Computer Vision: Cameras

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TTI Chicago

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Readings

- Chapter 2.1, 3.6, 4.3 and 6.1 of Szeliski's book
- Chapter 1 of Forsyth & Ponce

What did we see in class last week?

Image Alignment Algorithm

Given images A and B

1. Compute image features for A and B
2. Match features between A and B
3. Compute homography between A and B using least squares on set of matches

Is there a problem with this?

[Source: N. Snavely]

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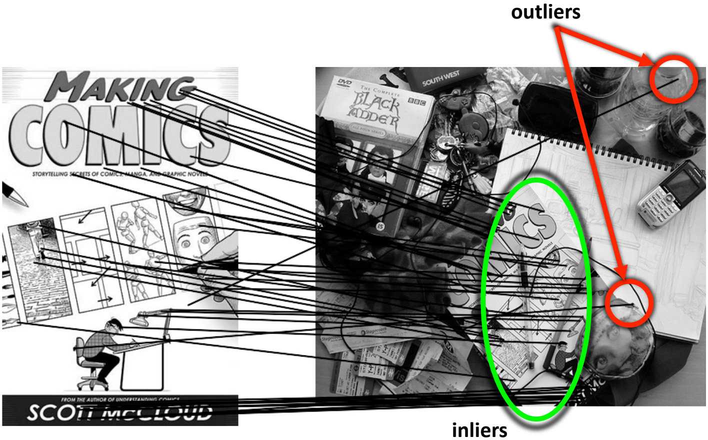
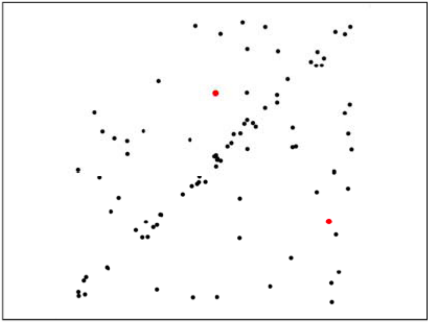
Robustness



[Source: N. Snavely]

RANSAC for line fitting example

1. Randomly select minimal subset of points
2. Hypothesize a model



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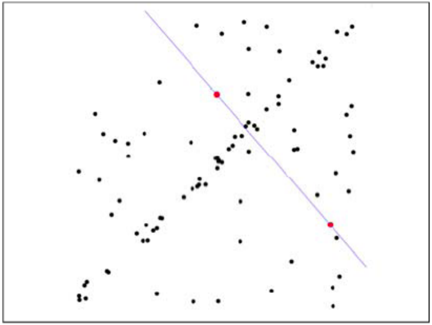

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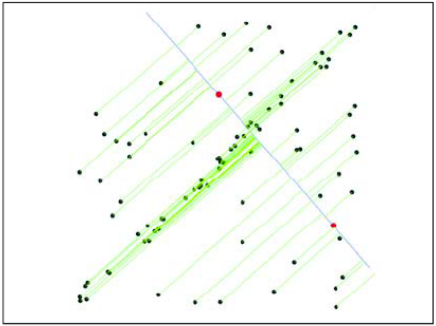

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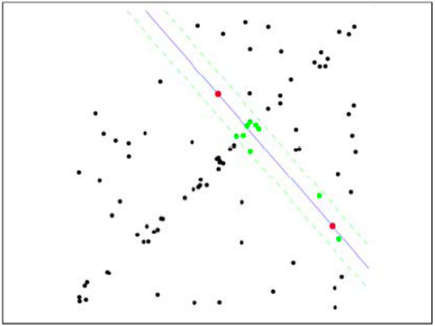

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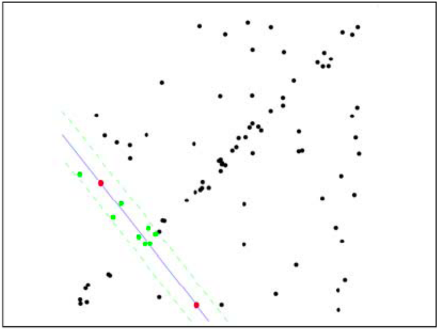

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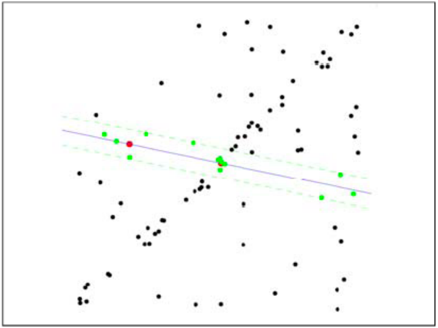

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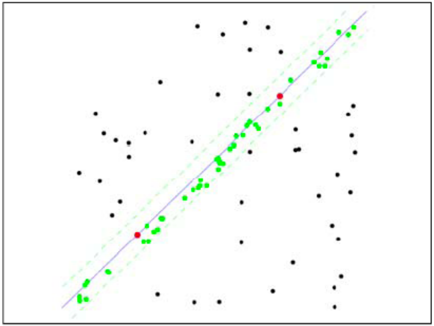

