# Visual Recognition: Filtering and Transformations

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TTI Chicago

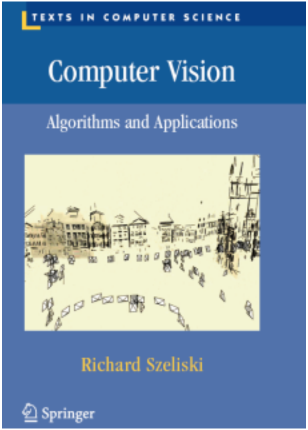
Jan 15, 2012

# Today's lecture ...

- More on Image Filtering
- Additional transformations

# Readings

- Chapter 2 and 3 of Rich Szeliski's book

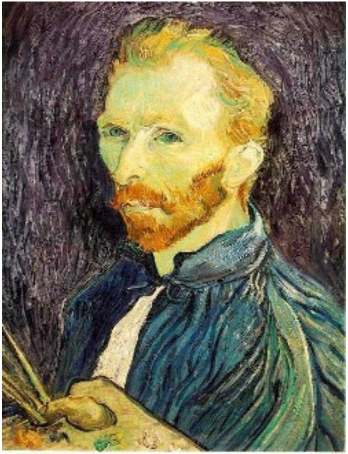



- Available online here

## Image Sub-Sampling

# Image Sub-Sampling

- Throw away every other row and column to create a 1/2 size image



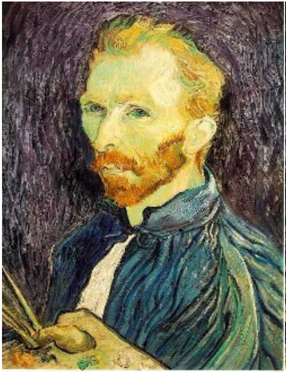
1/4

1/8

[Source: S. Seitz]

# Image Sub-Sampling

- Why does this look so crufty?



1/2

1/4 (2x zoom)

1/8 (4x zoom)

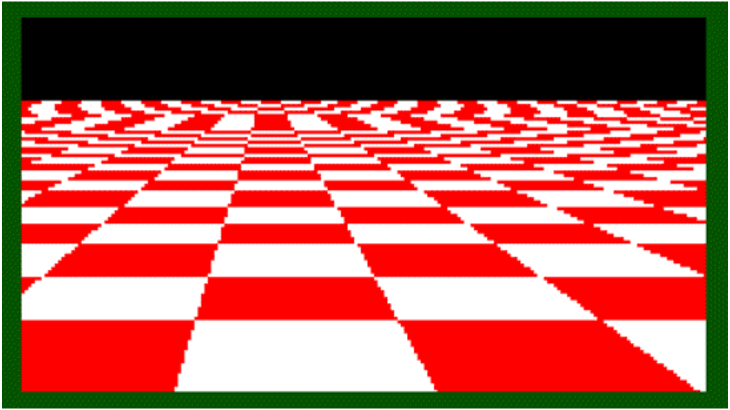
[Source: S. Seitz]

# Image Sub-Sampling

[Source: F. Durand]

# Even worse for synthetic images

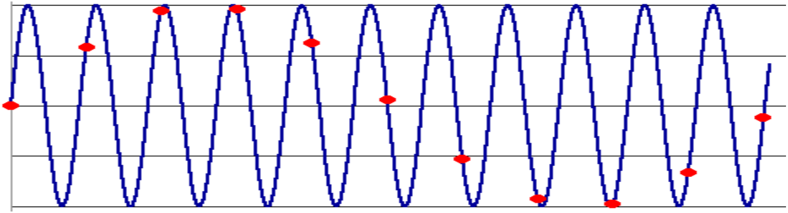
- What's happening?



[Source: L. Zhang]

# Aliasing

- Occurs when your sampling rate is not high enough to capture the amount of detail in your image



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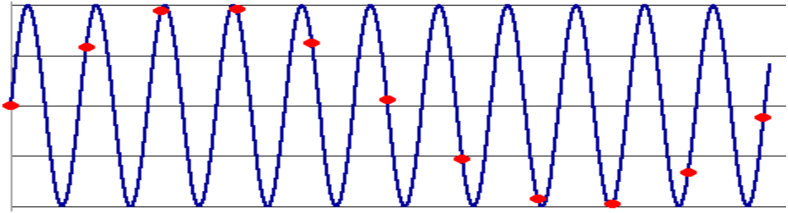

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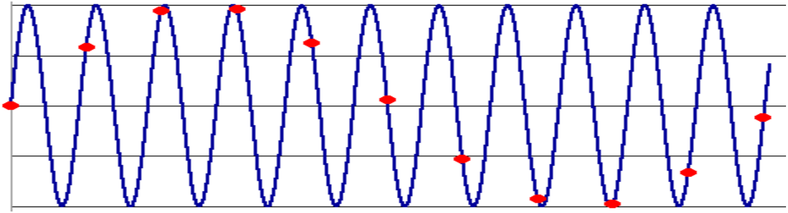

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# Aliasing problems

- **Shannons Sampling Theorem** shows that the minimum sampling

$$f_s \geq 2f_{max}$$

- If you haven't seen this... take a class on Fourier analysis... everyone should have at least one!

