CSC 411: Lecture 17: Ensemble Methods I

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Today

- Ensemble Methods
- Bagging
- Boosting

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- Classifiers are different due to different sampling of training data, or randomized parameters within the classification algorithm
- Aim: take simple mediocre algorithm and transform it into a super classifier without requiring any fancy new algorithm

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- Notes:
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  - Typically applied to weak models, such as decision stumps (single-node decision trees), or linear classifiers

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- Variance-bias decomposition is a way of analyzing the generalization error as a sum of 3 terms: variance, bias and irreducible error (resulting from the problem itself)

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    2. Bias reduction: for simple models, average of models has much greater capacity than single model (e.g., hyperplane classifiers, Gaussian densities).
        - Averaging models can reduce bias substantially by increasing capacity, and control variance by fitting one component at a time (e.g., boosting)

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- Probability that majority vote wrong: error under distribution where more than $N/2$ wrong

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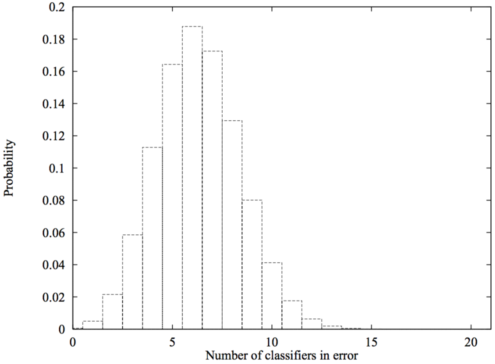

Figure : Example: The probability that exactly K (out of 21) classifiers will make an error assuming each classifier has an error rate of $\epsilon = 0.3$ and makes its errors independently of the other classifier. The area under the curve for 11 or more classifiers being simultaneously wrong is 0.026 (much less than ϵ).

[Credit: T. G Dietterich, Ensemble Methods in Machine Learning]

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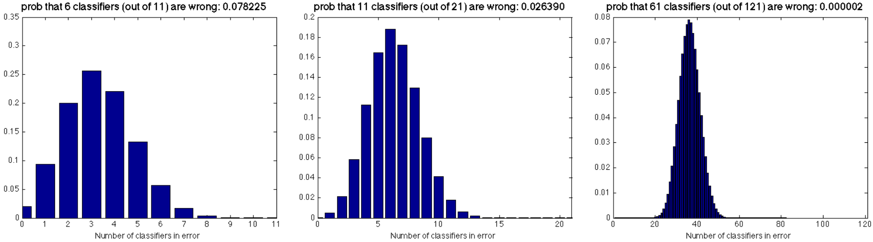

Figure : $\epsilon = 0.3$: (**left**) $N = 11$ classifiers, (**middle**) $N = 21$, (**right**) $N = 121$.

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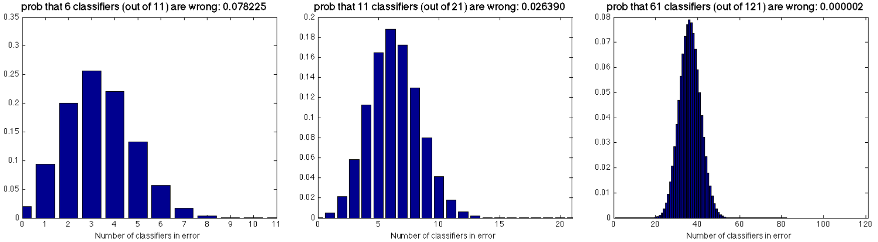

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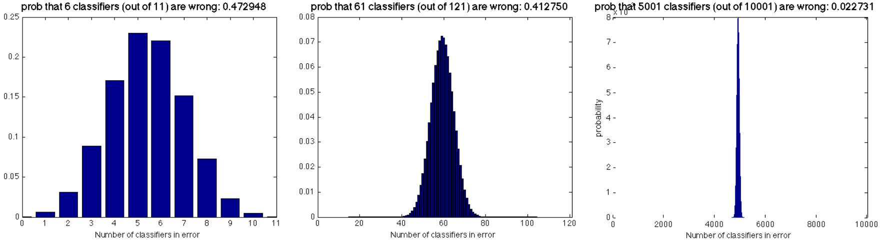

Figure : $\epsilon = 0.49$: (**left**) $N = 11$, (**middle**) $N = 121$, (**right**) $N = 10001$.