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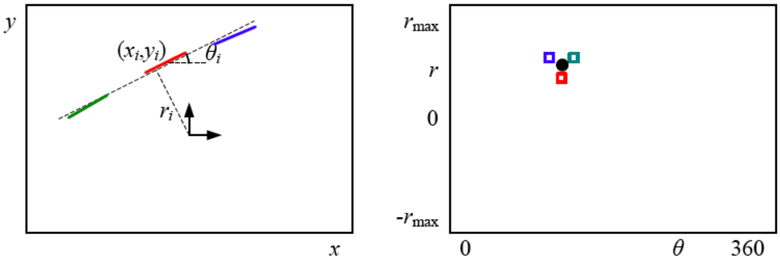

Figure: Images from Szeliski's book

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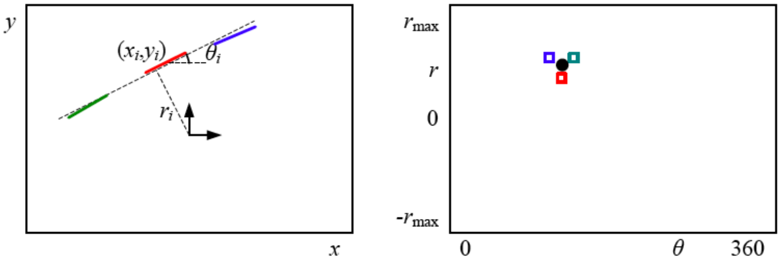

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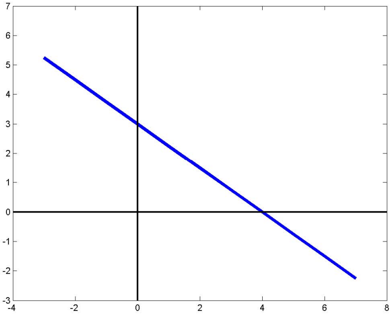
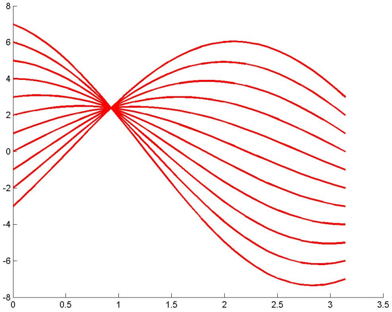
Example Hough Transform

With the parameterization $x \cos\theta + y \sin\theta = r$

- Points in picture represent sinusoids in parameter space
- Points in parameter space represent lines in picture
- Example $0.6x + 0.4y = 2.4$, Sinusoids intersect at $r = 2.4$, $\theta = 0.9273$



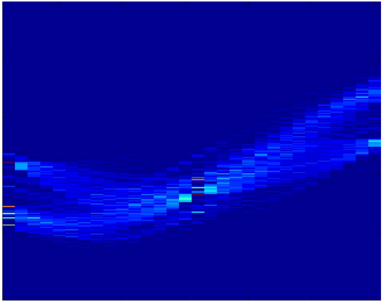


[Source: M. Kazhdan]

Hough Transform Algorithm

With the parameterization $x \cos\theta + y \sin\theta = r$

- Let $r \in [-R, R]$ and $\theta \in [0, \pi)$
- For each edge point (x_i, y_i), calculate: $\hat{r} = x_i \cos\hat{\theta} + y_i \sin\hat{\theta} \quad \forall\hat{\theta} \in [0, \pi)$
- Increase accumulator $A(\hat{r}, \hat{\theta}) = A(\hat{r}, \hat{\theta}) + 1$



- Threshold the accumulator values to get parameters for detected lines

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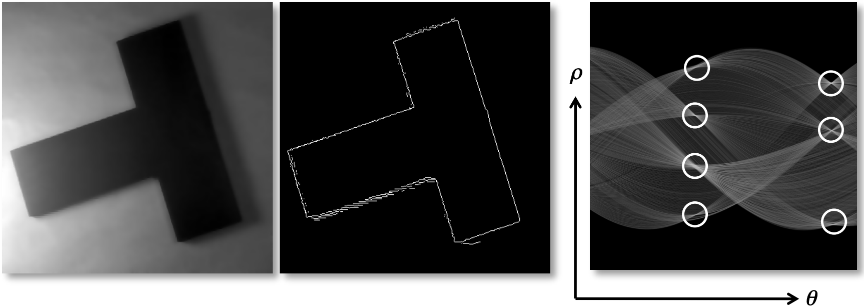

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Coming back to our problem ...

Generalized Hough Transform vs Ransac

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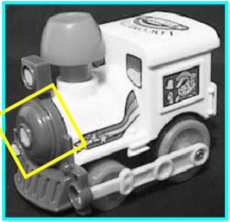
Generalized Hough Transform

- If we use scale, rotation, and translation invariant local features, then each feature match gives an alignment hypothesis (for scale, translation, and orientation of model in image).



