Computer Vision: Image Alignment

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TTI Chicago

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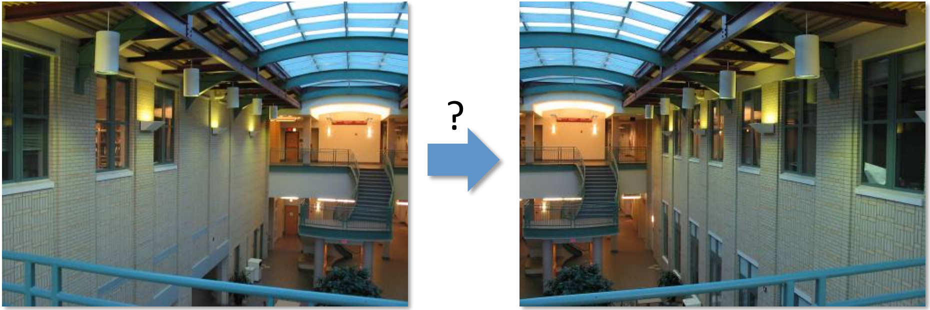
Readings

- Chapter 2.1, 3.6, 4.3 and 6.1 of Szeliski's book
- Chapter 1 of Forsyth & Ponce

What did we see in class last week?

What is the geometric relationship between these images?



[Source: N. Snavely]

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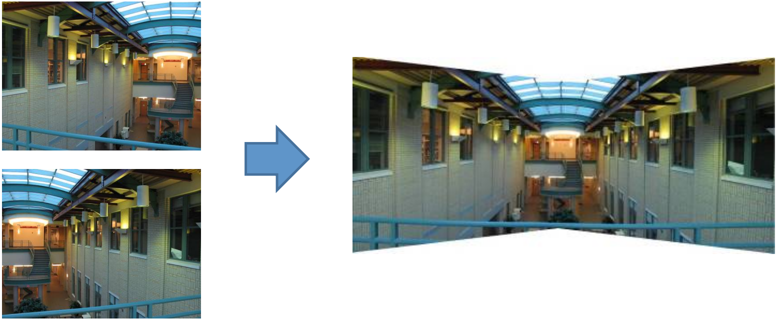


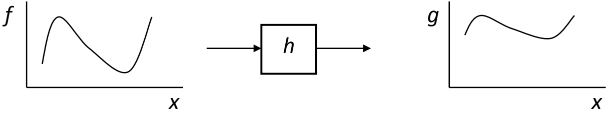
Very important for creating mosaics!

[Source: N. Snavely]

Image Warping

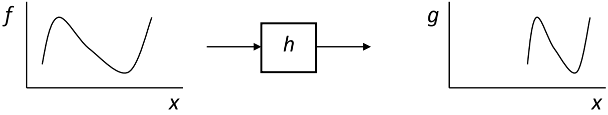
- **Image filtering**: change *range* of image

$$g(x) = h(f(x))$$

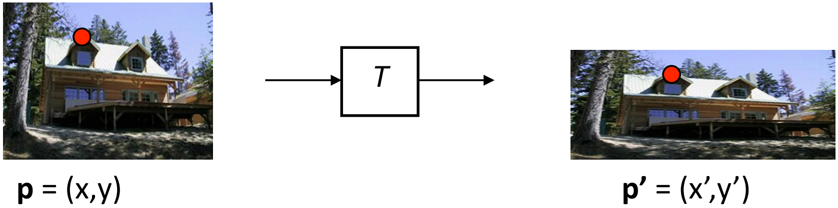


- **Image warping**: change *domain* of image

$$g(x) = f(h(x))$$



[Source: R. Szeliski]



- Transformation T is a coordinate-changing machine:

$$p' = T(p)$$

- What does it mean that T is global?
    - Is the same for any point p
    - Can be described by just a few numbers (parameters)

[Source: N. Snavely]

Forward and Inverse Warping

- **Forward Warping**: Send each pixel $f(x)$ to its corresponding location $(x', y') = T(x, y)$ in $g(x', y')$

> **procedure** *forwardWarp*$(f, \boldsymbol{h}, \textbf{out } g)$:
>
> For every pixel $\boldsymbol{x}$ in $f(\boldsymbol{x})$
>
> 1. Compute the destination location $\boldsymbol{x}' = \boldsymbol{h}(\boldsymbol{x})$.
> 2. Copy the pixel $f(\boldsymbol{x})$ to $g(\boldsymbol{x}')$.

