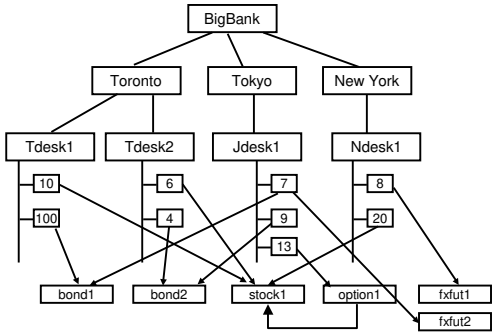
Ass #2/3 - PortfolioManager



PortfolioMangaer

% java PortfolioManager infile

mark-to-market: $230,942,340

CADdown: -$3,456,333

Irdown: +$2,456

…

instruments
portfolio/positions
market data
scenarios

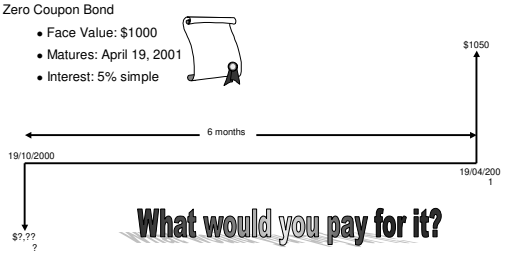
ASCII, XML

- Zero coupon bonds
- FX futures
- Equities
- European equity call options

Interest Rates - Pricing a Bond

Zero Coupon Bond

- Face Value: $1000
- Matures: April 19, 2001
- Interest: 5% simple

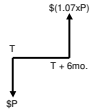


What would you pay for it?

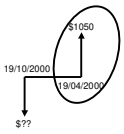
Interest Rates - Relative Pricing

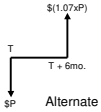
It depends on what other investments are available.

Assume only other investment is a US T-Bill returning 7% each half-year.



Interest Rates - Pricing the First Coupon





Alternate investment.

$1050 = 1.07 x P

P = $981.31

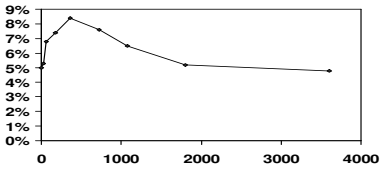
Supply and Demand will bring prices in-line

Interest Rates - Adding More Realism

- Actually,
  - T-Bills are priced by the market like anything else.
  - There are alternative investments at all sorts of maturities out to 30 years.

Interest Rates - The Spot Zero Curve

- The "spot zero curve" captures these rates of return in one concise curve.



- Gives YTM (yield-to-maturity) for non coupon bearing bonds of various maturities.
- Better to use a concept called "discount factors"

Interest Rates - Units

- Discount Factors
  - converts future dollars to present dollars
- Can express equivalently as *interest rates* which are considerably more intuitive.

Say 5yr. discount factor is 0.50835

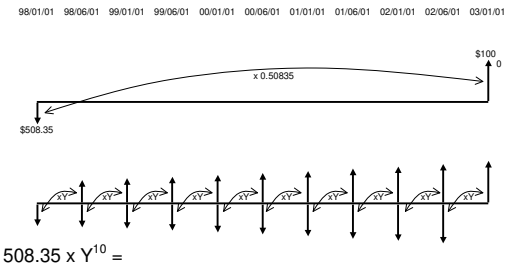
Bond worth $1000 five years from now costs $508.35 today.

Can express YTM of bond in units of annualised interest compounded annually.

Can also express in units of annualised interest compounded semi-annually.

All the same!

Interest Rates - Compounded Units



$508.35 \times Y^{10} = \$1000$

$Y = 1.07$

YTM = 7% semi-annual, semi-annually compounded
YTM = 14% annualised, semi-annually compounded

Interest Rates - Daycount Basis

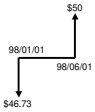
- Glossed over issue of units of time.
- Actually, all units are in days, although they seem to be quoted in years!
- Missing bit of information is the "daycount basis".

$$\frac{\text{Time in days}}{\text{Daycount}} = \text{Time in years}$$

- Examples of daycount bases:

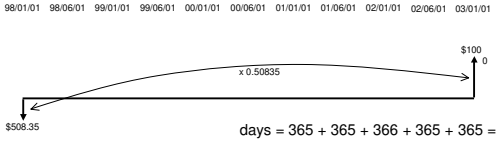
  ACT/360, ACT/365, ACT/ACT

Interest Rates - Years in Daycount of Bond



- Years between 98/01/01 and 98/06/01are computed as follows:
  - Days between = 31 + 28 + 31 + 30 + 31 + 30 = 181
  - For a ACT/360 daycount, Time in years = 181/360 = 0.50278
  - For a ACT/365 daycount, Time in years = 181/365 = 0.49589
  - For a ACT/ACT daycount, Time in years = 181/365 = 0.49589

Interest Rates - Converting using Daycount



days = 365 + 365 + 366 + 365 + 365 = 1826

$$\$508.38 = \frac{1}{\left(1 + \frac{YTM_5}{2}\right)^{2*days/360}} \times \$1000$$

