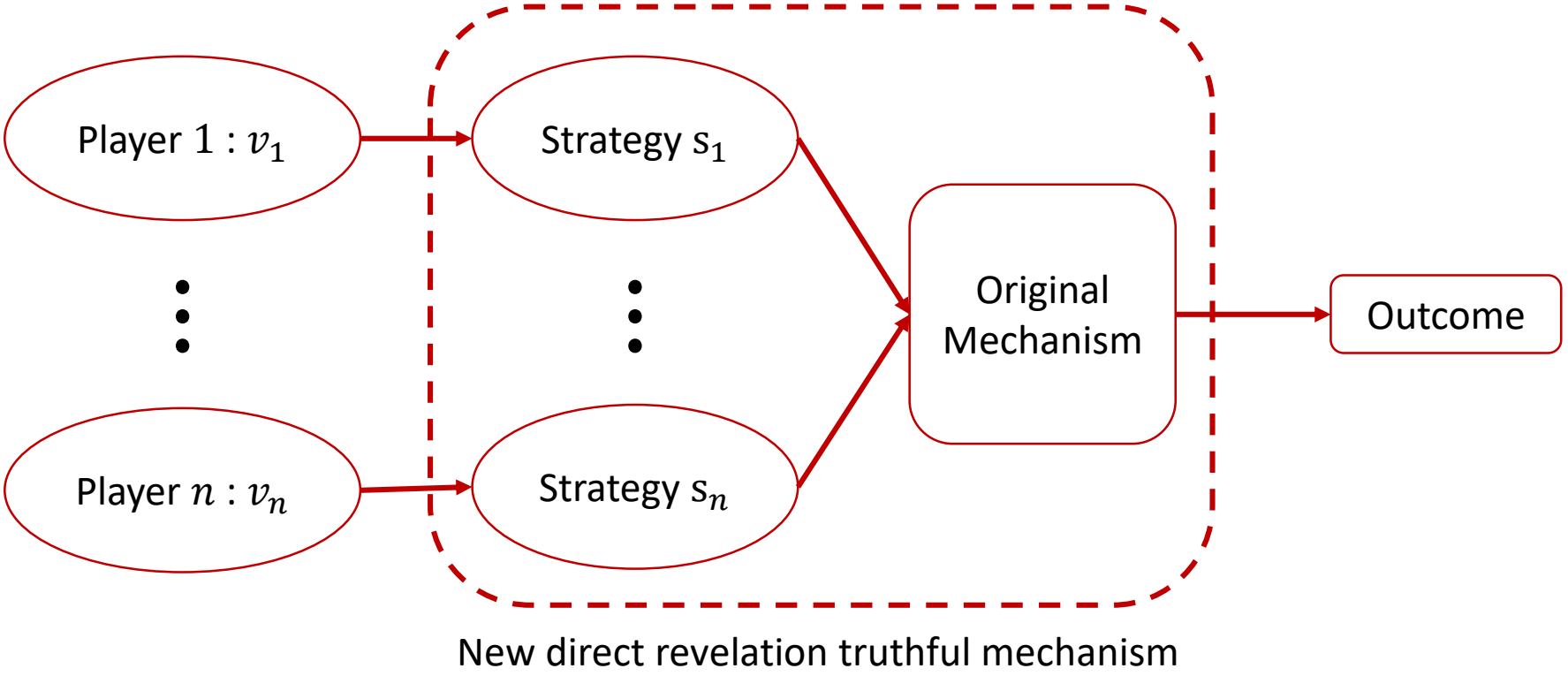
- Outcome = (allocation, payments)

- Strategyproof version [Gibbard, '73]
  - If a mechanism implements an outcome in dominant strategies, there's a direct revelation strategyproof mechanism implementing the same outcome.

- BNIC version [Dasgupta et al. '79, Holmstrom '77, Myerson '79]
  - If a mechanism implements an outcome as BNE, there's a direct revelation BNIC mechanism implementing the same outcome.

Revelation Principle

- Informal proof:



Applying Revelation Principle

- We already saw…
  - Sealed-bid 1st price auction
  - 2 agents with valuations drawn from $U[0,1]$
  - Each player halving his value was a BNE
  - Not naturally BNIC (players don't report value)

- Q: What is the BNIC variant of sealed-bid 1st price auction that we get using the revelation principle?

- Can also be used on non-direct-revelation mechs

Revenue of Auction Mechanisms
&
Revenue Equivalence

1^{st} Price Auction

- For n players with iid valuations from U[0,1], "shadowing" the bid by a factor of $(n-1)/n$ is a BNE

- $E[\text{Revenue}]$ to the auctioneer?
  - $E_{\{v_i \sim U[0,1]\}_{i=1}^{n}} \left(\frac{n-1}{n}\right) * \max_i v_i = \frac{n-1}{n+1}$ (Exercise!)

- Interestingly, this is equal to E[Revenue] from 2^{nd} price auction
  - $E_{\{v_i \sim U[0,1]\}_{i=1}^{n}} [2^{\text{nd}} \text{ highest } v_i] = \frac{n-1}{n+1}$ (Exercise!)

Revenue Equivalence

- If two BNIC mechanisms A and B:
  1. Always produce the same allocation;
  2. Have the same expected payment to agent i for some type v_i^0 (e.g., "zero value for all" → zero payment);
  3. Have agent valuations drawn from distributions with "path-connected support sets";
- Then they:
  - Charge the same expected payment to all agent types;
  - Have the same expected total revenue.

Revenue Equivalence

- Informally…
  - If two BNIC mechanisms always have the same allocation, then they have the same E[payments] and E[revenue].
  - Very powerful as it applies to any pair of BNIC mechanism

- 1st price (BNIC variant) and 2nd price auctions
  - Have the same allocation:
    Item always goes to the agent with the highest valuation
  - Thus, also have the same revenue

Non-Direct-Revelation Auctions

- Ascending auction (a.k.a. English auction)
  - All agents + auctioneer meet in a room.
  - Auctioneer starts the price at 0.
  - All agents want the item, and have their hands raised.
  - Auctioneer raise the price continuously.
  - Agents drop out when price > value for them

- Descending auction (a.k.a. Dutch auction)
  - Start price at a very high value.
  - Keep decreasing the price until some agent agrees to buy.