Name: *Student ID:*

STA 414/2104 — Second Test — 2006-04-07

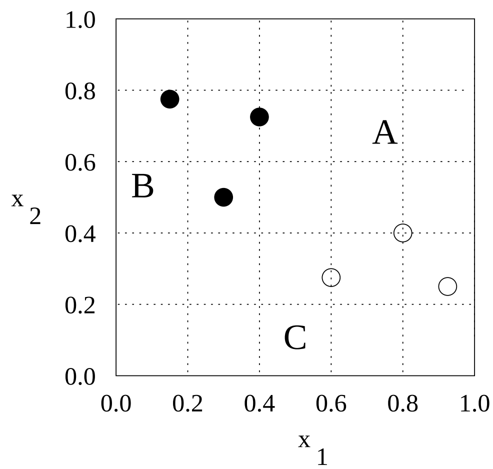
No books, notes, or calculators are allowed.

1/33	
2/32	
3/35	
T/100	

Question 1: [33 Marks] Consider a classification problem with two input variables, both having values in the interval $(0, 1)$, and a binary class variable, with classes represented by white circles ($\circ$) and black circles ($\bullet$). Suppose we believe that cases in these two classes can be perfectly separated by a line that is either horizontal or vertical. Suppose that our prior distribution says that:

- The line separating the classes is equally likely to be horizontal or vertical.
- The point where the line separating the classes intersects the horizontal or vertical axis is uniformly distributed over the interval $(0, 1)$.
- The $\bullet$ class is equally likely to be above or below the separating line, if it is a horizontal line, and equally likely to be right or left of the separating line, if it is a vertical line.

Suppose we have six training cases, three in each class, with input values indicated by their positions in the plot below:



We wish to make predictions for the class in three test cases, A, B, and C, for which the values of the inputs are given by the locations marked in the plot above.

For each of these three test cases, find the probability that it is in the $\bullet$ class, given the data from the six training cases, and assuming the model and prior distribution described above.

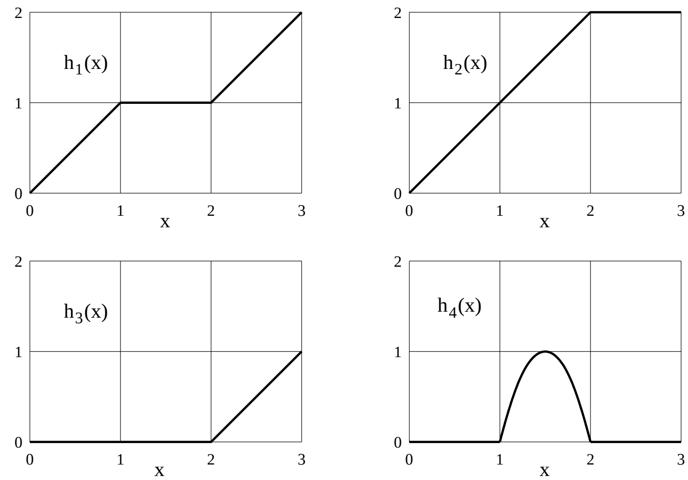
Test case A:

Test case B:

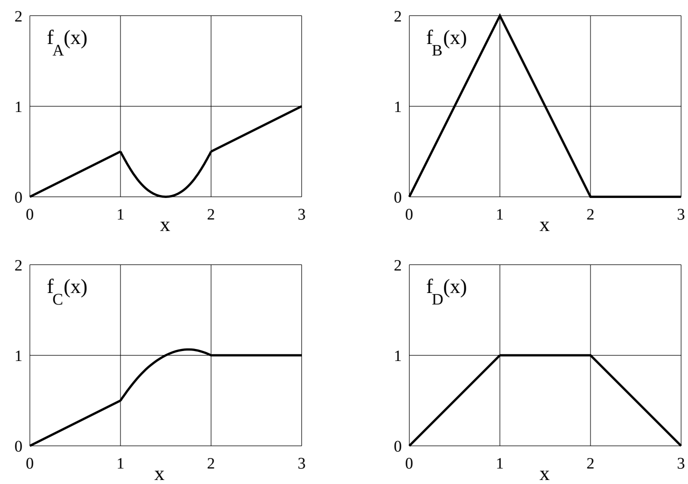
Test case C:

Question 2: [32 Marks]

Suppose we model functions of x over the interval $(0, 2)$ as linear combinations of the following four basis functions:



Write down a linear combination of $h_1(x)$, $h_2(x)$, $h_3(x)$, and $h_4(x)$ that produces each of the following four functions. (Note that there is no additional 'intercept' term in this model.)