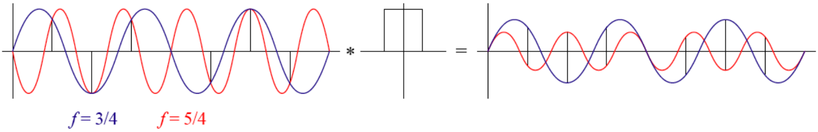
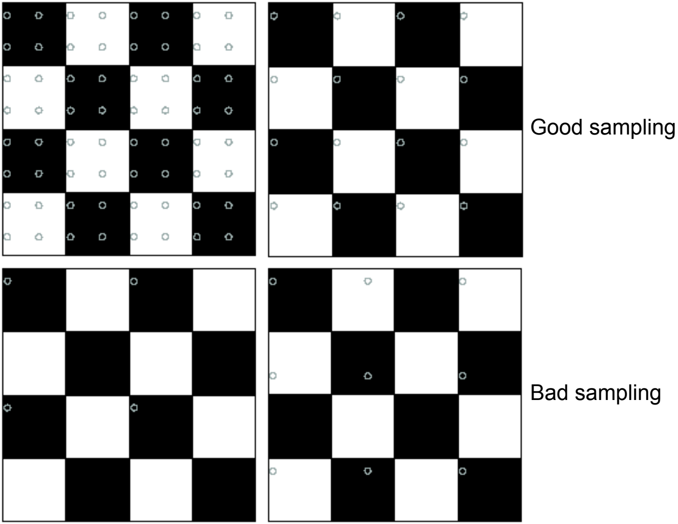



Figure: example of a 1D signal [R. Szeliski et al.]

# Nyquist limit 2D example




[Source: N. Snavely]

# Going back to Downsampling ...

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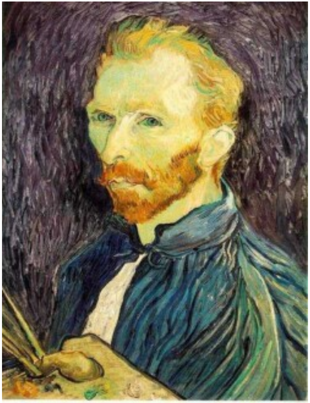
# Gaussian pre-filtering

- Solution: filter the image, then subsample



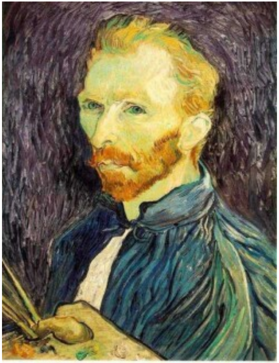
Gaussian 1/2

G 1/4

G 1/8

[Source: S. Seitz]

# Subsampling with Gaussian pre-filtering



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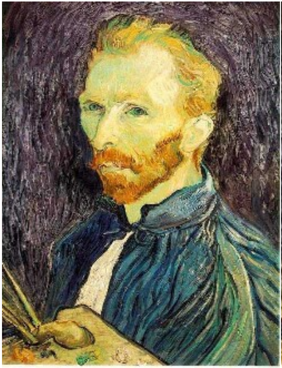
# Compare with ...



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[Source: S. Seitz]

# And in 2D...

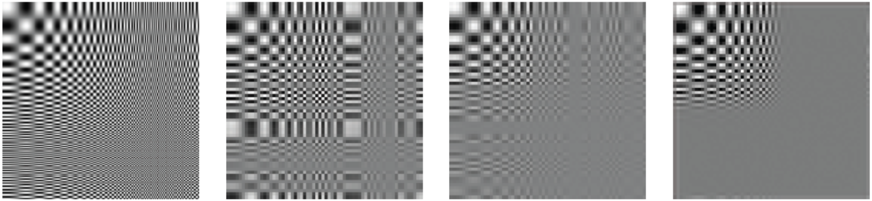

Figure: (a) Example of a 2D signal. (b–d) downsampled with different filters

[Source: R. Szeliski]

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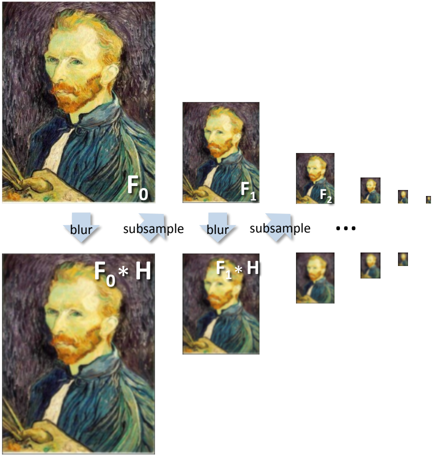


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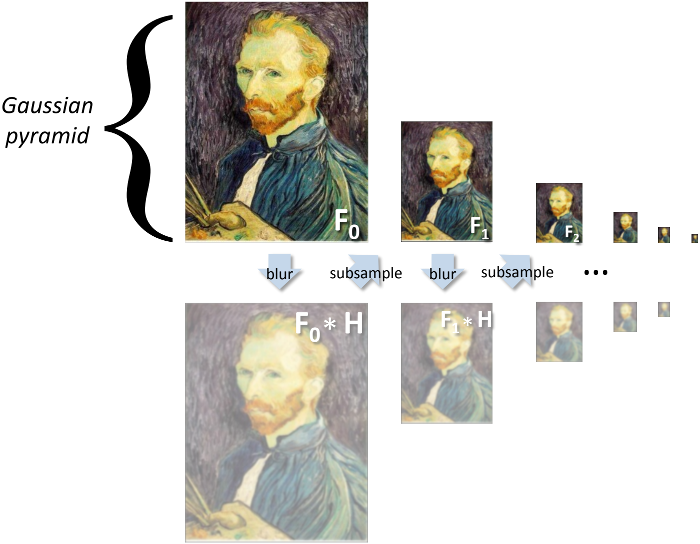
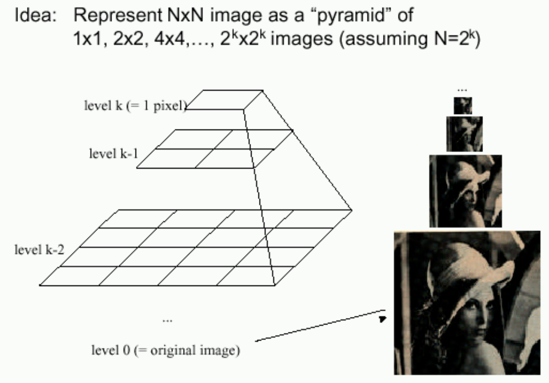


