STA 3000, Problem Set 5. Due in class February 13.

1. Consider the following hypothetical experiment. Subjects are shown one of two short films, which differ only in that in one film, the characters are shown eating apples, whereas in the other film, they are shown eating pears. The film to show is chosen at random. After viewing one of these films, each subject is interviewed about other aspects of the film, while being casually offered the choice of an apple or a pear to eat. The objective was to see whether this choice was influenced by what fruit the subject saw in the film.

   Fourteen subjects chose one fruit or the other to eat. The data for these subjects was as follows, with 0 representing viewing or eating an apple, and 1 representing viewing or eating a pear:

Fruit in film:	0	1	1	0	0	1	1	1	0	1	0	1	1	1
Fruit chosen:	0	1	1	0	1	1	1	0	0	1	0	1	0	1

   Two models for this data are being considered. In both models, the data pairs, (x_i, y_i), are considered to be IID given the model parameters, and the distribution of the film viewed is assumed to be uniform (ie, the film chosen is assumed not to affect whether the subject choose to eat one or the other fruit). In Model A, the fruit seen in the film is assumed to have no influence on the subject's choice of fruit to eat. This model has a single parameter, p, which is the probability of the subject choosing a pear. In other words,

   $$P(x_i = x,\, y_i = y \,|\, p) \;=\; (1/2)\, p^y\, (1-p)^{1-y}$$

   In Model B, the fruit seen in the film may affect which fruit the subject chooses. This is expressed by having two model parameters, p_0 and p_1, which are the probabilities of choosing a pear when the film showed apples or pears, respectively. In other words,

   $$P(x_i = x,\, y_i = y \,|\, p) \;=\; (1/2)\, p_x^y\, (1-p_x)^{1-y}$$

   (a) Suppose that the prior distribution for p in Model A is uniform over $(0, 1)$, and that the prior distribution for p_0 and p_1 in Model B is uniform over $(0, 1) \times (0, 1)$. Find the Bayes factor for comparing Model A with Model B.

   (b) Suppose instead that the prior for p in Model A is uniform, but in Model B, the prior for p_0 is Beta$(1, 2)$ and the prior for p_1 is Beta$(2, 1)$, with p_0 and p_1 being independent. Find the Bayes factor for comparing Model A with Model B using these priors.

   (c) Based on your common-sense knowledge of the situation, do you think either of the priors above capture all the prior information that would be reasonable in this problem? If not, what would be the effect on the Bayes factor of using better priors?

2. Do exercise 17 in chapter 4.

3. Do exercise 18 in chapter 4.

4. In class, I sketched a reason why UMP level α tests of a simple hypothesis versus a composite alternative will be admissible. Actually, this isn't quite true in general — in some unusual circumstances, such a test can be inadmissible.

   (a) Give an example of a testing situation with a simple hypothesis and a composite alternative for which there exists, for some α, a UMP level α test that is inadmissible.

   (b) State and prove as strong a theorem as you can regarding admissibility of UMP level α tests of a simple hypothesis, which contains a premise that eliminates examples such as you gave in (a).

   Note: It's possible that you might find it easier to start by trying to do part (b), since trying to find a proof without an extra premise may point to an example to use for (a).

5. Suppose $\mathcal{X} = \{-1, 0, +1\}$, $\Omega = [-1/3, +1/3]$, and $P(X = x \mid \Theta = \theta) \;=\; (1/3) + \theta x$.

   (a) For every α in $(0, 1)$, does there exist a UMP level α test for $H : \theta = -1/3$ versus $A : \theta > -1/3$? If so, find it.

   (b) For every α in $(0, 1)$, does there exist a UMPU level α test for $H : \theta = -1/3$ versus $A : \theta > -1/3$? If so, find it.

   (c) For every α in $(0, 1)$, does there exist a UMP level α test for $H : \theta = 0$ versus $A : \theta \neq 0$? If so, find it.

   (d) For every α in $(0, 1)$, does there exist a UMPU level α test for $H : \theta = 0$ versus $A : \theta \neq 0$? If so, find it.

   (e) Say what you can about whether the existence of the UMP tests and UMPU tests in the questions above changes if we observe not a single X value but rather a sample of n values, $X_1, \ldots, X_n$, that are IID with $P(X_i = x \mid \Theta = \theta) \;=\; (1/3) + \theta x_i$.