

CSC304 Lecture 2

Game Theory (Basic Concepts)

Game Theory

- How do rational, self-interested agents act?
- Each agent has a set of possible actions
- Rules of the game:
 - Rewards for the agents as a function of the actions taken by different agents
- We focus on noncooperative games
 - No external force or agencies enforcing coalitions

Normal Form Games

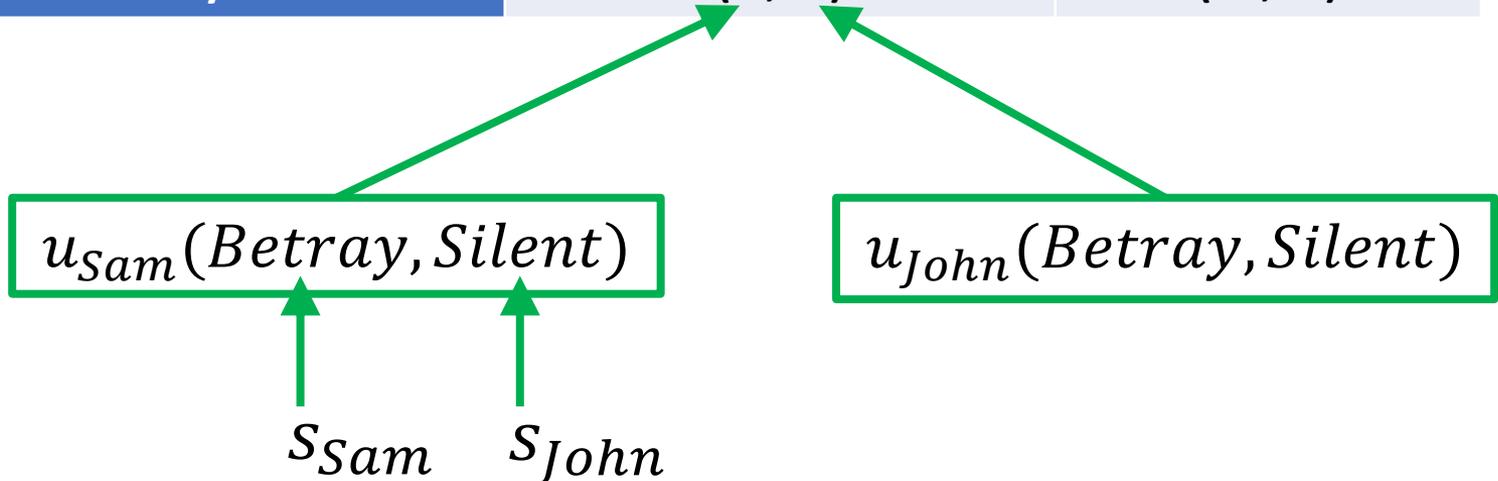
- A set of players $N = \{1, \dots, n\}$
- A set of actions S
 - Action of player $i \rightarrow s_i$
 - Action profile $\vec{s} = (s_1, \dots, s_n)$
- For each player i , utility function $u_i: S^n \rightarrow \mathbb{R}$
 - Given action profile $\vec{s} = (s_1, \dots, s_n)$, each player i gets reward $u_i(s_1, \dots, s_n)$

Normal Form Games

Recall: Prisoner's dilemma

$$S = \{\text{Silent}, \text{Betray}\}$$

		John's Actions	
		Stay Silent	Betray
Sam's Actions	Stay Silent	$(-1, -1)$	$(-3, 0)$
	Betray	$(0, -3)$	$(-2, -2)$



Player Strategies

- Pure strategy
 - Choose an action to play
 - E.g., “Betray”
 - For our purposes, simply an action.
 - In repeated or multi-move games (like Chess), need to choose an action to play at every step of the game based on history.
- Mixed strategy
 - Choose a probability distribution over actions
 - Randomize over pure strategies
 - E.g., “Betray with probability 0.3, and stay silent with probability 0.7”

Domination among Strategies

- s_i dominates s'_i if player i is always “better off” playing s_i than s'_i , regardless of the strategies of other players.
- Two variants: weak and strict domination
 - $u_i(s_i, \vec{s}_{-i}) \geq u_i(s'_i, \vec{s}_{-i}), \forall \vec{s}_{-i}$ (needed for both)
 - Strict inequality for **some** \vec{s}_{-i} ← s_i weakly dominates s'_i
 - Strict inequality for **all** \vec{s}_{-i} ← s_i strictly dominates s'_i

Example

P1 \ P2	b_1	b_2
a_1	(2, 3)	(4, 1)
a_2	(2, 5)	(6, 3)
a_3	(3, 1)	(5, 2)

- P1
 - a_1 vs a_2 ?
 - a_1 vs a_3 ?
 - a_2 vs a_3 ?
- P2
 - b_1 vs b_2 ?