# Gaussian Pyramids [Burt and Adelson, 1983]

- In computer graphics, a *mip map* [Williams, 1983]
- A precursor to wavelet transform

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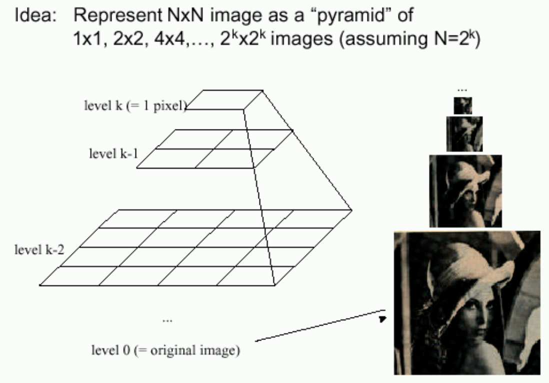


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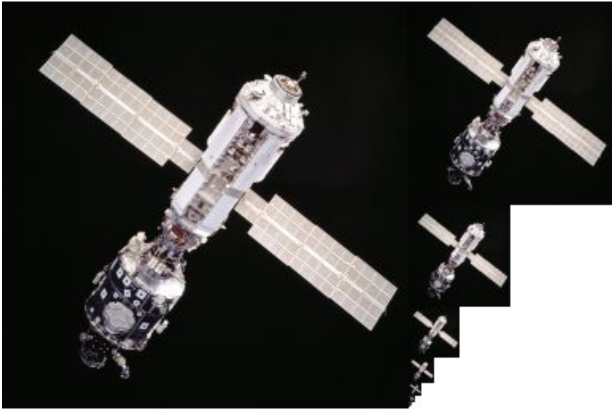
# Example of Gaussian Pyramid



[Source: N. Snavely]

# Decimation or Sub-sampling

- **Decimation**: reduces resolution

$$g(i,j) = \sum_{k,l} f(k,l)h(i - k/r, j - l/r)$$

with $r$ the down-sampling rate.

- Different filters exist to do this.

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Image Up-Sampling

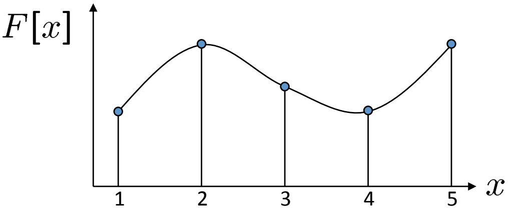
# Image Up-Sampling

- This image is too small, how can we make it 10 times as big?

- Simplest approach: repeat each row and column 10 times (Nearest neighbor interpolation)

[Source: N. Snavely]

# Image Interpolation




d = 1 in this example

Recall how a digital image is formed

$$F[x, y] = quantize\{f(xd, yd)\}$$

- It is a discrete point-sampling of a continuous function
- If we could somehow reconstruct the original function, any new image could be generated, at any resolution and scale

[Source: N. Snavely, S. Seitz]

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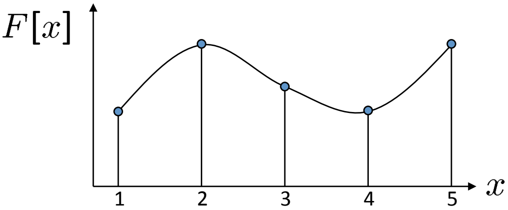


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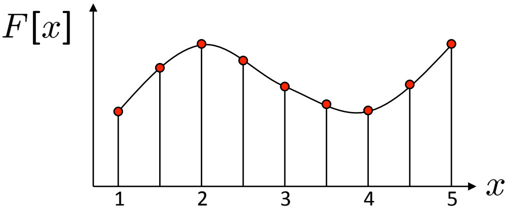


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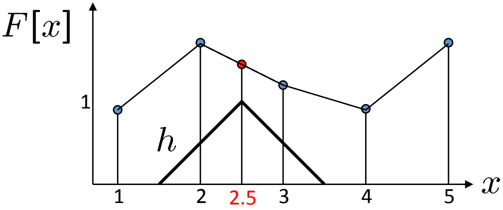


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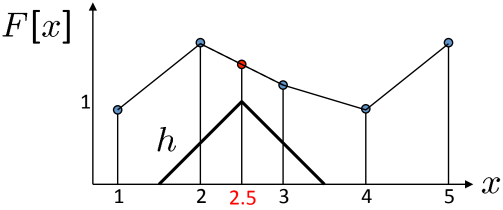


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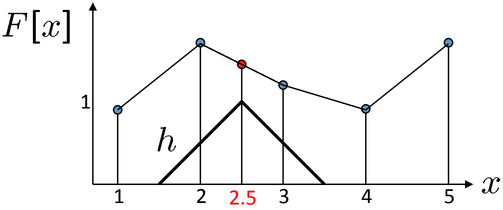


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[Source: N. Snavely, S. Seitz]

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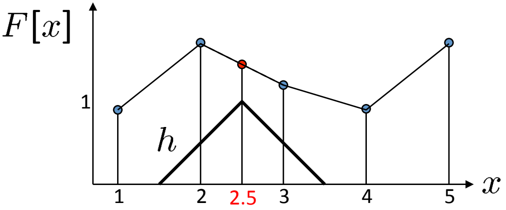


