Family name: *Given name:* *Student ID:*

STA 247 — Quiz #3, 2001-11-11, 3:10pm – 35 minutes long

No books, no notes, and no calculators may be used.

All numerical answers must be actual numbers (decimals such as 0.15 or simple fractions such as 3/13 or 4/3), not just a formula. If this requires arithmetic on numbers bigger than 1000, you've either made a mistake, or you should think of an easier way to solve the problem.

Q1: [50 total marks] You roll a fair six-sided die. Let the random variable X be -3 if the die shows 1 or 2, 0 if the die shows 3, and $+4$ if the die shows 4, 5, or 6. You also flip a fair coin nine times. Let Y be the number of times the coin lands heads in these nine flips. Define $Z = X - Y$. Answer the following, showing your work.

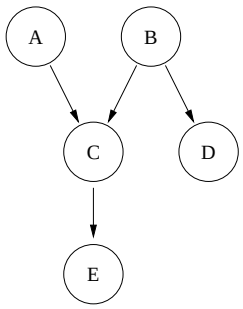
a) [10 marks] Find the expectation and variance of X.

b) [10 marks] Find the expectation and variance of Y.

c) [15 marks] Find the expectation, variance, and standard deviation of Z.

d) [15 marks] Find an upper bound on $P(Z \leq -8)$ using Chebyshev's inequality.

Q2: [24 marks] Consider the directed graphical model below:



Say whether each independence or conditional independence relationship below can be inferred based on this graphical model. Write "Yes" if the graphical model implies the relationship holds, or "No" if one cannot conclude that the relationship holds based just on the graphical model. No explanations are required. Marking: +4 for each correct answer, −3 for each incorrect answer, 0 for no answer, minimum 0.

a) D is independent of E.

b) A is independent of D.

c) A is conditionally independent of D given E.

d) A is conditionally independent of E given C.

e) C is conditionally independent of D given B.

f) A is conditionally independent of D given C and B.

Q3: [26 marks] Suppose that the joint distribution for A, B, C, D, and E in the graphical model of Q2 is defined as follows:

- The distribution of A is Bernoulli(1/2).
- The distribution of B is binomial(2, 1/4).
- Given $A = a$ and $B = b$, C is defined to be their product, $a \times b$. In other words, $P(C = a \times b \,|\, A = a, B = b) = 1$.
- Given $B = b$, the distribution of D is binomial(b, 1/2).
- Given $C = c$, the distribution of E is Bernoulli(c/2).

Find $P(A = 1, E = 0 \,|\, C = 1)$. Show your work and justify your answer.