Dominant Strategies

- s_i is a strictly (weakly) dominant strategy for player i if it strictly (weakly) dominates **every other strategy**
- Strict dominance is a strong concept
 - A player who has a strictly dominant strategy has no reason *not* to play it
 - If every player has a strictly dominant strategy, such strategies will very likely dictate the outcome of the game

Example

P1 \ P2	b_1	b_2
a_1	(2, 3)	(4, 1)
a_2	(2, 5)	(6, 3)
a_3	(3, 1)	(5, 2)

- Does either player have a dominant strategy?

Example

P1 \ P2	b_1	b_2	b_3
a_1	(2, 3)	(4, 1)	(2, 3)
a_2	(2, 5)	(6, 3)	(3, 5)
a_3	(3, 1)	(5, 2)	(4, 3)

- How about now?

Example

P1 \ P2	b_1	b_2	b_3
a_1	(2, 3)	(4, 1)	(2, 4)
a_2	(2, 5)	(6, 3)	(3, 6)
a_3	(3, 1)	(5, 2)	(4, 3)

- How about now?

Example: Prisoner's Dilemma

- Recap:

		John's Actions	
		Stay Silent	Betray
Sam's Actions	Stay Silent	$(-1, -1)$	$(-3, 0)$
	Betray	$(0, -3)$	$(-2, -2)$

- Betraying is a strictly dominant strategy for each player

Iterated Elimination

- What if there are no dominant strategies?
 - No single strategy dominates every other strategy
 - But some strategies might still be dominated
- Assuming everyone knows everyone is rational...
 - Can remove their dominated strategies
 - Might reveal a newly dominant strategy
- Two variants depending on what we eliminate:
 - Only strictly dominated? Or also weakly dominated?

Iterated Elimination

- Toy example:
 - Microsoft vs Startup
 - Enter the market or stay out?

	Startup	
Microsoft		
Enter	(2 , -2)	(4 , 0)
Stay Out	(0 , 4)	(0 , 0)

- Q: Is there a dominant strategy for startup?
- Q: Do you see a rational outcome of the game?

Iterated Elimination

- More serious: “Guess $2/3$ of average”
 - Each student guesses a real number between 0 and 100 (inclusive)
 - The student whose number is the closest to $2/3$ of the average of all numbers wins!
- In-class poll!
- Recall: We have a unique optimal strategy only if everyone is rational, and everyone thinks everyone is rational, and so on.

Nash Equilibrium

- What if we don't find a unique outcome after iterated elimination of dominated strategies?

		Professor	
		Attend	Be Absent
Students	Attend	(3 , 1)	(-1 , -3)
	Be Absent	(-1 , -1)	(0 , 0)

Nash Equilibrium

- **Nash Equilibrium**

- A strategy profile \vec{s} is in Nash equilibrium if s_i is the best action for player i given that other players are playing \vec{s}_{-i}

$$u_i(s_i, \vec{s}_{-i}) \geq u_i(s'_i, \vec{s}_{-i}), \forall s'_i$$

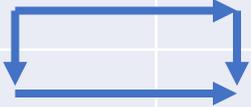


No quantifier on \vec{s}_{-i}

- Each player's strategy is only best *given* the strategies of others, and not *regardless*.

Recap: Prisoner's Dilemma

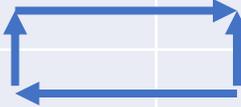
		John's Actions	
		Stay Silent	Betray
Sam's Actions	Stay Silent	$(-1, -1)$	$(-3, 0)$
	Betray	$(0, -3)$	$(-2, -2)$



- Nash equilibrium?
- Food for thought:
 - What is the relation between iterated elimination of weakly/strictly dominated strategies and Nash equilibria?

Recap: Microsoft vs Startup

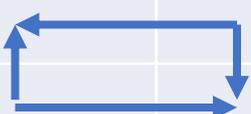
		Startup	
		Enter	Stay Out
Microsoft	Enter	(2, -2)	(4, 0)
	Stay Out	(0, 4)	(0, 0)



- Nash equilibrium?

Recap: Attend or Not

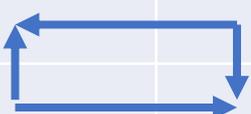
		Professor	
		Attend	Be Absent
Students	Attend	(3, 1)	(-1, -3)
	Be Absent	(-1, -1)	(0, 0)



- Nash equilibrium?

Example: Stag Hunt

		Hunter 1	
		Stag	Hare
Hunter 2	Stag	(4, 4)	(0, 2)
	Hare	(2, 0)	(1, 1)



- Game:
 - Each hunter decides to hunt stag or hare
 - Stag = 8 days of food, hare = 2 days of food
 - Catching stag requires both hunters, catching hare requires only one
 - If they catch one animal together, they share
- Nash equilibrium?