ACT/360 daycount basis

annualised rates w/ semi-annual compounding

Interest Rates - Same Rate, Different Units

YTM
- annualised rates, semi-annually compounded, ACT/360 daycount
  13.793%
- annualised rates, semi-annually compounded, ACT/365 daycount
  13.991%
- annualised rates, semi-annually compounded, ACT/ACT daycount
  13.999%
- annualised rates, annually compounded, ACT/ACT daycount
  14.489%
- annualised rates, daily compounded, ACT/365 daycount
  13.526%
- annualised rates, continuously compounded, ACT/365 daycount
  13.523%

Interest Rates - Continuous Compounding

$$\$508.38 = \frac{1}{\left(1 + \frac{YTM_5}{m}\right)^{m*days/365}} \times \$1000$$

In the limit as 'm' (number of compounding periods in a year) goes to infinity:

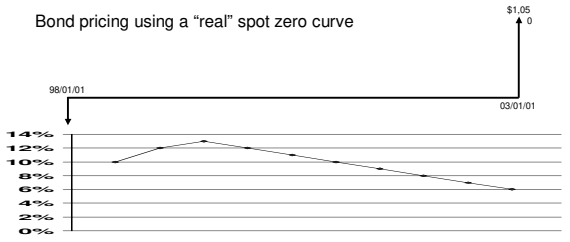
$$\$508.38 = e^{-YTM \times days/365} \times \$1000$$

YTM = -ln(508.38/1000)*365/1826

= 13.523% cont. ACT/365

Interest Rates - Bond Pricing w/ a Zero Curve

Bond pricing using a "real" spot zero curve



Units are annualised rates, continuously compounded, on an ACT/365 daycount basis

$P = \$1050 \times e^{-0.06 \times 5.03}$

P = $776.46

Interest Rates - Parity

- Each distinct currency has its own zero curve.
- No reason borrowing in USD should be the same rate as borrowing in CAD.

  USD 1yr. rate = 10% ANNU ACT/ACT

  CAD 1yr. rate = 5% ANNU ACT/ACT

- Q. Why not convert into USD and invest there?

  A. Because exchange rates could move in 1yr. and kill you.

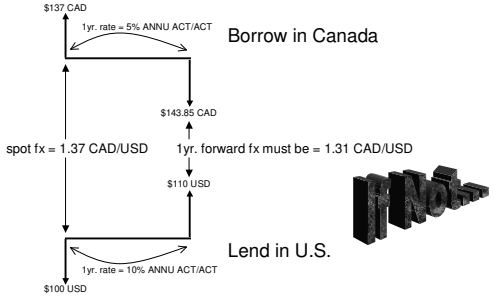
- But, by using FX Futures contracts, I can lock in a rate today and know exactly what the exchange rate will be in 1yr.'s time.

Interest Rates - Parity

- This leads to a relationship between
  - the CAD-USD spot fx rate,
  - the USD 1yr. spot IR rate,
  - the CAD 1yr. spot IR rate,
  - the CAD-USD 1yr. forward fx rate.

- If this relationship is broken, arbitrageurs working at large banks will trade and make instantaneous risk-free profits.
- Forces of supply and demand will force the prices back into alignment.

Interest Rates - Parity



Interest Rates - IR Parity Arbitrage

Say 1yr. future fx rate was 1.37 and not 1.31.

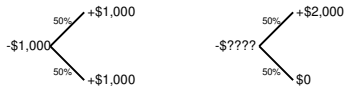
- Borrow $100 CAD at 5% (owe $105 CAD in 1yr.'s time)
- Buy $4.55 CAD worth of candy bars.
- Convert $95.45 CAD at 1.37 to $69.67 USD
- Loan $69.67 USD at 10%
- Enter into 1 yr. fx forward contract at 1.37 CAD/USD

- In 1 yr.'s time
  - Get back $76.64 USD
  - Use forward contract to convert to $105 CAD at 1.37 CAD/USD
  - Pay back $105 CAD dept in its entirety

- Net result: Ahead 4 candy bars! No risk taken!

Option Pricing

- Deals with the valuation of risky securities.

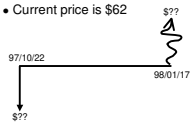


Q. How much would you pay?
A. It depends.

Option Pricing - Stock Call Option

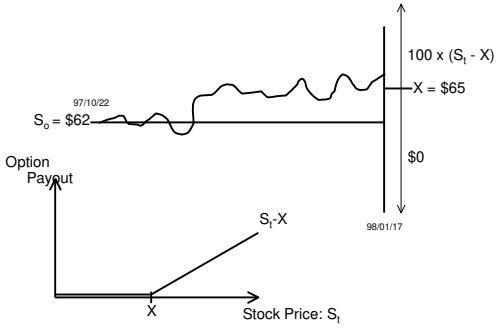
Call Option

- Option to purchase 100 shares of IBM stock
- On Feb.17, 1998
- At a strike of $65 per share
- Current price is $62

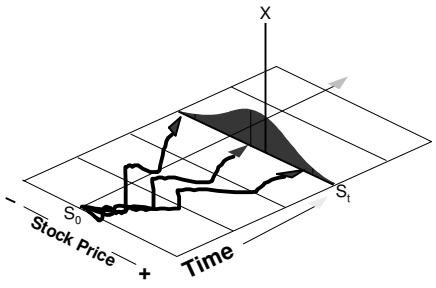


What would you pay for it?

