

Probability and Anthropic Reasoning in
Small, Large, and Infinite Universes

Radford M. Neal, Dept. of Statistics, University of Toronto

Probability:

Events A, B, C, \dots , subsets of S .

Random variables X, Y, Z, \dots define events like $X = 0$ or $Y \in (1.2, 1.7)$.

Conditional probability: $P(B|A) = P(A \cap B)/P(A)$.

Standard axioms:

1) $P(A) \geq 0$, for all events A .

2) $P(S) = 1$.

3) For any sequence of events A_1, A_2, A_3, \dots for which $A_i \cap A_j = \emptyset$ for $i \neq j$,

$$P(A_1 \cup A_2 \cup A_3 \cup \dots) = P(A_1) + P(A_2) + P(A_3) + \dots$$

Axiom 3 is known as “countable additivity”. Weakening it to apply only to a finite collection of events is a variation that is sometimes considered. It is mathematically consistent, but allows “paradoxical” situations:

T. Seidenfeld, M. J. Schervish, and J. B. Kadane, Non-comglomerability of finite-valued, finitely additive probability, 1998.

P. Bartha and C. Hitchcock, The shooting room paradox and conditionalizing on measurably challenged sets, 1999.

Bayesian inference:

Probabilities are “degrees of belief”.

If we have observed data D , we make predictions conditional on D :

Know $X = 0 \rightarrow$ predict A is true with probability $P(A|X = 0)$.

If we have various theories, T_1, T_2, \dots , we judge their plausibility by $P(T_i|D)$, or look at ratios like $P(T_1|D)/P(T_2|D)$, where D is *all* relevant evidence.

Key point: If we're not sure what's relevant, D should be everything we know.

Bayes' rule:

$$P(A|D) = \frac{P(A) P(D|A)}{P(D)}. \quad \text{In ratio form: } \frac{P(T_1|D)}{P(T_2|D)} = \frac{P(T_1)}{P(T_2)} \frac{P(D|T_1)}{P(D|T_2)}.$$

Prediction of A if we *assume* theory T_i is true is done with

$$P(A|D \cap T_i) = \frac{P(A|T_i) P(D|A \cap T_i)}{P(D|T_i)}$$

Brian Greene

The Hidden Reality, 2011

p149: ...these astrophysical processes have produced planets throughout the cosmos, orbiting their respective suns at a vast assortment of distances. We find ourselves on one such planet situated 93 million miles from our sun because that's a planet on which our form of life *could* evolve. Failure to take account of this selection bias would lead one to search for a deeper answer. But that's a fool's errand.

... No one took exception to this element of Carter's argument...

Anthropic “Self-Sampling Assumption” (SSA):

You should imagine that you are a randomly selected observer, chosen uniformly from all observers in some “reference class” at all times and places.

My claim:

In a “small” universe, where it’s unlikely you have a duplicate, there is no need for anthropic reasoning based on SSA. Indeed, it is incorrect.

We can make predictions or assess theories by the standard method of looking at probabilities conditional on all that we know — ie, *all* our memories. I call this “Full Non-indexical conditioning” (FNC).

The “Self-Indication Assumption” (SIA):

The probability of a theory should be adjusted in proportion to the number of observers in the reference class it predicts exists.

Assuming SIA along with SSA gives the same result as FNC, but FNC has a much clearer justification.

Anthony Aguirre

On making predictions in a multiverse: conundrums, dangers, and coincidences
arxiv:astro-ph/0506519v1 2005

p10: Imagine that I have a theory in which the cosmological constant Λ is (with very high probability) much higher than we observe, and the dark matter particle mass m_{DM} is almost certainly > 1000 GeV. I condition on our observed Λ , simply accepting that I am in an unusual universe. Now say I measure $m_{DM} = 1$ GeV. I would like to say my theory is ruled out. Fine, but here is where it gets odd: according to top-down reasoning, I should also have already ruled it out if I had done my calculation in 1997, before Λ was measured. And someone who invented the very same theory next week - but had not been told that I have already ruled it out - would *not* rule it out, but instead just take the low value of m_{DM} (along with the observed Λ) as part of the conditionalization!

Stephen Hawking

Cosmology from the top down

arxiv:astro-ph/0305562, 2003

...most physicists are very reluctant to appeal to the anthropic principle. They would rather believe that there is some mechanism that causes all but four of the dimensions to compactify spontaneously... I'm sorry to disappoint these hopes... We live in a universe that appears four dimensional, so we are interested only in amplitudes for surfaces with three large dimensions. This may sound like the anthropic principle argument that the reason we observe the universe to be four dimensional, is that life is possible only in four dimensions. But the argument here is different, because it doesn't depend on whether four dimensions, is the only arena for life. Rather it is that the probability distribution over dimensions is irrelevant, because we have already measured that we are in four dimensions.

