

UNIVERSITY OF TORONTO  
Faculty of Arts and Science

APRIL 2011 EXAMINATIONS

CSC384H1S  
Introduction to Artificial Intelligence  
Instructor: Sheila McIlraith

Duration — 3 hours

Examination Aids: Calculator permitted, but not necessary.

**PLEASE HAND IN**

Student Number: \_\_\_\_\_

Last Name: \_\_\_\_\_

First Name: \_\_\_\_\_

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*Do **not** turn this page until you have received the signal to start.*  
(In the meantime, please fill out the identification section above,  
and read the instructions below *carefully*.)

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This final examination consists of 8 questions on 21 pages (including this one). *When you receive the signal to start, please make sure that your copy of the examination is complete.* Answer each question directly on the examination paper, in the space provided, and use *the reverse side of the pages for rough work*. (If you need more space for one of your solutions, use the reverse side of the page and indicate **clearly** which part of your work should be marked.)

# 1: \_\_\_\_\_/ 15

# 2: \_\_\_\_\_/ 20

# 3: \_\_\_\_\_/ 24

# 4: \_\_\_\_\_/ 10

# 5: \_\_\_\_\_/ 12

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# 7: \_\_\_\_\_/ 12

# 8: \_\_\_\_\_/ 21

*Good Luck!*

TOTAL: \_\_\_\_\_/134

**Question 1. Search [15 MARKS]**

Give *short* answers to the following questions.

(a) [5 MARKS] Depth-first iterative deepening search seems to be less efficient because every time the depth-bound is increased it starts its search from scratch. However, it actually is not any less efficient than breadth-first search. Explain.

(b) [5 MARKS] If  $h()$  is admissible and  $s$  is the start node, how is  $f^*(s)$  related to the cost of the solution found by A\* search?

(c) [5 MARKS] It can be shown that when  $h(n) = h^*(n)$ , A\* only expands nodes that lie on an optimal path to a goal. When this condition on  $h(n)$  holds does it imply that A\* will always take time linear in the solution length to find an optimal solution? Explain your answer.

**Question 2. CSP [20 MARKS]**

**The Musicians** There are three different musicians, John, Mark, and Sam. They each come from a different country, one comes from the United States, one from Australia, and one from Japan. They each play a different musical instrument, one plays the piano, one the saxophone, and one the violin. They take turns playing a solo piece of music (that is, they take turns playing alone) and each musician plays only once. The pianist plays first. John plays the saxophone and plays before the Australian. Mark comes from the United States and plays before the violinist.

Set up this problem as a CSP problem using the following variables `john`, `mark`, `sam`, `violin`, `sax`, `piano`, `aust`, `us`, `japan`. Each of these variables has domain 1,2,3 except `piano` which has domain 1. The value of each variable indicates the order in which that variable plays in the band. For example, if `aust` is assigned the value of 1, then the Australian plays first, if `piano` is assigned the value of 1, then the piano is played first, if `sam` is assigned the value of 3, then Sam plays third, etc.

(a) [5 MARKS] Translate the problem into a set of *binary* constraints between the variables. That is each constraint must be between two variables. List all of the constraints, giving:

- The two variables the constraint is over.
- The type of constraint. You may use the symbols  $=$ ,  $\neq$ ,  $<$ ,  $>$ ,  $\leq$ ,  $\geq$ , to indicate what kind of constraints hold between two variables. (*Hint: The constraints in this problem are a subset of these types.*)

(b) [12 MARKS] Find the *first solution* to this CSP by using the *Forward Checking* algorithm with *dynamic variable reordering* using the heuristic that we always instantiate next that variable with smallest remaining elements in its domain. Thus, `piano` must be the first variable instantiated during search. After `piano` then always instantiate next the variable with the least number of remaining domain values at this point in the search. *Break ties* by always instantiating the variable that lies first in the ordering:

$$\text{john} > \text{mark} > \text{sam} > \text{violin} > \text{sax} > \text{aust} > \text{us} > \text{japan}$$

When exploring the possible values of a variable, always explore the lowest value first. *If you want full marks make sure you follow these ordering rules!*

Draw the search tree explored by the above algorithm. **At each node** indicate:

- The variable being instantiated, and the value it is being assigned.
- A list of the variables that have had at least one of their values pruned by the new assignment and for such variable a list of its remaining legal values. *Note, you must follow the forward checking algorithm precisely: only prune values that would be pruned by the algorithm.*
- Mark any node where a deadend occurs because of domain wipe out (use the symbol DWO).

