CSC364 **Problem Set 1** **Winter/2002**

You may consult with others concerning the general approach for solving problems on assignments, but you must write up all solutions entirely on your own. Anything else is *cheating* and is subject to prosecution. Due Date: Wednesday, February 6 at the beginning of tutorial. Late assignments will not be accepted. This homework will help to prepare you for the first test, during tutorial on February 13.

(1) (20 points) Design a multi-tape Turing Machine that takes a string w over $\{0, 1, \#\}$ as input, and accepts the string if and only if it is of the form $x\#y\#z$ where x, y, z are binary strings such that x plus y equals z. Give both an informal description as well as a complete formal specification of the Turing Machine.

(2) (10 points) Prove that, for each Turing machine that halts under all inputs, there is an equivalent Turing machine (accepting the same language) that never moves its head to the left of its starting position. Give a complete formal description of the simulation.

(3) (10 points) Prove that the following set, S, is countable by providing a one-to-one mapping from the set to the natural numbers. The set S consists of all triples, (i, j, k), where i, j, k are natural numbers.

(4) Label each of the following problems as either (i) recursive; (ii) recursively enumerable but not recursive; (iii) not recursively enumerabe. Justify your answer. (If recursive, or r.e., give a high-level description of a Turing machine or algorithm, and if not recursive or not r.e., prove it using a reduction.) You may assume that all Turing Machines under consideration have input alphabet $\{0, 1\}$. Also, when a Turing Machine M is part of the input, it is given by its encoding.

   (a) (10 points) Given a Turing Machine M does it halt on the empty string?

   (b) (10 points) Given a Turing Machine M and input w, does M on input w ever move its tape head left?

   (c) (10 points) Given two Turing Machines, M_1 and M_2, do they accept the same languages?

   (d) (10 points) Given a number x in binary, and a Turing Machine M, is M the lexicographically first Turing Machine (accepting strings over 0,1) that accepts the number x?

   (e) (10 points) Given a number x in binary, and a Turing Machine M, is M the lexicographically first Turing Machine that accepts the number x within x steps?

(5) (10 points) Show that if a language L is recursively enumerable and the complement of L, $\overline{L}$, is also recursively enumerable, then L is recursive.