When should we ignore information?

Anthropic reasoning (SSA without SIA) *ignores* information about how many observers in the reference class exist, when evaluating how plausible a theory is. (At least for a theory that predicts at least one observer.)

An even more extreme view is to ignore *all* information when evaluating how plausible a theory is!

A possible source of confusion:

We should evaluate theories with $P(T_i|D) \propto P(D|T_i)$.

If $D = D_1 \cap D_2 \cap D_3 \cap \dots \cap D_K$, with the D_k independent given T_i :

$$P(D|T_i) = P(D_1|T_i) P(D_2|T_i) P(D_3|T_i) \dots P(D_K|T_i)$$

We can view the probability of the data in terms of successive predictions.

When predicting something *assuming* T_i , the success of previous predictions is irrelevant. But not when assessing the total evidence for T_i .

Leonard Susskind

The Cosmic Landscape: String Theory and the Illusion of Intelligent Design, 2006

p15: To Victor's [a friend's] question, "Was it not God's infinite kindness and love that permitted our existence?" I would have to answer with Laplace's reply to Napoléon: "I have no need of this hypothesis." *The Cosmic Landscape* is my answer...

p359-360: The properties of emergent systems are not very flexible. There may be an enormous variety of starting points for the microscopic behavior of atoms, but... they tend to lead to a very small number of large-scale endpoints. ...

This insensitivity to the microscopic starting point is the thing that condensed-matter physicists like best about emergent systems. But the probability that out of the small number of possible fixed points (endpoints) there should be one with the incredibly fine-tuned properties of our anthropic world is negligible. ... A universe based on conventional condensed-matter emergence seems to me to be a dead-end idea.

Assessing the evidence for theories with free parameters:

If theory T_1 has a free parameter ϕ , for which we have some prior density, f , we evaluate the probability of the data given T_i by

$$P(D|T_1) = \int P(D|\phi = x \cap T_1) f(x) dx$$

If D is all our memories, then in a small multiverse model, T_2 , with no free parameters, but where universes have a random ϕ chosen with density f , we get a similar expression.

Same also for model T_3 in which ϕ is fixed, but we haven't managed to compute its value yet (but our best guess is given by f).

Once we manage to compute the unique ϕ_0 determined by T_3 , its probability will get much higher or much lower, depending on how large $P(D|\phi = \phi_0 \cap T_3)$ is.

What about large or infinite universes/multiverses?

In a universe large enough to have duplicate observers, the probability of some being with your memories existing may approach one, and will equal one for a suitable infinite universe.

It seems we can't compare such theories based on $P(D|T_i)$, where D is all your memories.

A conservative approach: Assume that when all the difficulties arising with such large universes are worked out, the result will be much the same as for a small universe — no anthropic reasoning required.

I think this is at least reasonable if you are tempted to apply anthropic reasoning to a non-cosmological problem.

The fallacy of making a fantastic assumption and then not taking it seriously...

Matthew Davenport and Ken D. Olum

Are there Boltzmann brains in the vacuum?

arxiv:1008.0808v1 2010

p2: ... it is possible for there to spontaneously appear a brain that is in exactly the state of your brain at this moment, and thus is apparently indistinguishable from you ... the number of such “Boltzmann brains” will grow without bound, while the number of normal observers is finite. Thus by anthropic reasoning you should believe with probability 1 that you are one of the Boltzmann brains.

Of course, no one really believes that he is a Boltzmann brain...

Furthermore, there is a simple test to see whether you are a Boltzmann brain. Wait 1 second and see if you still exist. Most Boltzmann brains are momentary fluctuations. So the prediction of the above argument is that you will vanish in the next second. When you don't, you conclude that this argument made a severely wrong prediction.¹

¹ If you are concerned with the fact that you could never observe your own ceasing to exist, you can change the argument to say that the theory that you are a Boltzmann brain predicts that your observations of the external world are coherent only by chance, and that subsequent observations will not remain coherent.

Andrea De Simone, Alan H. Guth, Andrei Linde, Mahdiyar Noorbala,
Michael P. Salem, and Alexander Vilenkin

Boltzmann brains and the scale-factor cutoff measure of the multiverse
arxiv:0808.3778v3 2010

p1: If the Boltzmann brains prevail, then a randomly chosen observer would be overwhelmingly likely to be surrounded by an empty world, where all but vacuum energy has redshifted away, rather than the rich structure that we observe.

p1–2: According to the theory there would be an infinite number of Boltzmann brains, distributed throughout the spacetime, that would happen to share exactly all her memories and thought processes at that moment... all predictions would be based on the proposition that she is a Boltzmann brain... the continued orderliness of the world that we observe is distinctly at odds with the predictions of a Boltzmann-brain-dominated cosmology.