Option Pricing - Call Option Payout



Option Pricing - Computing Option Value

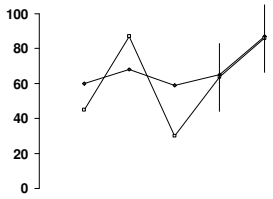


Option Pricing - Model of Stock Prices

- To compute distribution of stock price in the future, we need a model of how stock prices will change through time.
- Model used is geometric Brownian motion.

Option Pricing - Markov Process

- A stochastic process where only the current value is relevant for predicting the next value
  - Past history is not taken into account.

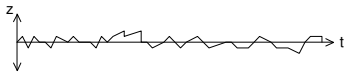


Option Pricing - Wiener Process

- Also called Brownian motion
  - Used in Physics to describe the motion of a particle that is subject to a large number of small molecular shocks.

$$dz = n \cdot \sqrt{dt}$$

where *n* is drawn from a standardised normal distribution N(0,1)



Option Pricing - The Generalised Wiener Process

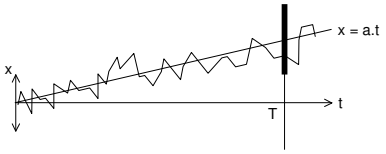
$dz = n \cdot \sqrt{dt}$ — expected drift rate

$dx = a.dt + b.dz$

mean of change in x = a.T

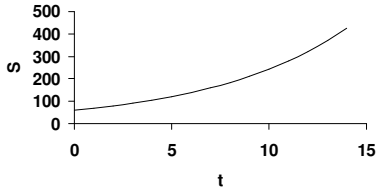
variance of change in $x = b^2.T$ — variance rate

where *a* and *b* are constants.



Option Pricing - Constant 14% Rate of Return

S = 60 . exp(0.14 x t)



Option Pricing - Ito Process

$dz = n \cdot \sqrt{dt}$

$dx = a(x,t).dt + b(x,t).dz$

where *a* and *b* are functions of *x* and *t*.

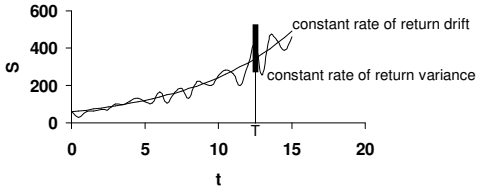
Option Pricing - Stock Process

S_0=60, μ=14%, σ=20%

$dS/S = \mu.dt + \sigma.dz$

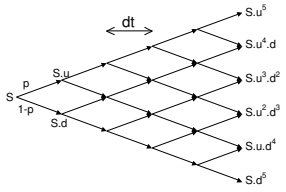
$dS = S.\mu.dt + S.\sigma.dz$

Ito Process for Stocks



Option Pricing - Lattices

- Can model this process as a lattice on stock prices.



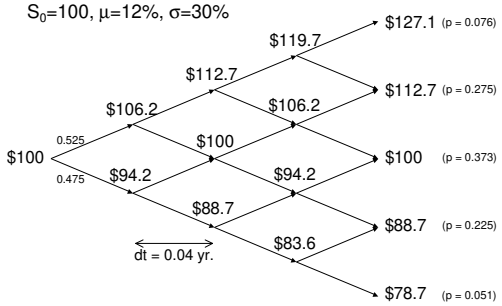
$u = e^{\sigma.sqrt(dt)}$

$d = 1/u$

$p = (e^{\mu.dt} - d)/(u-d)$

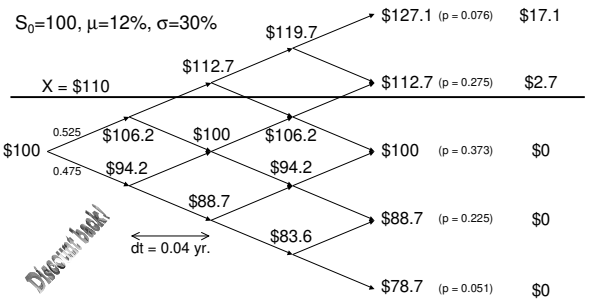
Option Pricing - Example Lattice

S_0=100, μ=12%, σ=30%



Option Pricing - Pricing an Option

S_0=100, μ=12%, σ=30%



Option expected value = 0.076 x $17.10 + 0.275 x $2.70 = $2

Option-Pricing - Black-Scholes

- In the limit as dt → 0, can derive a closed-form solution for the expected value of a European option.
- Black-Scholes equation.

Nobel Prize

$c = S.N(d_1) - X.e^{-r.(T-t)}.N(d_2)$

$d_2 = d_1 - \sigma.sqrt(T-t)$

$d_1 = (ln(S/X) + (r+\sigma^2/2).(T-t)) / \sigma.sqrt(T-t)$