- **Inverse Warping:** Each pixel at the destination is sampled from the original image

> **procedure** *inverseWarp*$(f, \boldsymbol{h}, \textbf{out } g)$:
>
> For every pixel $\boldsymbol{x}'$ in $g(\boldsymbol{x}')$
>
> 1. Compute the source location $\boldsymbol{x} = \hat{\boldsymbol{h}}(\boldsymbol{x}')$
> 2. Resample $f(\boldsymbol{x})$ at location $\boldsymbol{x}$ and copy to $g(\boldsymbol{x}')$

All 2D Linear Transformations

Linear transformations are combinations of

- Scale,
- Rotation
- Shear
- Mirror

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

[Source: N. Snavely]

All 2D Linear Transformations

Properties of linear transformations:

- Origin maps to origin
- Lines map to lines

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- Closed under composition

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} \begin{bmatrix} i & j \\ k & l \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

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[Source: N. Snavely]

Affine Transformations

Affine transformations are combinations of

- Linear transformations, and
- Translations

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

Properties of affine transformations:

- Origin does not necessarily map to origin
- Lines map to lines
- Parallel lines remain parallel
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[Source: N. Snavely]

Projective Transformations

- Affine transformations and Projective warps

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

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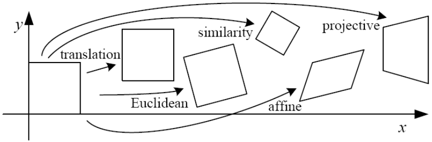
2D Image Tranformations



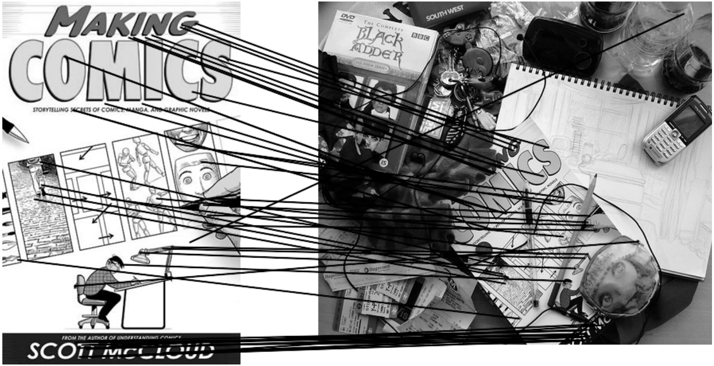
Transformation	Matrix	# DoF	Preserves	Icon
translation	$\begin{bmatrix} \boldsymbol{I} \mid \boldsymbol{t} \end{bmatrix}_{2\times3}$	2	orientation	
rigid (Euclidean)	$\begin{bmatrix} \boldsymbol{R} \mid \boldsymbol{t} \end{bmatrix}_{2\times3}$	3	lengths	
similarity	$\begin{bmatrix} s\boldsymbol{R} \mid \boldsymbol{t} \end{bmatrix}_{2\times3}$	4	angles	
affine	$\begin{bmatrix} \boldsymbol{A} \end{bmatrix}_{2\times3}$	6	parallelism	
projective	$\begin{bmatrix} \tilde{\boldsymbol{H}} \end{bmatrix}_{3\times3}$	8	straight lines	

- These transformations are a nested set of groups
- Closed under composition and inverse is a member

Computing transformations

Given a set of matches between images A and B

- How can we compute the transform T from A to B?
- Find transform T that best agrees with the matches



[Source: N. Snavely]