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  - *"Our experience is that most efforts should be concentrated in deriving substantially different approaches, rather than refining a simple technique."*
  - *"We strongly believe that the success of an ensemble approach depends on the ability of its various predictors to expose different complementing aspects of the data. Experience shows that this is very different than optimizing the accuracy of each individual predictor."*

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- Bagging: bootstrap aggregation (Breiman 1994)

Bagging

- Simple idea: generate M bootstrap samples from your original training set. Train on each one to get y_m, and average them

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- Each bootstrap sample is drawn with replacement, so each one contains some duplicates of certain training points and leaves out other training points completely

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- Final classifier: weighted sum of component classifiers

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  - That seems like a weak assumption but beware!
- Can you apply this learning module many times to get a strong learner that can get close to zero error rate on the training data?
  - Theorists showed how to do this and it actually led to an effective new learning procedure (Freund & Shapire, 1996)

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- Weight on example n for classifier m: $\mathbf{w}_n^m$
- Cost function for classifier m

$$J_m = \sum_{n=1}^{N} w_n^m \underbrace{[y_m(\mathbf{x}^n) \neq t^{(n)}]}_{1 \text{ if error, } 0 \text{ o.w.}} = \sum \text{weighted errors}$$

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- The weights for the next round are then

$$w_n^{m+1} = \exp\left(-\frac{1}{2} t^{(n)} \sum_{i=1}^{m} \alpha_i y_i(\mathbf{x}^{(n)})\right) = w_n^m \exp\left(-\frac{1}{2} t^{(n)} \alpha_m y_m(\mathbf{x}^{(n)})\right)$$

How to make predictions using a committee of classifiers

- Weight the binary prediction of each classifier by the quality of that classifier:

$$y_M(\mathbf{x}) = \text{sign}\left(\sum_{m=1}^{M} \frac{1}{2}\alpha_m y_m(\mathbf{x})\right)$$

- This is how to do inference, i.e., how to compute the prediction for each new example.

AdaBoost Algorithm

- Input: $\{\mathbf{x}^{(n)}, t^{(n)}\}_{n=1}^{N}$, and WeakLearn: learning procedure, produces classifier $y(\mathbf{x})$
- Initialize example weights: $w_n^m(\mathbf{x}) = 1/N$
- For m=1:M
  - $y_m(\mathbf{x}) = WeakLearn(\{\mathbf{x}\}, \mathbf{t}, \mathbf{w})$, fit classifier by minimizing

$$J_m = \sum_{n=1}^{N} w_n^m [y_m(\mathbf{x}^n) \neq t^{(n)}]$$

  - Compute unnormalized error rate

$$\epsilon_m = \frac{J_m}{\sum w_n^m}$$

  - Compute classifier coefficient $\alpha_m = \log \frac{1-\epsilon_m}{\epsilon_m}$
  - Update data weights

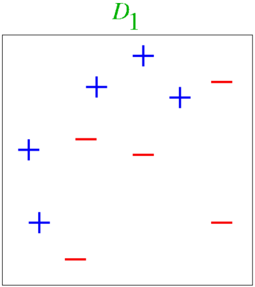
$$w_n^{m+1} = w_n^m \exp\left(-\frac{1}{2} t^{(n)} \alpha_m y_m(\mathbf{x}^{(n)})\right)$$

- Final model

$$Y(\mathbf{x}) = sign(y_M(\mathbf{x})) = sign(\sum_{m=1}^{M} \alpha_m y_m(\mathbf{x}))$$

AdaBoost Example

- Training data



[Slide credit: Verma & Thrun]

