Computer Science CSC 238H
St. George Campus

Wednesday July 22 1998
University of Toronto

Quiz #7

First Name: _____

Last Name: _____

Student Number: _____

1. [1 mark] In lecture last week, we saw that the function T defined by the recurrence $T(n) = a_1 T(\lceil n/b \rceil) + a_2 T(\lfloor n/b \rfloor) + cn^d$ (where a_1, a_2, b, c and d are constants satisfying $b^d > a_1 + a_2 \geq 1, b \geq 2$ and $c > 0$) has the property $T(n) \in \Theta(n^d)$.

Let S be the function defined by the recurrence

$$S(0) = 1$$
$$S(n) = 2S(\lfloor n/3 \rfloor) + n^3 + 3n^2 + 7n \text{ (for } n > 0).$$

Let U be the function defined by the recurrence

$$U(0) = 1$$
$$U(n) = 2U(\lfloor n/3 \rfloor) + 11n^3 \text{ (for } n > 0).$$

First prove that $S(n) \leq U(n)$ for all $n \in \mathbb{N}$. Then use the fact mentioned above to show that $S(n) \in O(n^3)$