(c) [3 MARKS] What is the solution?

**Question 3. Logic [24 MARKS]****Logical Translation:**

John Doe was murdered last night and Detective Sam Spade was called in to solve the case. Detective Spade made the following observations:

1. *Everyone who had the opportunity and was not tall was questioned by some local detective.*
2. *Some of the murderers had the opportunity and they were only questioned by murderers.*
3. *No murderers were tall.*

Based on these observations, Detective Spade was able to verify an important **suspicion**:

4. *Some of the local detectives were murderers!*

(a) [6 MARKS] Represent Detective Spade's three observations and his one suspicion in First Order Logic. Use the following list of predicates. (If you want full marks you *must* use these predicate names.)

- $Ld(x)$ ,  $x$  is a local detective.
- $M(x)$ ,  $x$  is one of the murders.
- $T(x)$ ,  $x$  is tall.
- $Ho(x)$ ,  $x$  had the opportunity to commit the murder.
- $Q(x, y)$ ,  $x$  questioned  $y$ .

(b) [6 MARKS] Convert the formulas (except the one to be proved – “the suspicion”) to clausal form. (Recall that the 8 steps are: eliminate implication, move negation inward, standardize variables, skolemize, convert to prenex, distribute  $\vee$  over  $\wedge$ , flatten conjunctions and disjunctions, convert to clauses.) **Clearly indicate any Skolem functions or constants used in the conversion.**

(c) [2 MARKS] Convert the negation of the statement to be proved (“the suspicion”) to clausal form. (Note that we are not asking you to do the proof in this question.)

**Logical Interpretations:**

(d) [4 MARKS] What is the difference between material implication ( $\Rightarrow$ ) and logical entailment ( $\models$ ). Please explain your answer.

**Most General Unifier:**

(e) [6 MARKS] For each of the pairs below, give the most general unifier (MGU) or state why no unifier exists. If a unifier exists, provide the expression that results from the unification. In all of the expressions that follow, variables are upper case.

1.  $\text{moved}(a, W, h(X))$  and  $\text{moved}(Y, f(Y, Z), Z)$

2.  $p(X, g(X), Z)$  and  $p(Z, Y, h(Y))$

3.  $f(X, h(b, X))$  and  $f(g(P, a), h(b, g(Q, Q)))$



**Question 4. Resolution [10 MARKS]**

The following English sentences:

- Everyone who loves all animals is loved by someone.
- Anyone who kills an animal is loved by no one.
- Jack loves all animals.
- Either Jack or Curiosity killed the cat, who is named Tuna.

Can be represented in clausal form, as follows, where uppercase letters indicate variables.

1.  $(\text{animal}(f(X)), \text{loves}(g(X), X))$
2.  $(\neg \text{loves}(X, f(X)), \text{loves}(g(X), X))$
3.  $(\neg \text{animal}(Y), \neg \text{kills}(X, Y), \neg \text{loves}(Z, X))$
4.  $(\neg \text{animal}(X), \text{loves}(\text{jack}, X))$
5.  $(\text{kills}(\text{jack}, \text{tuna}), \text{kills}(\text{curiosity}, \text{tuna}))$
6.  $\text{cat}(\text{tuna})$
7.  $(\neg \text{cat}(X), \text{animal}(X))$

and the negation of the query “Did Curiosity kill the cat?”