Least squares formulation

- For each point (x_i, y_i) we have

$$\begin{aligned} x_i + x_t &= x_i' \\ y_i + y_t &= y_i' \end{aligned}$$

- We define the residuals as

$$\begin{aligned} r_{x_i}(x_t) &= x_i + x_t - x_i' \\ r_{y_i}(y_t) &= y_i + y_t - y_i' \end{aligned}$$

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- **Goal:** minimize sum of squared residuals

$$C(x_t, y_t) = \sum_{i=1}^{n} (r_{x_i}(x_t)^2 + r_{y_i}(y_t)^2)$$

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[Source: N. Snavely]

Matrix Formulation

- We can also write as a matrix equation

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ & \vdots \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} x'_1 - x_1 \\ y'_1 - y_1 \\ x'_2 - x_2 \\ y'_2 - y_2 \\ \vdots \\ x'_n - x_n \\ y'_n - y_n \end{bmatrix}$$

$$\underset{2n \times 2}{\mathbf{A}} \quad \underset{2 \times 1}{\mathbf{t}} = \underset{2n \times 1}{\mathbf{b}}$$

- Solve for **t** by looking at the fixed-point equation

Affine Transformations

When we are dealing with an affine transformation

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

- How many unknowns?

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Affine Transformation Cost Function

- We can write the residuals as

$$\begin{aligned} r_{x_i}(a,b,c,d,e,f) &= (ax_i + by_i + c) - x'_i \\ r_{y_i}(a,b,c,d,e,f) &= (dx_i + ey_i + f) - y'_i \end{aligned}$$

- Cost function

$$C(a,b,c,d,e,f) = \sum_{i=1}^{N} \left(r_{x_i}(a,b,c,d,e,f)^2 + r_{y_i}(a,b,c,d,e,f)^2 \right)$$

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- And in matrix form ...

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[Source: N. Snavely]

Matrix form

$$
\begin{bmatrix}
x_1 & y_1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & x_1 & y_1 & 1 \\
x_2 & y_2 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & x_2 & y_2 & 1 \\
 & & & \vdots & & \\
x_n & y_n & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & x_n & y_n & 1
\end{bmatrix}
\begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix}
=
\begin{bmatrix} x'_1 \\ y'_1 \\ x'_2 \\ y'_2 \\ \vdots \\ x'_n \\ y'_n \end{bmatrix}
$$

$$
\underset{2n \times 6}{\mathbf{A}} \quad \underset{6 \times 1}{\mathbf{t}} = \underset{2n \times 1}{\mathbf{b}}
$$

[Source: N. Snavely]

General Formulation

- Let $x' = f(x; p)$ be a parametric transformation
- In the case of translation, similarity and affine, there is a linear relationship between the amount of motion $\Delta x = x' - x$ and the unknown parameters

$$\Delta x = x' - x = \mathbf{J}(x)\mathbf{p}$$

with $\mathbf{J} = \frac{\partial f}{\partial p}$ is the **Jacobian** of the transformation $\mathbf{f}$ with respect to the motion parameters $\mathbf{p}$

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General Formulation

Transform	Matrix	Parameters p	Jacobian J
translation	$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \end{bmatrix}$	(t_x, t_y)	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
Euclidean	$\begin{bmatrix} c_\theta & -s_\theta & t_x \\ s_\theta & c_\theta & t_y \end{bmatrix}$	(t_x, t_y, θ)	$\begin{bmatrix} 1 & 0 & -s_\theta x - c_\theta y \\ 0 & 1 & c_\theta x - s_\theta y \end{bmatrix}$
similarity	$\begin{bmatrix} 1+a & -b & t_x \\ b & 1+a & t_y \end{bmatrix}$	(t_x, t_y, a, b)	$\begin{bmatrix} 1 & 0 & x & -y \\ 0 & 1 & y & x \end{bmatrix}$
affine	$\begin{bmatrix} 1+a_{00} & a_{01} & t_x \\ a_{10} & 1+a_{11} & t_y \end{bmatrix}$	$(t_x, t_y, a_{00}, a_{01}, a_{10}, a_{11})$	$\begin{bmatrix} 1 & 0 & x & y & 0 & 0 \\ 0 & 1 & 0 & 0 & x & y \end{bmatrix}$

- Let's do a couple on the board!

