CSC 364 **Quiz #2** **November 8/2000**

Name _____ Student No. _____

NO AIDS ALLOWED

Answer ALL questions on test paper. Use backs of sheets for scratch work.

Total Marks: 35

(1) (5 points) Define the complexity class NP. Be as specific as possible.

(2) (10 points) Design a Sipser-basic Turing machine (one head, and one tape infinite to the right) that takes a string of 0's and 1's as input and produces the reverse of the string as output. For example, if the tape is initially 11001 (followed by blanks), then after halting the tape should be 10011 (followed by blanks). Give the high-level description of the Turing Machine, and specify the states that you will use. You do not need to give a full formal description of the Turing Machine.

(3) The complement of a language $L \subseteq \{0,1\}^*$ consists of all strings $x \in \{0,1\}^*$ that are not in L.

a. (5 points) Suppose that an arbitrary langauge L is in P. Explain why the complement of L, $\overline{L}$ is also in P.

b. (5 points) If L is in NP, then does it also follow that the complement of L is in NP? Justify your answer briefly.

(10) IND-SET is the following problem. The input is an undirected graph $G = (V, E)$ and a number k. The input is accepted if and only if G contains an independent set of size k. An independent set of size k is a subset $V' \subseteq V$, $|V'| = k$, such that all edges within V' (that is, all edges (u', v'), such that $u', v' \in V'$) are NOT present in G. For example, the graph G1 below would be accepted for $k = 4$ and $G2$ would be rejected for $k = 4$.

a. (4 points) Show that IND-SET is in NP.

b. (6 points) Show that IND-SET is NP-complete by giving a polynomial-time reduction from the CLIQUE problem to IND-SET. Briefly justify your reduction. (Recall that an instance of CLIQUE is a graph $G = (V, E)$ and a number k. The instance is accepted if and only if G contains a clique of size k, where a clique of size k is a subset $V' \subset V$, $|V'| = k$ such that all edges within V' are present in G.)