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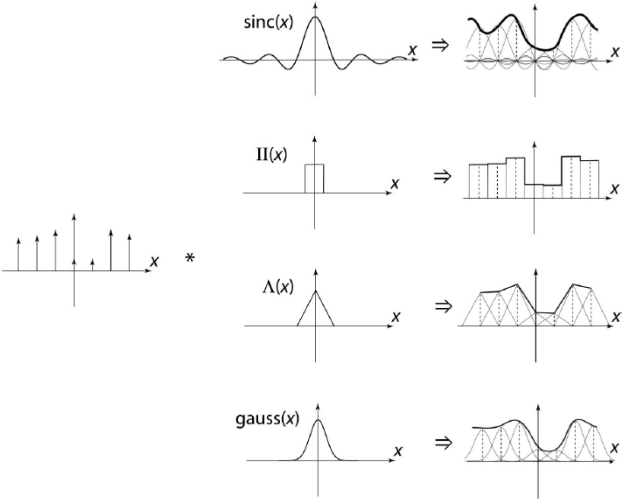



"Ideal" reconstruction

Nearest-neighbor interpolation

Linear interpolation

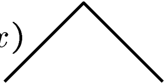
Gaussian reconstruction

Source: B. Curless

# Reconstruction filters

- What does the 2D version of this hat function look like?

$h(x)$



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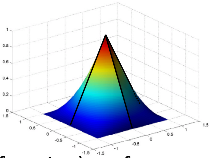

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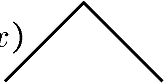

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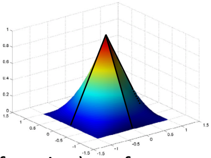

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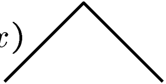

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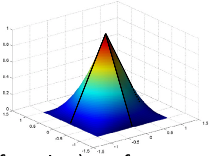

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Original image

Interpolation results

Nearest-neighbor interpolation

Bilinear interpolation

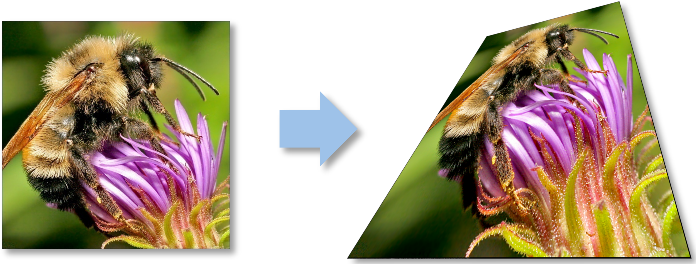
Bicubic interpolation

[Source: N. Snavely]

# Image Interpolation

What operation have we done?

Also used for *resampling*



[Source: N. Snavely]

# Depixelating Pixel Art

- Published by [Kopt et al., SIGGRAPH 2011]


Nearest-neighbor result (original: 40×16 pixels)


Our result

# More Examples

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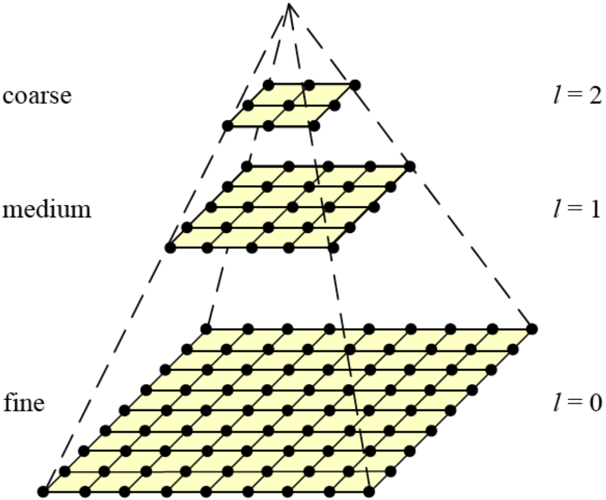


[Source: R. Szeliski]

# Interpolation and Decimation

- To **interpolate** (or upsample) an image to a higher resolution, we need to select an **interpolation kernel** with which to convolve the image

$$g(i,j) = \sum_{k,l} f(k,l)h(i-rk, j-rl)$$

  with $r$ the up-sampling rate.

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The most used one is the **Laplacian pyramid**:

- We first **blur** and **subsample** the original image by a factor of two and store this in the next level of the pyramid.
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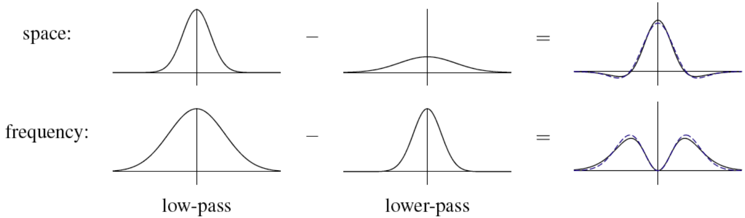


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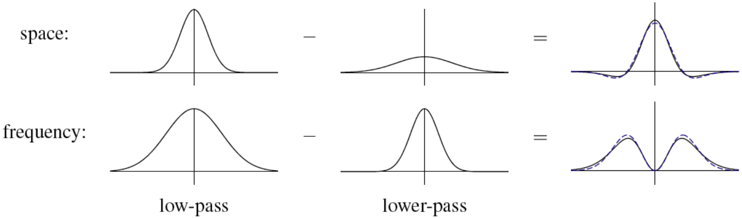


[Source: R. Szeliski]