General Formulation

- The sum of square residuals is then

$$
\begin{aligned}
E_{LLS} &= \sum_i ||\mathbf{J}(\mathbf{x}_i)\mathbf{p} - \Delta\mathbf{x}_i||_2^2 \\
&= \mathbf{p}^T[\sum_i \mathbf{J}^T(\mathbf{x}_i)\mathbf{J}(\mathbf{x}_i)]\mathbf{p} - 2\mathbf{p}^T[\sum_i \mathbf{J}^T(\mathbf{x}_i)\Delta\mathbf{x}_i)] + \sum_i ||\Delta\mathbf{x}_i||_2
\end{aligned}
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- We can compute the solution by looking for a fixed point, yielding

$$\mathbf{A}\mathbf{p} = \mathbf{b}$$

with $\mathbf{A} = \sum_i \mathbf{J}^T(\mathbf{x}_i)\mathbf{J}(\mathbf{x}_i)$ the **Hessian** and $\mathbf{b} = \sum_i \mathbf{J}^T(\mathbf{x}_i)\Delta\mathbf{x}_i$

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Uncertainty Weighting

- The above solution assumes that all feature points are matched with same accuracy.
- If we associate a scalar variance σ_i^2 with each correspondence, we can minimize the **weighted least squares** problem

$$E_{WLS} = \sum_i \sigma_i^{-2} ||\mathbf{r}_i||_2^2$$

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with Σ, the matrix containing for each observation the noise level

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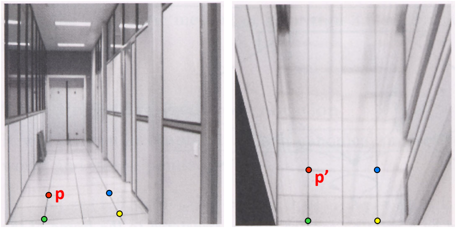

To unwarp (rectify) and image

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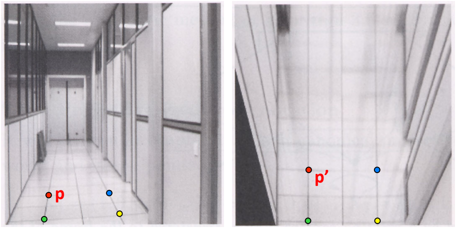

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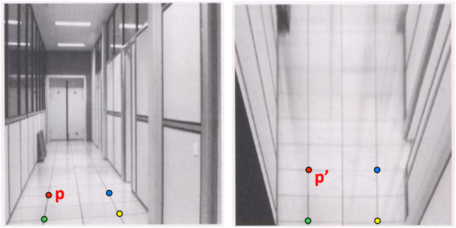

To unwarp (rectify) and image

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- solve equations of the form: $\mathbf{wp}' = \mathbf{Hp}$
  - linear in unknowns: $\mathbf{w}$ and coefficients of $\mathbf{H}$

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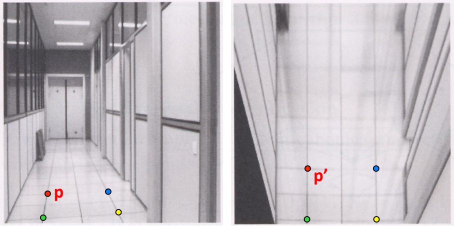

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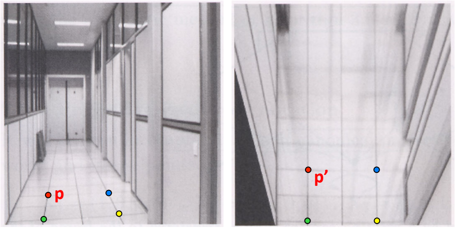
- solve equations of the form: $\mathbf{w}\mathbf{p}' = \mathbf{H}\mathbf{p}$
  - linear in unknowns: $\mathbf{w}$ and coefficients of $\mathbf{H}$
  - $\mathbf{H}$ is defined up to an arbitrary scale factor

