Computer Science CSC 238H
St. George Campus

Wednesday June 24 1998
University of Toronto

Midterm Test

First Name: ______________________________

Last Name: ______________________________

Student Number: ______________________

Tutorial (circle one):

Joe	Geneviève	Alex	Steve	Eugenia
SF 1101	GB 244	GB 119	GB 120	SF 3109
(remedial)	(Erin, New, PHE)	(Trin, Inn, Scar)	(Vic, SMC, Eng)	(Wood, UC)

This test lasts 50 minutes. *Aids allowed: one handwritten sheet of notes measuring* $8\frac{1}{2}$ *by 11 inches. Make sure your test has four pages, including this cover page.*
Answer in the space provided, using the reverse side of the pages for rough work. (If you need more space, use the reverse side of the page and indicate ***clearly*** *which part of your work should be marked.)*

Question 1	/13
Question 2	/10
Question 3	/14
Question 4	/13
Total	/50

1. [13 marks] Consider the following symbols with their intended meanings. (The domain is the set of all people.)

Predicate	Meaning
$A(x)$	x is an artist
$G(x)$	x goes mad
$P(x)$	x is a poet
$M(x)$	x is a mathematician
$F(x, y)$	x is a friend of y

Constant	Meaning
d	Leonardo da Vinci

[8] **(a)** Write predicate formulae that represent the following statements.

(i) Poets and artists do not go mad but some mathematicians do.

(ii) Leonardo da Vinci is an artist and mathematician, not a poet, although he is the friend of a poet.

[5] **(b)** What is the meaning of the following predicate formula? You should give your answer in *plain* English.

$$\forall x[M(x) \rightarrow (P(x) \vee A(x))] \wedge \forall x \sim [G(x) \wedge M(x)]$$

2. [10 marks] Is the following formula valid, unsatisfiable or neither? Justify your answer.

$$\forall x[A(x) \leftrightarrow \exists y B(x,y)] \wedge \forall x[B(x, f(x))]$$

3. [14 marks] Beside each function $f(n)$ in the left column, write down the number of the **first** function $g(n)$ in the right column such that $f(n) \in O(g(n))$.

(a) $n^{1.9} \log_2 n$ ______

(b) $1/n$ ______

(c) $(n+7)^3$ ______

(d) 2^{n+5} ______

(e) $\log_4(n^2 + 42n + 3)$ ______

(f) $10 \cdot 2^{\log_4 n}$ ______

(g) $\sum_{i=1}^{n} \sqrt{i} \log_2 i$ ______

1. 1
2. $\log_2 n$
3. $(\log_2 n)^2$
4. $\sqrt{n}$
5. $\sqrt{n} \log_2 n$
6. n
7. $n \log_2 n$
8. n^2
9. n^3
10. 2^n
11. n^n

4. [13 marks] Let α be a *propositional* formula. We shall say that α is a *special* formula if each symbol appearing in α is either a propositional variable, (,), $\top$, $\vee$ or $\wedge$.

[11] **(a)** Prove that every special formula is satisfiable. Hint: use mathematical induction on the length of α and try to think of one particular tva that will satisfy any special formula.

[2] **(b)** Give an example of a propositional formula that is not equivalent to any special formula.