Computer Science CSC238H | Wednesday May 27 1998
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Predicate Logic

Basic Predicate Logic Equivalences

1. Renaming Bound Variables:

   - $\forall x\ \alpha \Leftrightarrow \forall y\ \alpha[y/x]$
   - $\exists x\ \alpha \Leftrightarrow \exists y\ \alpha[y/x]$

   where α is any predicate wff in which Y is not free and $\alpha[y/x]$ is obtained from α by replacing every free occurrence of x by y.

2. Order of quantifiers:

   - $\forall x \forall y\ \alpha \Leftrightarrow \forall y \forall x\ \alpha$
   - $\exists x \exists y\ \alpha \Leftrightarrow \exists y \exists x\ \alpha$

   where α is any predicate wff.

   Note that in general, $\forall x \exists y\ \alpha \not\Leftrightarrow \exists y \forall x\ \alpha$.

3. Negation:

   - $\sim \forall x\ \alpha \Leftrightarrow \exists x \sim \alpha$
   - $\sim \exists x\ \alpha \Leftrightarrow \forall x \sim \alpha$

   where α is any predicate wff.

4. Conjunction and Disjunction:

   - $\forall x\ \alpha \wedge \forall x\ \beta \Leftrightarrow \forall x[\alpha \wedge \beta]$
   - $\exists x\ \alpha \vee \exists x\ \beta \Leftrightarrow \exists x[\alpha \vee \beta]$

   where α and β are any predicate wffs.

   More generally, if α and β are any predicate wffs such that x does not occur free in α,

   - $\alpha \wedge \forall x\ \beta \Leftrightarrow \forall x[\alpha \wedge \beta]$
   - $\alpha \vee \forall x\ \beta \Leftrightarrow \forall x[\alpha \vee \beta]$
   - $\alpha \wedge \exists x\ \beta \Leftrightarrow \exists x[\alpha \wedge \beta]$
   - $\alpha \vee \exists x\ \beta \Leftrightarrow \exists x[\alpha \vee \beta]$

Basic Valid Predicate WFFs

- $\forall x\ \alpha \rightarrow \alpha[t/x]$
  where α is any predicate wff, x is any variable and t is any term.
- $\alpha[t/x] \rightarrow \exists x\ \alpha$
  where α is any predicate wff, t is any term and x is any variable.