Homographies



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[Source: N. Snavely]

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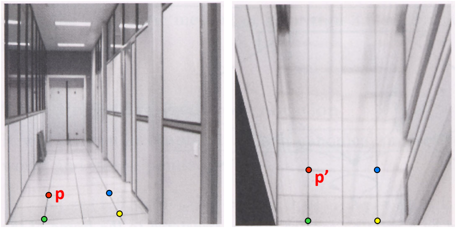

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[Source: N. Snavely]

Solving for Homographies

$$\begin{bmatrix} ax'_i \\ ay'_i \\ a \end{bmatrix} = \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

- To get to non-homogenous coordinates

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- This is still linear in the unknowns

$$\begin{bmatrix} x_i & y_i & 1 & 0 & 0 & 0 & -x_i'x_i & -x_i'y_i & -x_i' \\ 0 & 0 & 0 & x_i & y_i & 1 & -y_i'x_i & -y_i'y_i & -y_i' \end{bmatrix} \begin{bmatrix} h_{00} \\ h_{01} \\ h_{02} \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{20} \\ h_{21} \\ h_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

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Image Alignment Algorithm

Given images A and B

1. Compute image features for A and B
2. Match features between A and B
3. Compute homography between A and B using least squares on set of matches

Is there a problem with this?

[Source: N. Snavely]

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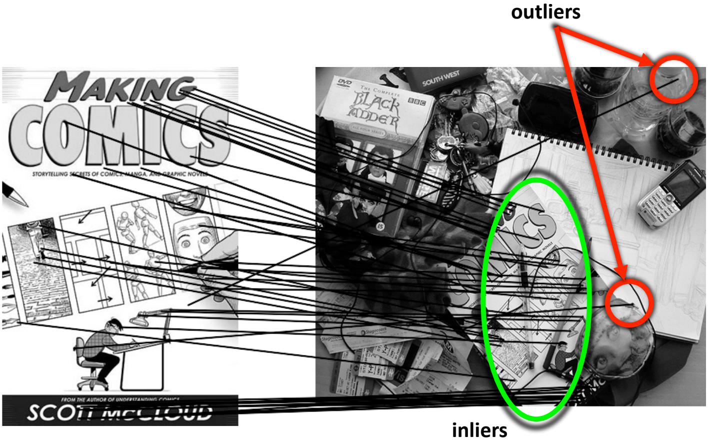
Robustness



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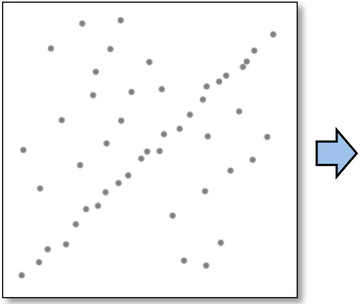
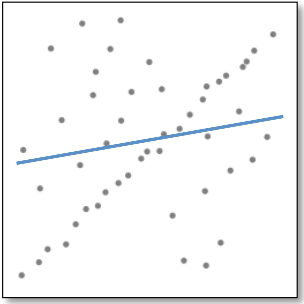
Simple case

- Lets consider a simpler example ... linear regression



Problem: Fit a line to these datapoints



Least squares fit

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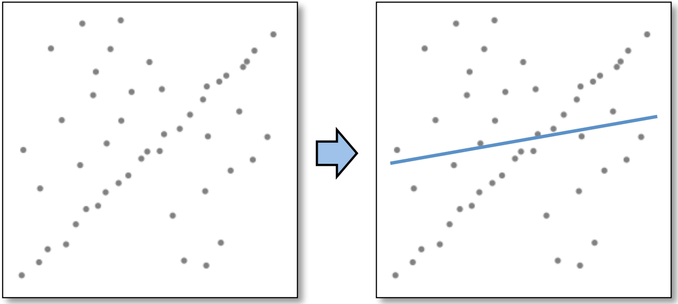