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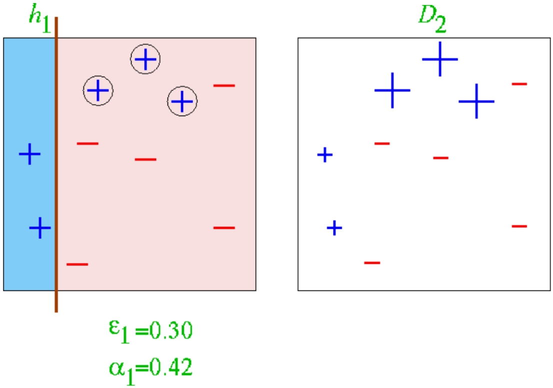

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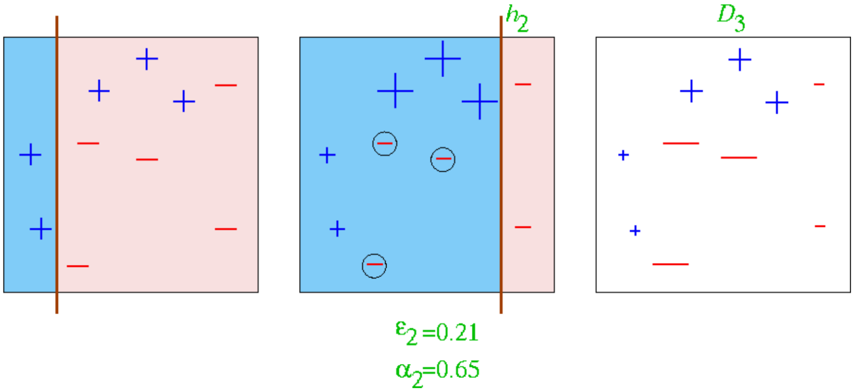
AdaBoost Example

- Round 2



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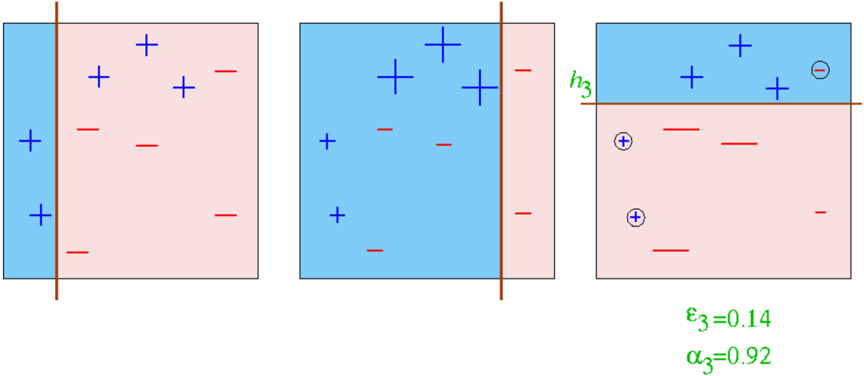
AdaBoost Example

- Round 3



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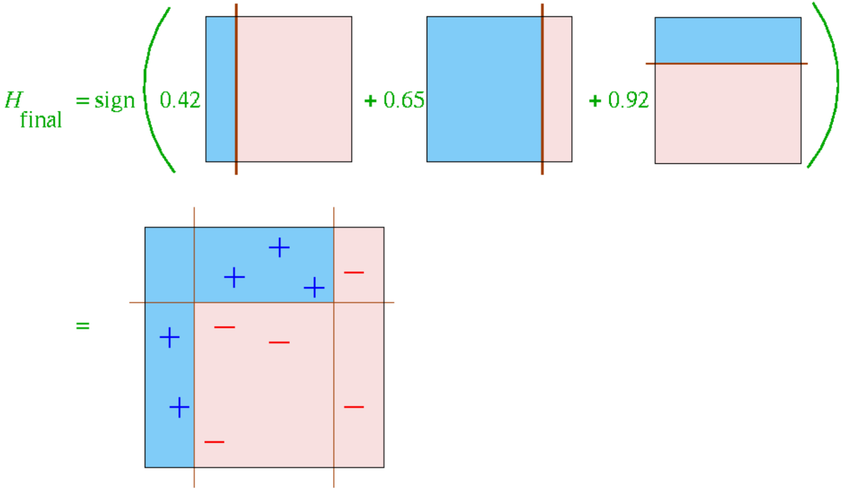
AdaBoost Example