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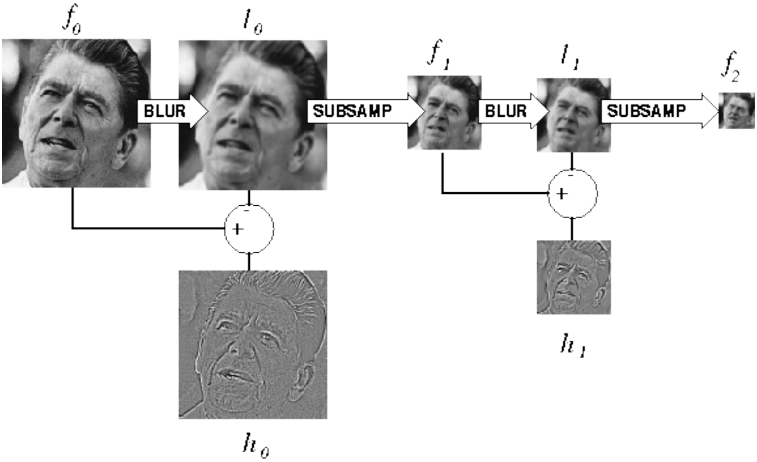


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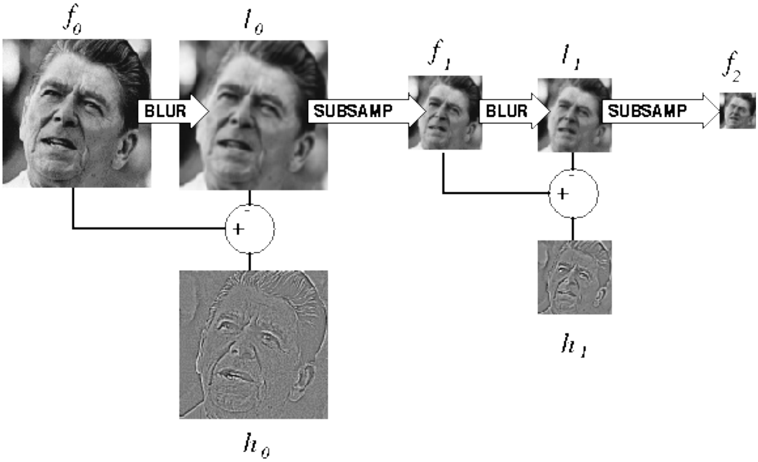
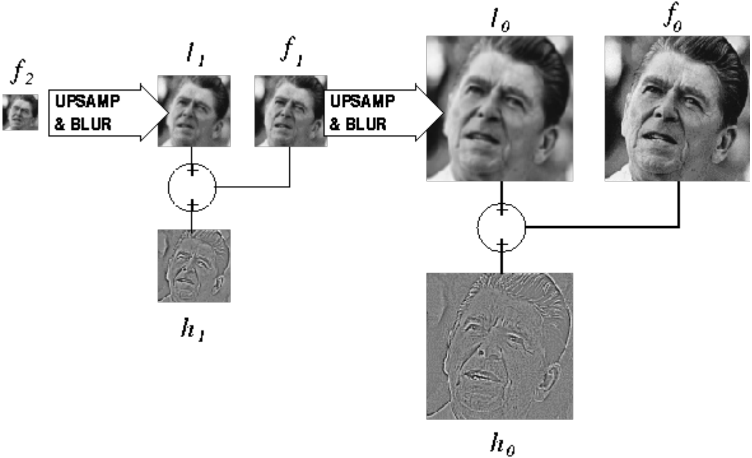



- How do we reconstruct back?

# Laplacian Pyramid Re-construction




- When is this useful?

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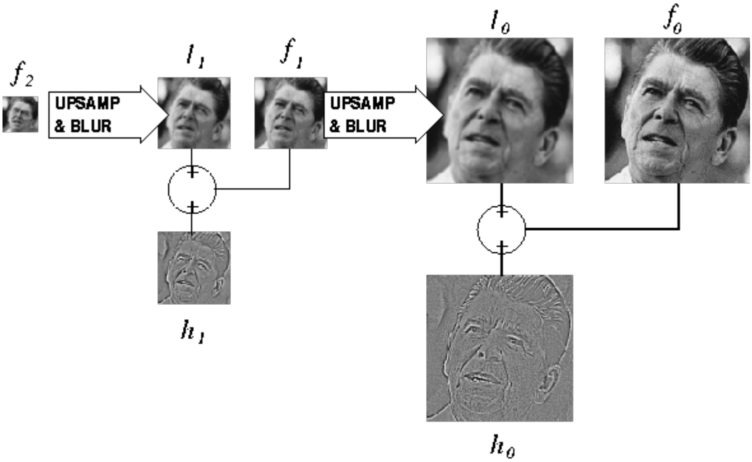


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# More Complex Filters

# Steerable Filters

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# Example of Steerable Filter

- 2D symmetric Gaussian with $\sigma = 1$ and assume constant is 1

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- The first derivative

$$G_1^0 = \frac{\partial}{\partial x} \exp\left(-x^2 + y^2\right) = -2x \exp\left(-x^2 + y^2\right)$$

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$$G_1^\theta = \cos\theta G_1^0 + \sin\theta G_1^{90}$$

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# More on steerable filters

- Because convolution is a linear operation, we can synthesize an image filtered at an arbitrary orientation by taking linear combinations of the images filtered with $G_1^0$ and $G_1^{90}$

$$\text{if} \quad R_1^0 = G_1^0 * I \quad \text{and} \quad R_1^{90} = G_1^{90} * I \quad \text{then} \quad R_1^\theta = \cos\theta R_1^0 + \sin\theta R_1^{90}$$

