**CSC 364** **Quiz #3** **December 6/2000**

Name ______________________ Student No. ______________________

NO AIDS ALLOWED

Answer ALL questions on test paper. Use backs of sheets for scratch work.

Total Marks: 35

(1) (5 points) Define the class of recursive languages.

(2) (5 points) Define the class of recursively enumerable languages.

(3) (5 points) Give an example of a language that is recursively enumerable but not recursive. You may not use any language mentioned in this quiz. You should describe the language (not just state its name).

(4) (5 points) Give an example of a language that is not recursively enumerable. You may not use any language mentioned in this quiz. Again, you should describe the language.

(5) (20 points, 10 each) Consider each of the following languages. State if they are (a) recursive; (b) recursively enumerable but not recursive; or (c) not recursively enumerable. Prove your answers. In your proofs you may use the fact that the language $TOTAL$ is not recursively enumerable, and $HB$ is not recursive. $TOTAL$ is the language consisting of strings $< M >$ such that $< M >$ encodes a Turing machine $M$, and such that $M$ halts on all inputs. $HB$ is the language consisting of strings $< M >$ such that $M$ halts when run on the blank input.

(a.) $ALL$ is the language over $\{0, 1\}^*$ consisting of strings $< M >$ such that $< M >$ encodes a Turing machine $M$, and such that $M$ accepts all strings.

(b.) $VALIDTM$ is the language over $\{0,1\}$ consisting of strings $< M >$ such that $< M >$ is a valid encoding of a Turing machine.

(6) (10 points) Assume that $P = NP$. Prove that under this assumption, there is a polynomial-time algorithm for $FINDCLIQUE$. $FINDCLIQUE$ is the following search problem. The input is a pair $(G = (V, E), k)$. The output is a set of vertices $V' \subseteq V$ such that $G$ contains a clique on $V'$, and $|V'| = k$, if such a clique exists. You may use the fact that $CLIQUE$ is in $NP$, where $CLIQUE$ is the language consisting of all pairs $(G = (V, E), k)$ such that $G$ is a graph containing a clique of size $k$.