CS 364 Exam 2

Please complete this section:

Name (please print): ______________________________

Student Number: ______________________________

No aids allowed. There are 4 questions worth a total of 54 marks. Answer all questions on the question paper. Use backs of pages for scratch work. Check that your exam has 4 pages.

(1) (10 points) Assume that $P = NP$. Prove that under this assumption, there is a polynomial-time algorithm for $FINDSAT$. $FINDSAT$ is the following search problem. The input is a CNF formula f with underlying variables $x_1, \cdots, x_n$. The output is a 0/1 assignment, α, to the underlying variables such that $f(\alpha) = 1$, if one exists. Otherwise (if no satisfying assignment exists), the output is "unsatisfiable".

(2) The complement of a language $L \subseteq \{0,1\}^*$ consists of all strings $x \in \{0,1\}^*$ that are not in L.

a. (5 points) Suppose that an arbitrary langauge L is in P. Explain why the complement of L, $\overline{L}$ is also in P.

b. (5 points) If L is in NP, then does it also follow that the complement of L is in NP? Justify your answer briefly.

(3) (6 points) Suppose the runtime of an algorithm is $T(n)$. Circle the expressions below for $T(n)$ which would make the algorithm polytime. NO JUSTIFICATION NECESSARY.

a. $2^{\sqrt{n}} + 5 \log n$

b. $3^{5 \log n}$

c. $8^{(7 \log n)^6}$

(4) (18 points) For each of the following problems, specify if the problem is (a) in P, (b) NP-complete, (c) recursive, but not known to be in NP (d) not recursive. NO JUSTIFICATION NECESSARY.

a. $3COL$: Given an undirected graph G, accept if and only if there is a coloring of the vertices such that each vertex is assigned exactly one color from Red, Blue, Green, and such that no two adjacent vertices are colored the same color.

b. UNIQUE-SAT: Given a boolean CNF formula f over variables $x_1, \cdots, x_n$ accept if and only if there is *exactly* one truth assignment α such that $f(\alpha) = 1$.

c. UCONN: The input is an undirected graph G and two distinguished vertices s and t. Accept if and only if there is a path from s to t in G.

d. HALT-BLANK: Given the encoding of a Turing machine, $< M >$, accept if and only if M halts on the blank tape.

e. 50-CLIQUE: Given an undirected graph $G = (V, E)$, accept if and only if G contains a clique of size exactly 50.

f. 50-MAXCLIQUE: Given an undirected graph $G = (V, E)$, accept if and only if the G contains a clique of size exactly 50, and there is no clique in G larger than 50.

(5) (10 points) At least one of the problems from question (4) is NP-complete. Pick one of the problems above that you said was NP-complete and prove it. You may use the fact that SAT and $3SAT$ are NP-complete in your reduction.