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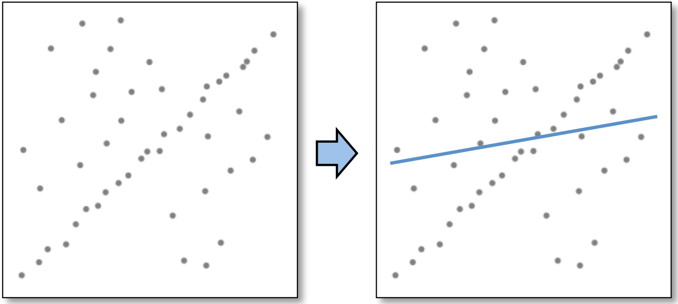

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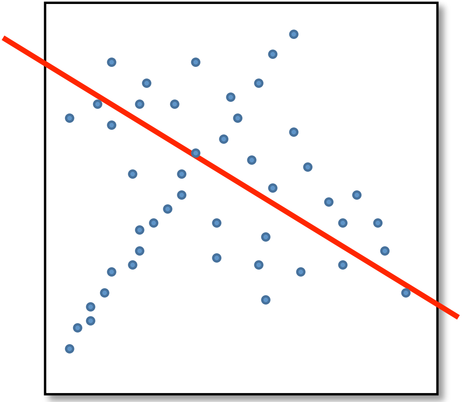

Inliers: 3

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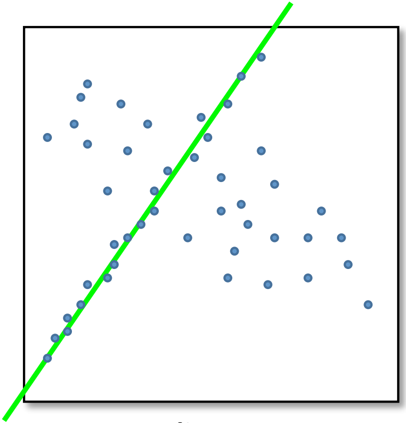

Inliers: 20

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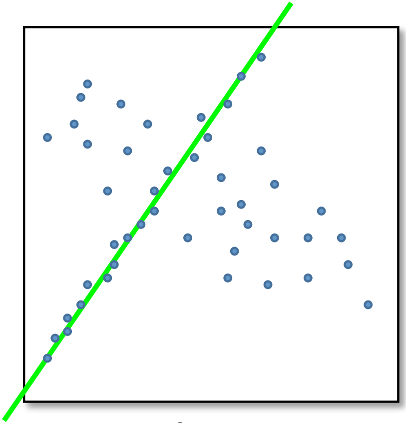

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What's the problem with this approach?

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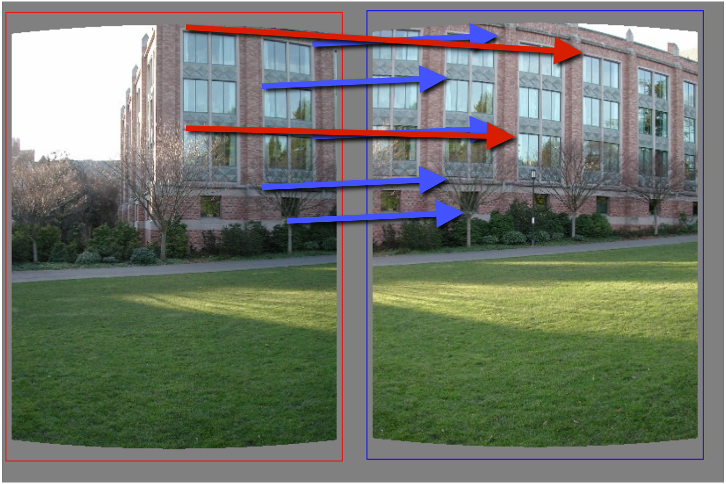
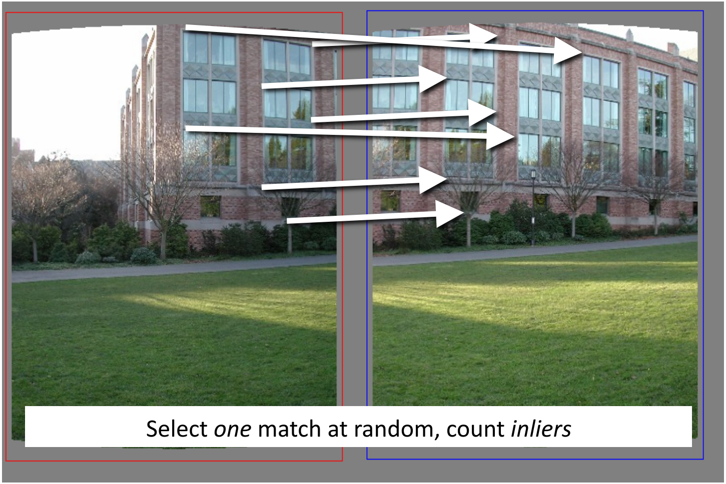
Translations