- Final classifier



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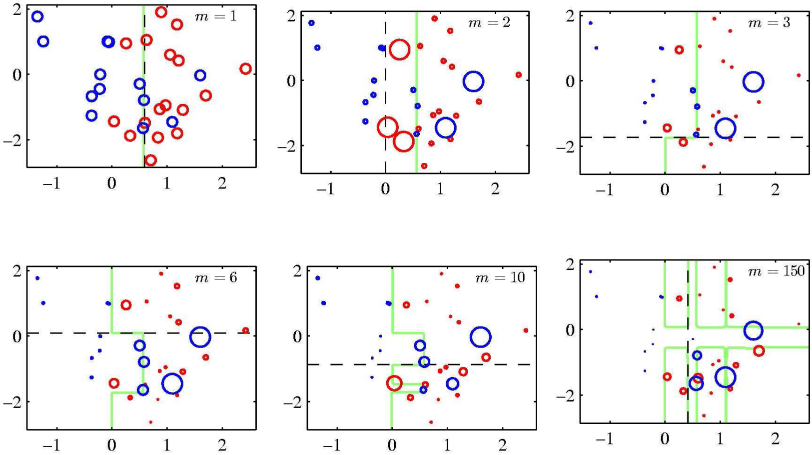

- Each figure shows the number *m* of base learners trained so far, the decision of the most recent learner (dashed black), and the boundary of the ensemble (green)

AdaBoost Applet: http://cseweb.ucsd.edu/~yfreund/adaboost/

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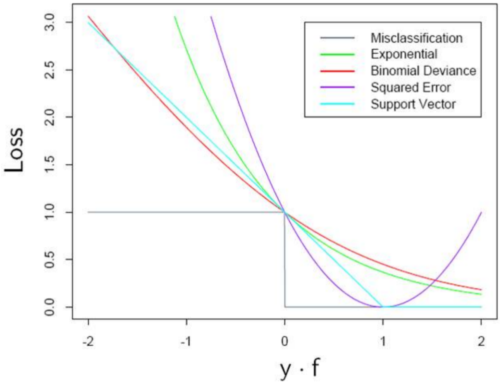
- We do this in a sequential manner, one classifier at a time

Loss Functions



- Misclassification: 0/1 loss
- Exponential loss: $\exp(-t \cdot f(x))$ (AdaBoost)
- Squared error: $(t - f(x))^2$
- Soft-margin support vector (hinge loss): $\max(0, 1 - t \cdot y)$

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$$\begin{aligned} E_{relevant} &= \sum_{n=1}^{N} \exp\left(-t^{(n)} f_{m-1}(\mathbf{x}^{(n)}) - \frac{1}{2} t^{(n)} \alpha_m y_m(\mathbf{x}^{(n)})\right) \\ &= \sum_{n=1}^{N} w_n^m \exp\left(-\frac{1}{2} t^{(n)} \alpha_m y_m(\mathbf{x}^{(n)})\right) \end{aligned}$$

with $w_n^m = \exp\left(-t^{(n)} f_{m-1}(\mathbf{x}^{(n)})\right)$

Continuing the derivation

$$E_{relevant} = \sum_{n=1}^{N} w_n^m \exp\left(-t^{(n)}\frac{\alpha_m}{2}y_m(\mathbf{x}^{(n)})\right)$$

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- The second term is constant w.r.t. $y_m(\mathbf{x})$
- Thus we minimize the weighted number of wrong examples