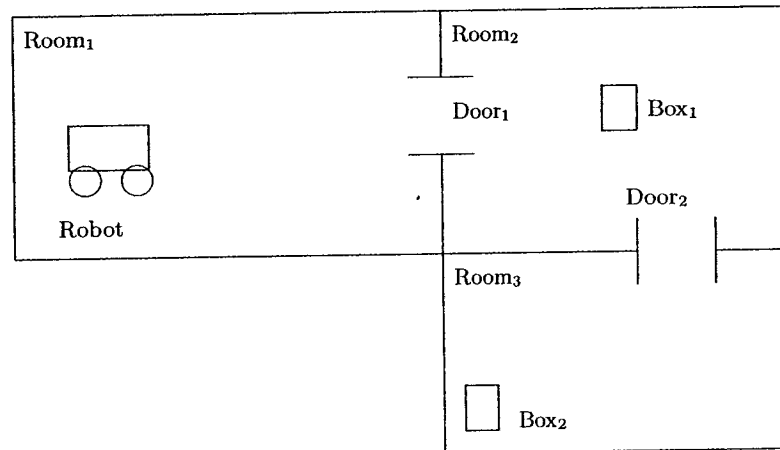
8.  $\neg \text{kills}(\text{curiosity}, \text{tuna})$

(a) [10 MARKS] Construct a proof of the empty clause by resolution refutation *using the notation developed in class*. That is, every new clause must be labeled by the resolution step that was used to generate it. For example, a clause labeled  $R[4c, 1d]\{x = a, y = f(b)\}$  means that it was generated by resolving literal  $c$  (the 3rd literal) of clause 4 against literal  $d$  (the 4th literal) of clause 1, using the MGU  $\{x = a, y = f(b)\}$ .

1.  $(\text{animal}(f(X)), \text{loves}(g(X), X))$
2.  $(\neg \text{loves}(X, f(X)), \text{loves}(g(X), X))$
3.  $(\neg \text{animal}(Y), \neg \text{kills}(X, Y), \neg \text{loves}(Z, X))$
4.  $(\neg \text{animal}(X), \text{loves}(\text{jack}, X))$
5.  $(\text{kills}(\text{jack}, \text{tuna}), \text{kills}(\text{curiosity}, \text{tuna}))$
6.  $\text{cat}(\text{tuna})$
7.  $(\neg \text{cat}(X), \text{animal}(X))$
8.  $\neg \text{kills}(\text{curiosity}, \text{tuna})$

**Question 5. Planning I** [12 MARKS]

Consider the following planning domain consisting of a robot pushing boxes between connected rooms.



We will represent this domain with the following symbols

- $b_1, b_2$  the two boxes;  $r_1, r_2, r_3$  the three rooms,  $d_1, d_2$  the two doors.
- $\text{open}(X)$ —door  $X$  is open.
- $\text{in}(X, Y)$ —box  $X$  is in room  $Y$ .
- $\text{robin}(X)$ —the robot is in room  $X$ .
- $\text{join}(X, Y, Z)$ —door  $X$  joins rooms  $Y$  and  $Z$ .

(a) [12 MARKS] Give a STRIPS (NOT ADL) representation of the following actions. In all cases use the above symbols to specify sensible preconditions and effects for each of the actions.

1. gothru(D,R1,R2) the robot goes from room R1 to R2 via door D.
2. pushthru(B,R1,R2,D) the robot pushes the box B from room R1 to room R2 via door D.
3. close(D) the robot closes door D.
4. open(D) the robot opens door D.

**Question 6. Planning II [20 MARKS]**

Consider the set of STRIPS actions:

Name	Pre	Adds	Del
T	c	b	c
S	b	a	b
R	a	b,c	

And the start state  $S_0 = \{c\}$ , and goal  $G = \{a, b, c\}$ .

(a) [6 MARKS] In order to compute a *relaxed plan heuristic* for automated planning we discussed, in class, a popular relaxation of the GraphPlan planning graph structure that could be used to compute a heuristic. Show the so-called reachability graph (the graph structure constructed from the relaxation of the planning problem) starting in  $S_0$  until the final state layer includes the goal  $G$ .

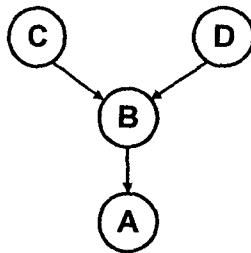
(b) [6 MARKS] If CountActions is being used as the heuristic, what will be the heuristic value of  $S_0$ . (That is, what is  $\text{CountActions}(G, S_f)$  in the reachability graph you just built, where  $S_f$  is the final state layer.) To get full marks you must show how you came up with your answer.