[Source: N. Snavely]

RAndom SAmple Consensus



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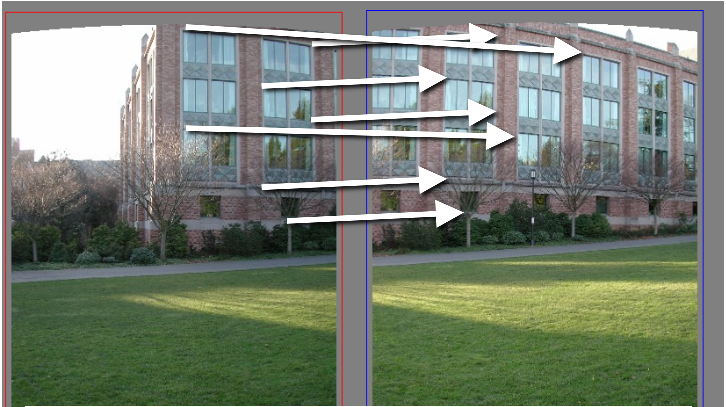

Select another match at random, count *inliers*

[Source: N. Snavely]

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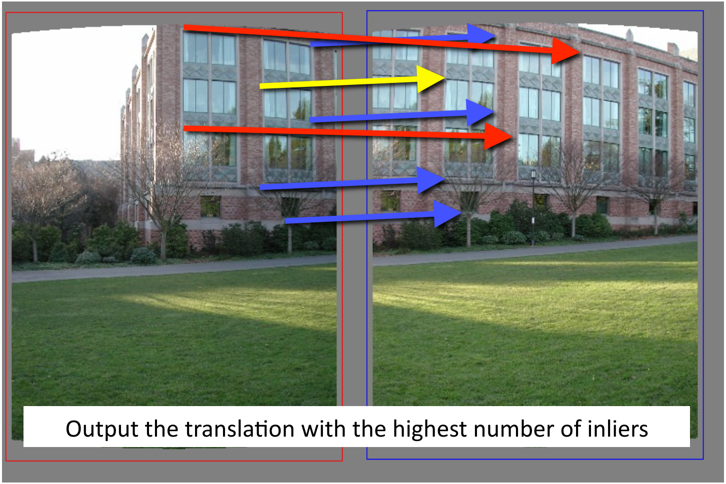

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RANSAC

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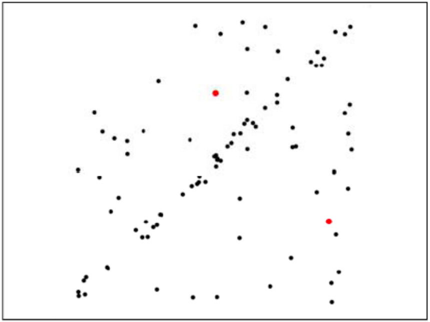
RANSAC for line fitting example

1. Randomly select minimal subset of points
2. Hypothesize a model



[Source: R. Raguram]