AdaBoost Algorithm

- Input: $\{\mathbf{x}^{(n)}, t^{(n)}\}_{n=1}^{N}$, and WeakLearn: learning procedure, produces classifier $y(\mathbf{x})$
- Initialize example weights: $w_n^m(\mathbf{x}) = 1/N$
- For m=1:M
  - $y_m(\mathbf{x}) = WeakLearn(\{\mathbf{x}\}, \mathbf{t}, \mathbf{w})$, fit classifier by minimizing

$$J_m = \sum_{n=1}^{N} w_n^m [y_m(\mathbf{x}^n) \neq t^{(n)}]$$

  - Compute unnormalized error rate

$$\epsilon_m = \frac{J_m}{\sum w_n^m}$$

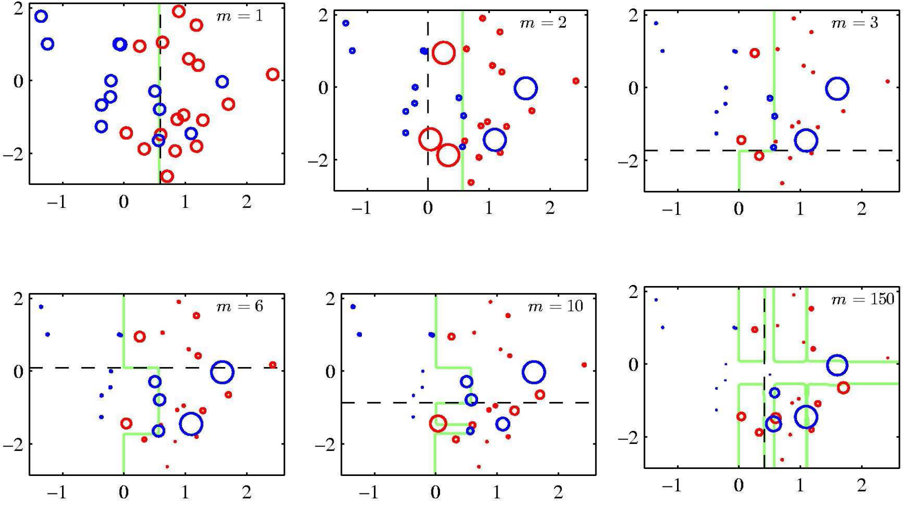
  - Compute classifier coefficient $\alpha_m = \log \frac{1-\epsilon_m}{\epsilon_m}$
  - Update data weights

$$w_n^{m+1} = w_n^m \exp\left(-\frac{1}{2} t^{(n)} \alpha_m y_m(\mathbf{x}^{(n)})\right)$$

- Final model

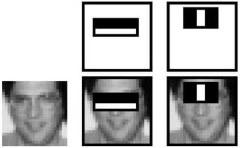
$$Y(\mathbf{x}) = sign(y_M(\mathbf{x})) = sign(\sum_{m=1}^{M} \alpha_m y_m(\mathbf{x}))$$

AdaBoost Example



An impressive example of boosting

- Viola and Jones created a very fast face detector that can be scanned across a large image to find the faces.



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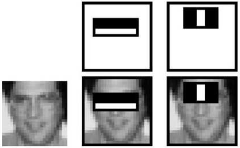

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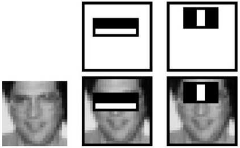

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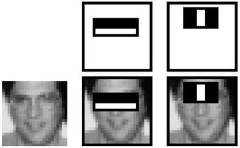

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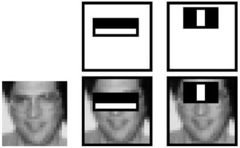

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  - There is a neat trick for computing the total intensity in a rectangle in a few operations.
    - So its easy to evaluate a huge number of base classifiers and they are very fast at runtime.
  - The algorithm adds classifiers greedily based on their quality on the weighted training cases.

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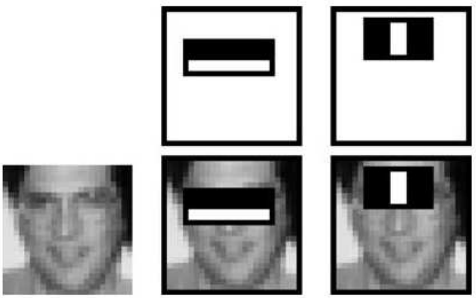
  - Change loss function for weak learners: false positives less costly than misses



AdaBoost Face Detection Results