- (c) [2 MARKS] The CountActions heuristic given in class is not admissible. Why?
- (d) [2 MARKS] Can this heuristic be made to be admissible? And if so what price do we pay?
- (e) [2 MARKS] If two facts  $q$  and  $p$  are mutex free at state level  $i$ , can they ever become mutex at some level  $k > i$ ? Why/Why not?
- (f) [2 MARKS] In the situation calculus representation of dynamical domains, representing the non-effects of an action is an issue. What is this problem called?

**Question 7. Bayes Net I [12 MARKS]**

(a) [3 MARKS] Given the variables  $A$ ,  $B$ ,  $C$ , and  $D$ , write the joint distribution,  $Pr(A, B, C, D)$ , as a product of conditional probabilities using the *chain rule*.

(b) [5 MARKS] Now consider the following Bayes Net structure over the variables.



Show the derivation of the Bayes Net product decomposition from your answer in (a). Justify the steps in the derivation that lead to the product decomposition.

(c) [2 MARKS] Say that variable  $A$  has 4 possible values,  $B$  has 3 possible values,  $C$  has 3 possible values, and variable  $D$  has 2 possible values. How many values will be contained in the conditional probability table for  $A$ , given the Bayes Net above?

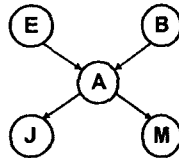
(d) [2 MARKS] In Bayes nets how are d-separation and conditional independence related.



**Question 8. Bayes Net II [21 MARKS]**

Consider a version of the burglary network given in class. Recall that a burglar can set the alarm off; an earthquake can set the alarm off; the alarm can cause Mary to call; and the alarm can cause John to call.

We model this “causal” knowledge within the topology of a Bayesian Network where Burglary (B), Earthquake (E), Alarm (A), JohnCalls (J), and MaryCalls (M) are *binary* variables taking on the value of true or false.



In what follows, we use upper case to indicate a variable and lower case to indicate the values of a variable. When you are asked to give a probability involving some *variables*, you must give the value of this probability for *all values* of the variables. *Hint, many of the questions can be answered directly without any numeric calculations.*

Please show your work including how you derive your answers and provide explanation for any mathematical simplifications that you may employ.

Let the conditional probability tables (CPTs) for the network be:

E	e	-e	B	b	-b	A	a	-a	J	j	-j	M	m	-m
	1/10	9/10		1/10	9/10	e, b	9/10	1/10	a	8/10	2/10	a	1/2	1/2
						e, -b	2/10	8/10	-a	2/10	8/10	-a	0	1
						-e, b	8/10	2/10						
						-e, -b	0	1						

(a) [3 MARKS] Given that Mary phones you ( $m$ ) what is the probability that the alarm went off ( $a$ )?

(b) [2 MARKS] Say that there was a burglary ( $b$ ) and but no earthquake ( $-e$ ), what is the expression specifying the posterior probability of John phoning you ( $j$ ) given the evidence. (You do not need to calculate a numeric answer, just give the probability expression.)

(c) [2 MARKS] What is  $Pr(M|A)$ ? (i.e., the four probability values  $Pr(m|a)$ ,  $Pr(-m|a)$ ,  $Pr(m|-a)$ ,  $Pr(-m|-a)$ ).

(d) [2 MARKS] What is  $Pr(M|A, J)$ ? (i.e., the 8 probability values  $Pr(m|a, j)$ ,  $Pr(m|a, -j)$ ,  $\dots$ ,  $Pr(-m|a, -j)$ ).

(e) [2 MARKS] What do these values tell us about the relationship between  $M$ ,  $J$  and  $A$ ?

(f) [8 MARKS] What is  $Pr(M|J)$ ? (i.e., the four probability values  $Pr(m|j)$ ,  $Pr(-m|j)$ ,  $Pr(m|-j)$ , and  $Pr(-m|-j)$ ).

(g) [2 MARKS] What do these values tell us about the relationship between  $M$  and  $J$ , and why does this relationship differ when we know  $A$ ?

THIS IS AN EXTRA PAGE FOR ROUGH WORK.

Total Marks = 134