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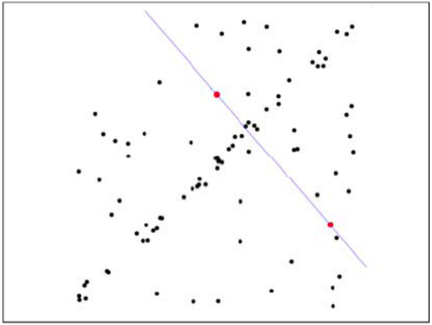

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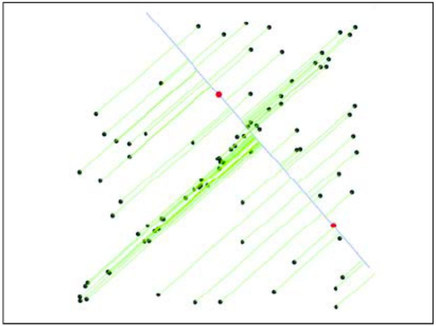

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RANSAC for line fitting example

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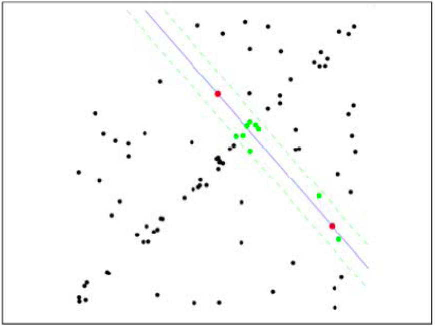

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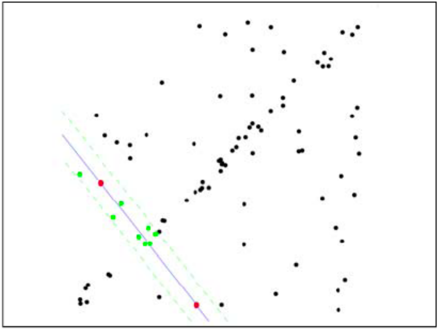

[Source: R. Raguram]

RANSAC for line fitting example

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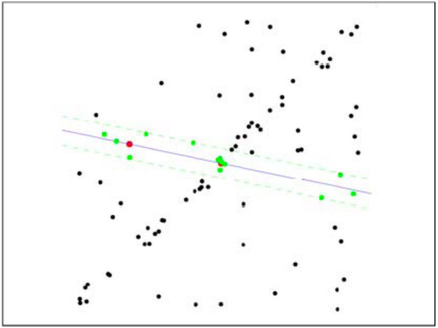

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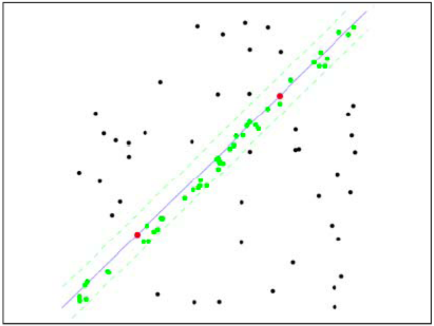

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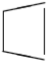
How big is the number of samples?

- For alignment, depends on the motion model
- Each sample is a correspondence (pair of matching points)

Transformation	Matrix	# DoF	Preserves	Icon
translation	$\begin{bmatrix} \boldsymbol{I} \mid \boldsymbol{t} \end{bmatrix}_{2\times 3}$	2	orientation	
rigid (Euclidean)	$\begin{bmatrix} \boldsymbol{R} \mid \boldsymbol{t} \end{bmatrix}_{2\times 3}$	3	lengths	
similarity	$\begin{bmatrix} s\boldsymbol{R} \mid \boldsymbol{t} \end{bmatrix}_{2\times 3}$	4	angles	
affine	$\begin{bmatrix} \boldsymbol{A} \end{bmatrix}_{2\times 3}$	6	parallelism	
projective	$\begin{bmatrix} \tilde{\boldsymbol{H}} \end{bmatrix}_{3\times 3}$	8	straight lines	

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Next class ... more on cameras and projection