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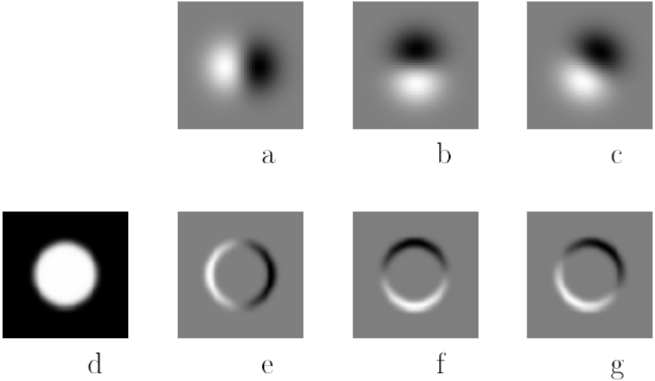


**Figure 2-1:** Example of steerable filters. (a) $G_1^{0°}$, first derivative with respect to $x$ (horizontal) of a Gaussian. (b) $G_1^{90°}$, which is $G_1^{0°}$, rotated by 90°. From a linear combination of these two filters, one can create $G_1^{\theta}$, an arbitrary rotation of the first derivative of a Gaussian. (c) $G_1^{30°}$, formed by $\frac{1}{2}G_1^{0°} + \frac{\sqrt{3}}{2}G_1^{90°}$. The same linear combinations used to synthesize $G_1^{\theta}$ from the basis filters will also synthesize the response of an image to $G_1^{\theta}$ from the responses of the image to the basis filters: (d) Image of circular disk. (e) $G_1^{0°}$ (at a smaller scale than pictured above) convolved with the disk, (d). (f) $G_1^{90°}$ convolved with (d). (g) $G_1^{30°}$ convolved with (d), obtained from $\frac{1}{2}$ [image e] $+\frac{\sqrt{3}}{2}$ [image f].

[Source: W. Freeman 91]

# More complex filters

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$$G_{\hat{\mathbf{u}}\hat{\mathbf{u}}} = u^2 G_{xx} + 2uvG_{x,y} + v^2 G_{y,y}$$

with $\hat{\mathbf{u}} = (u, v)$

## Other transformations

# Integral Images

- If an image is going to be repeatedly convolved with different box filters, it is useful to compute the **summed area table**.
- It is the running sum of all the pixel values from the origin

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$$S([i_0, i_1] \times [j_0, j_1]) = s(i_1, j_1) - s(i_1, j_0 - 1) - s(i_0 - 1, j_1) + s(i_0 - 1, j_0 - 1)$$

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# Example of Integral Images

| | | | | |
|---|---|---|---|---|
| 3 | 2 | 7 | 2 | 3 |
| 1 | 5 | 1 | 3 | 4 |
| 5 | 1 | **3** | 5 | 1 |
| 4 | 3 | 2 | 1 | 6 |
| 2 | 4 | 1 | 4 | 8 |

(a) S = 24

| | | | | |
|---|---|---|---|---|
| 3 | 5 | 12 | 14 | 17 |
| 4 | *11* | **19** | 24 | 31 |
| 9 | **17** | 28 | 38 | 46 |
| 13 | 24 | 37 | 48 | 62 |
| 15 | 30 | 44 | 59 | 81 |

(b) s = 28

| | | | | |
|---|---|---|---|---|
| **3** | 5 | 12 | *14* | 17 |
| 4 | 11 | 19 | 24 | 31 |
| 9 | 17 | 28 | 38 | 46 |
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(c) S = 24

**Figure 3.17** Summed area tables: (a) original image; (b) summed area table; (c) computation of area sum. Each value in the summed area table $s(i, j)$ (red) is computed recursively from its three adjacent (blue) neighbors (3.31). Area sums $S$ (green) are computed by combining the four values at the rectangle corners (purple) (3.32). Positive values are shown in **bold** and negative values in *italics*.

# Non-linear filters: Median filter

- We have seen **linear filters**, i.e., their response to a sum of two signals is the same as the sum of the individual responses.

$$h \circ (f + g) = h \circ f + h \circ g$$

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|---|---|---|---|---|
| 1 | 2 | 1 | 2 | 4 |
| 2 | 1 | 3 | 5 | 8 |
| 1 | 3 | 7 | 6 | 9 |
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$$w(i,j,k,l) = \exp\left(-\frac{(i-k)^2 + (j-l)^2}{2\sigma_d^2} - \frac{||f(i,j) - f(k,l)||^2}{2\sigma_r^2}\right)$$

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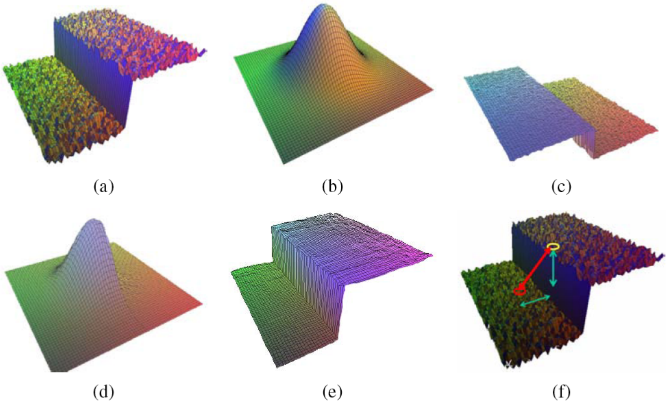


Figure: Bilateral filtering [Durand & Dorsey, 02]. (a) noisy step edge input. (b) domain filter (Gaussian). (c) range filter (similarity to center pixel value). (d) bilateral filter. (e) filtered step edge output. (f) 3D distance between pixels

[Source: R. Szeliski]

