COMS 6998 Communication Complexity and Applications,
Spring 2022
Time/Location: Wednesdays 2:10-4

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Website: www.cs.toronto..edu/ ∼ toni/Courses/CommComplexity2022/cc2022.html
Refer to this site periodically for important announcements and handouts, including lecture notes and paper links.

Course Description. One of the most basic challenges in the theory of computation is proving computational limitations for well-studied important problems in a variety of models. Communication complexity has played a central role in understanding information bottlenecks for many computational models as well as to many seemingly unrelated problems. For example, communication complexity has been the main tool used to establish state-of-the-art lower bounds in the following areas: graph theory, circuit complexity, game theory, proof complexity, property testing, extension complexity, distributed computing and learning theory.

In this course we will cover the foundations of communication complexity, and applications to a wide variety of other areas. Prerequisites are a solid background in linear algebra, and an undergraduate course in complexity theory or theory of computing.

Tentative Course Outline.

(I) Introduction to two-player communication complexity. Basic concepts and definitions; motivation; important cc problems (equality, set disjointness, inner product); a brief introduction to the vast applications of communication complexity.

(II) Flavors of communication

1. Deterministic

2. Randomized (Distributional Complexity and Yao's theorem, Public versus Private coins, Discrepancy method
3. Nondeterministic communication complexity
4. Relationships between the models.

(III) Lower bounds and lower bound methods.

1. Fooling sets
2. rank and logrank conjecture
3. discrepancy method
4. corruption (set disjointness lower bound)
5. lifting (query complexity and polynomial degree to communication complexity, strong link to TFNP classes and proof complexity.)
6. information complexity
7. important matrix norms and duality.

(IV) Applications (We will cover the first 4 topics and selected additional ones based on your preferences.)

1. Lifting query complexity to circuit complexity (KW games)
2. Lifting query complexity to extended formulations
3. Learning theory (via lifting)
4. Graph theory (Clique versus independent set)
5. Branching programs
6. Data structures
7. Streaming
8. Differential privacy
9. Mechanism Design
10. Property Testing

(V) (Time Permitting) Number-on-forehead model of multiparty communication complexity. Randomized, nondeterministic complexity in this model. Lower bounds via the BNS discrepancy method and generalized

discrepancy. Applications including: Proof complexity lower bounds via NOF communication complexity, ACC circuit lower bounds, connections with additive combinatorics.

Grading and Assignments. Grading will be based on 2-3 assignments which will be handed out during the semester, plus a short presentation. More details on the presentation will be given in the first few weeks of the class. You may work alone or in small groups, and I will give a list of suggested topics/papers.

Class attendence is manditory and you are encouraged to ask questions in class. I will present many open problems during the course and hope that some of you will solve some of these problems! It is a great area with lots of connections to other problems, and a wealth of interesting open problems.

Course Materials. Each lecture will include lecture notes and supplemental reading material, available on the website. Supplementary recommended books include: *Communication Complexity* by Kushilevitz and Nisan, and *Communication Complexity and Applications* by Rao and Yehudayoff. The course webpage will have links to all lecture notes and additional materials (such as links to papers and lecture notes from similar courses.)