$$f_A(x) \quad =$$

$$f_B(x) \quad =$$

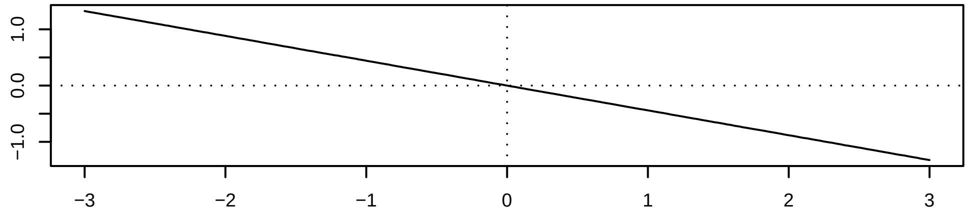
$$f_C(x) \quad =$$

$$f_D(x) \quad =$$

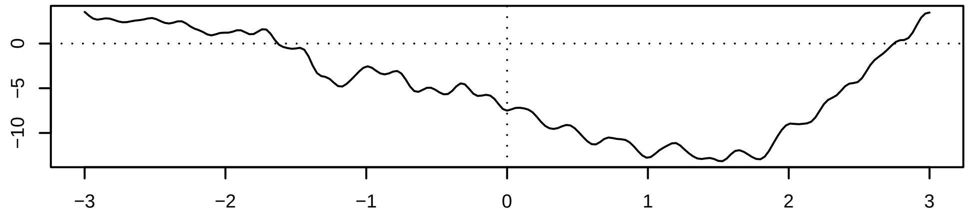
Question 3: [35 Marks]

Below are five functions randomly drawn from five different Gaussian processes. For all five Gaussian processes, the mean function is zero. The covariance functions are one of those listed below.

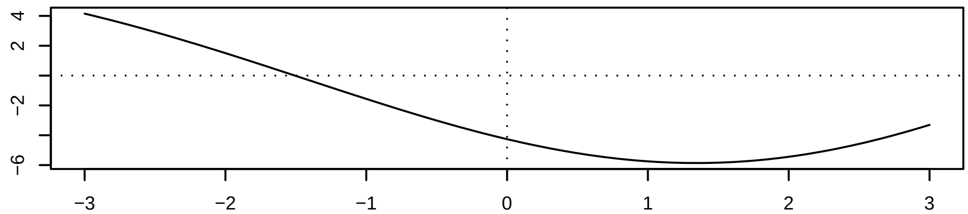
a)



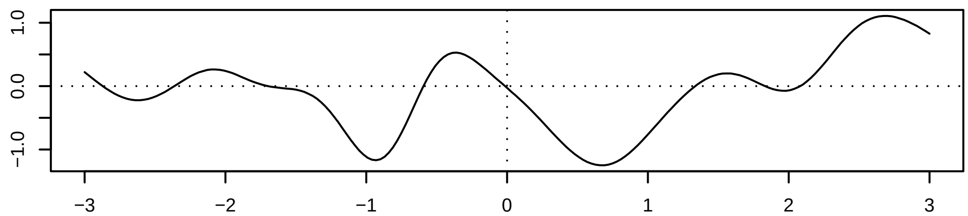
b)



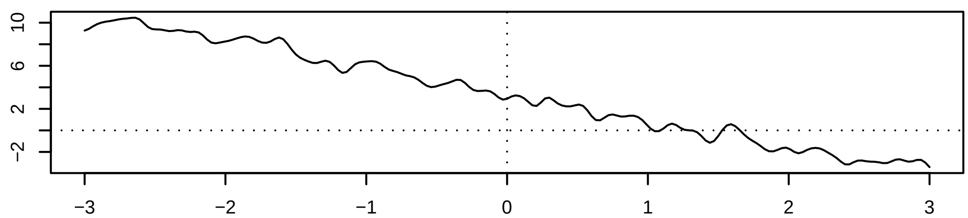
c)



d)



e)



For each of the five covariance functions below, indicate which of the five functions above is most likely to have been drawn from the Gaussian process with that covariance function.

1) $\mathrm{Cov}(y_{i_1}, y_{i_2}) \;=\; 0.5^2 \exp(-((x_{i_1} - x_{i_2})\,/\,0.5)^2\,)$

2) $\mathrm{Cov}(y_{i_1}, y_{i_2}) \;=\; x_{i_1}\, x_{i_2}$

3) $\mathrm{Cov}(y_{i_1}, y_{i_2}) \;=\; 5^2 \;+\; 5^2\, x_{i_1}\, x_{i_2} \;+\; 0.5^2 \exp(-((x_{i_1} - x_{i_2})\,/\,0.1)^2\,)$

4) $\mathrm{Cov}(y_{i_1}, y_{i_2}) \;=\; 0.7^2 \exp(-((x_{i_1} - x_{i_2})\,/\,0.1)^2\,) \;+\; 8^2 \exp(-((x_{i_1} - x_{i_2})\,/\,2)^2\,)$

5) $\mathrm{Cov}(y_{i_1}, y_{i_2}) \;=\; 8^2 \exp(-((x_{i_1} - x_{i_2})\,/\,5)^2\,)$