# Distance Transform

- Useful to quickly precomputing the distance to a curve or a set of points.
- Let $d(k, l)$ be some distance metric between pixel offsets, e.g., Manhattan distance

$$d(k, l) = |k| + |l|$$

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- The distance transform $D(i,j)$ of a binary image $b(i,j)$ is defined as

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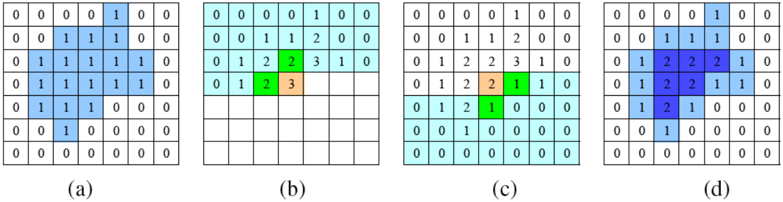


Figure: City block distance transform: (a) original binary image; (b) top to bottom (forward) raster sweep: green values are used to compute the orange value; (c) bottom to top (backward) raster sweep: green values are merged with old orange value; (d) final distance transform.

[Source: R. Szeliski]

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| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

(a)

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 2 | 0 | 0 |
| 0 | 1 | 2 | 2 | 3 | 1 | 0 |
| 0 | 1 | 2 | 3 | | | |
| | | | | | | |
| | | | | | | |
| | | | | | | |

(b)

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 2 | 0 | 0 |
| 0 | 1 | 2 | 2 | 3 | 1 | 0 |
| 0 | 1 | 2 | 2 | 1 | 1 | 0 |
| 0 | 1 | 2 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

(c)

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| 0 | 1 | 2 | 2 | 2 | 1 | 0 |
| 0 | 1 | 2 | 2 | 1 | 1 | 0 |
| 0 | 1 | 2 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

(d)

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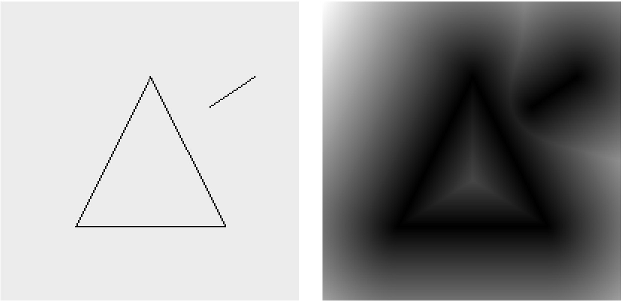
[Source: R. Szeliski]

# Example of Distance Transform

- More complicated in the Euclidean case.
- Example of a distance transform



- The ridges is the **skeleton** or **medial axis**.
- Extension: Signed distance transform.

[Source: P. Felzenszwalb]

# Fourier Transform

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# Filtering and Fourier

- Convolution can be expressed as a weighted summation of shifted input signals (sinusoids); so it is just a single sinusoid at that frequency.

$$o(x) = h(x) * s(x) = A \sin(\omega x + \phi_o)$$

  $A$ is the **gain** or **magnitude** of the filter, while the phase difference $\Delta\phi = \phi_o - \phi_i$ is the **shift** or **phase**

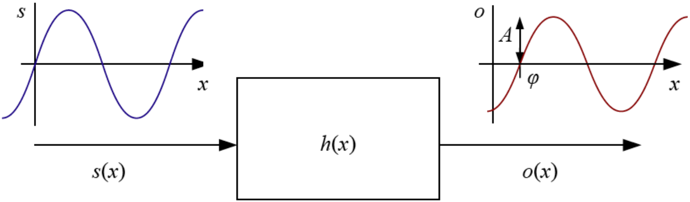


**Figure 3.24** The Fourier Transform as the response of a filter $h(x)$ to an input sinusoid $s(x) = e^{j\omega x}$ yielding an output sinusoid $o(x) = h(x) * s(x) = Ae^{j\omega x + \phi}$.

# Complex notation

- The sinusoid is express as $s(x) = e^{j\omega x} = \cos\omega x + j\sin\omega x$ and the filter sinusoid as

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where N is the length of the signal.

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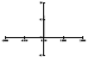
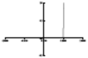
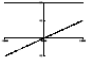
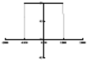
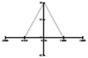
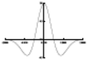
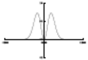
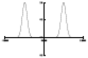
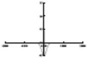
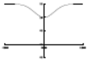
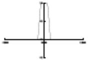
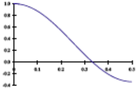
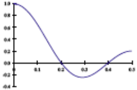
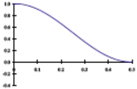
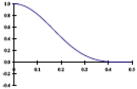
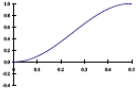
# Properties Fourier Transform

| Property | Signal | | Transform |
|---|---|---|---|
| superposition | $f_1(x) + f_2(x)$ | | $F_1(\omega) + F_2(\omega)$ |
| shift | $f(x - x_0)$ | | $F(\omega)e^{-j\omega x_0}$ |
| reversal | $f(-x)$ | | $F^*(\omega)$ |
| convolution | $f(x) * h(x)$ | | $F(\omega)H(\omega)$ |
| correlation | $f(x) \otimes h(x)$ | | $F(\omega)H^*(\omega)$ |
| multiplication | $f(x)h(x)$ | | $F(\omega) * H(\omega)$ |
| differentiation | $f'(x)$ | | $j\omega F(\omega)$ |
| domain scaling | $f(ax)$ | | $1/aF(\omega/a)$ |
| real images | $f(x) = f^*(x)$ | $\Leftrightarrow$ | $F(\omega) = F(-\omega)$ |
| Parseval's Theorem | $\sum_x [f(x)]^2$ | $=$ | $\sum_\omega [F(\omega)]^2$ |

[Source: R. Szeliski]

| Name | Signal | | | Transform | |
|---|---|---|---|---|---|
| impulse |  | $\delta(x)$ | $\Leftrightarrow$ | $1$ |  |
| shifted impulse |  | $\delta(x-u)$ | $\Leftrightarrow$ | $e^{-j\omega u}$ |  |
| box filter |  | $\text{box}(x/a)$ | $\Leftrightarrow$ | $a\text{sinc}(a\omega)$ |  |
| tent |  | $\text{tent}(x/a)$ | $\Leftrightarrow$ | $a\text{sinc}^2(a\omega)$ |  |
| Gaussian |  | $G(x;\sigma)$ | $\Leftrightarrow$ | $\frac{\sqrt{2\pi}}{\sigma}G(\omega;\sigma^{-1})$ |  |
| Laplacian of Gaussian |  | $(\frac{x^2}{\sigma^4}-\frac{1}{\sigma^2})G(x;\sigma)$ | $\Leftrightarrow$ | $-\frac{\sqrt{2\pi}}{\sigma}\omega^2 G(\omega;\sigma^{-1})$ |  |
| Gabor |  | $\cos(\omega_0 x)G(x;\sigma)$ | $\Leftrightarrow$ | $\frac{\sqrt{2\pi}}{\sigma}G(\omega\pm\omega_0;\sigma^{-1})$ |  |
| unsharp mask |  | $(1+\gamma)\delta(x)$ $-\gamma G(x;\sigma)$ | $\Leftrightarrow$ | $(1+\gamma)-$ $\frac{\sqrt{2\pi}\gamma}{\sigma}G(\omega;\sigma^{-1})$ |  |
| windowed sinc |  | $\text{rcos}(x/(aW))$ $\text{sinc}(x/a)$ | $\Leftrightarrow$ | (see Figure 3.29) |  |

[Source: R. Szeliski]

| Name | Kernel | Transform | Plot |
|---|---|---|---|
| box-3 | $\frac{1}{3}\begin{array}{\|c\|c\|c\|}\hline 1 & 1 & 1 \\ \hline\end{array}$ | $\frac{1}{3}(1+2\cos\omega)$ |  |
| box-5 | $\frac{1}{5}\begin{array}{\|c\|c\|c\|c\|c\|}\hline 1 & 1 & 1 & 1 & 1 \\ \hline\end{array}$ | $\frac{1}{5}(1+2\cos\omega+2\cos 2\omega)$ |  |
| linear | $\frac{1}{4}\begin{array}{\|c\|c\|c\|}\hline 1 & 2 & 1 \\ \hline\end{array}$ | $\frac{1}{2}(1+\cos\omega)$ |  |
| binomial | $\frac{1}{16}\begin{array}{\|c\|c\|c\|c\|c\|}\hline 1 & 4 & 6 & 4 & 1 \\ \hline\end{array}$ | $\frac{1}{4}(1+\cos\omega)^2$ |  |
| Sobel | $\frac{1}{2}\begin{array}{\|c\|c\|c\|}\hline -1 & 0 & 1 \\ \hline\end{array}$ | $\sin\omega$ |  |
| corner | $\frac{1}{2}\begin{array}{\|c\|c\|c\|}\hline -1 & 2 & -1 \\ \hline\end{array}$ | $\frac{1}{2}(1-\cos\omega)$ |  |

[Source: R. Szeliski]

# 2D Fourier Transform

- Same as 1D, but in 2D. Now the sinusoid is

$$s(x, y) = \sin(\omega_x x + \omega_y y)$$

- The 2D Fourier in continuous domain is then

$$H(\omega_x, \omega_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x, y) e^{-j(\omega_x x + \omega_y y)} dxdy$$

and in the discrete domain

$$H(k_x, k_y) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} h(x, y) e^{-2\pi j \frac{k_x x + k_y y}{MN}}$$

where M and N are the width and height of the image.

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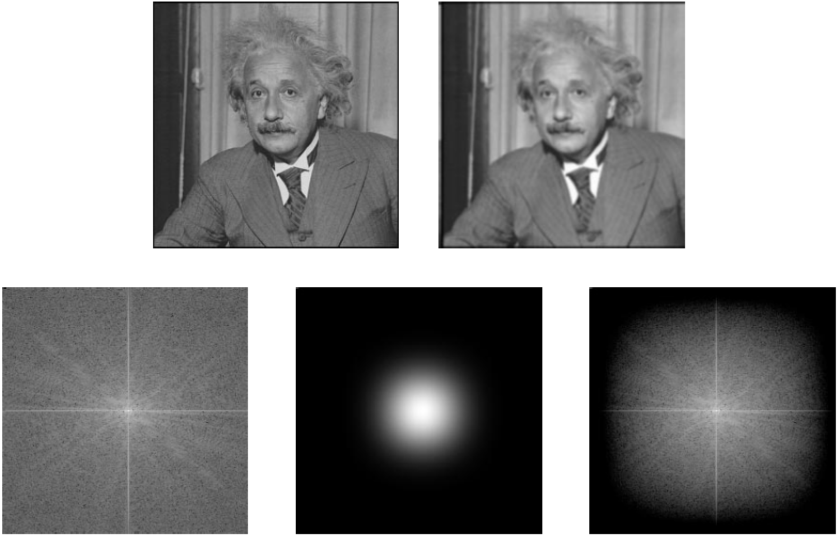
# Example of 2D Fourier Transform



[Source: A. Jepson]

Next class ... image features