[Source: S. Lazebnik]

Generalized Hough Transform

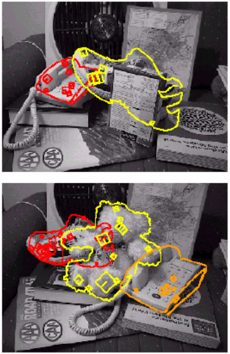
- A hypothesis generated by a single match is in general unreliable,
- Let each match vote for a hypothesis in Hough space.

[Source: K. Grauman]

Recognition Example

Background subtract for model boundaries

Objects recognized,



Recognition in spite of occlusion

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Panoramas

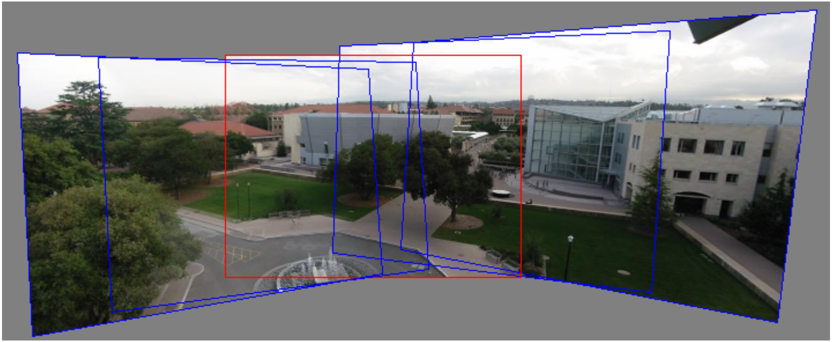
Given two images:

1. Detect features
2. Match features
3. Compute a homography using RANSAC
4. Combine the images together (somehow)

What if we have more than two images?

Creating Panoramas

- Can we use homographies to create a 360 panorama?



- In order to figure this out, we need to learn what a camera is

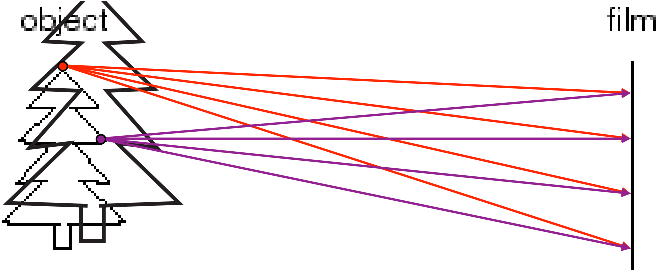
[Source: N. Snavely]

360 Panorama

[Source: N. Snavely]

Let's look at cameras

Image Formation



Lets design a camera

- Idea 1: put a piece of film in front of an object
- Do we get a reasonable image?

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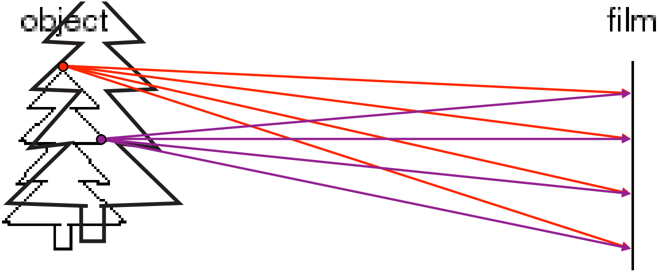

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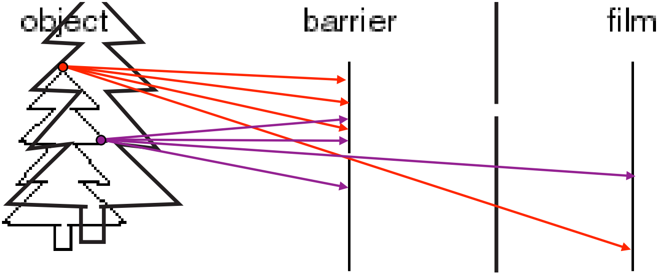
Pinhole Camera



Add a barrier to block off most of the rays

- This reduces blurring
- The opening known as the aperture

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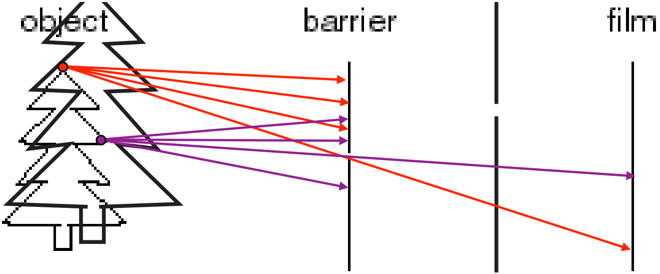

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[Source: N. Snavely]

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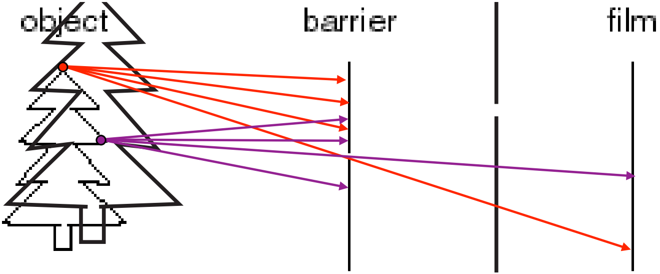

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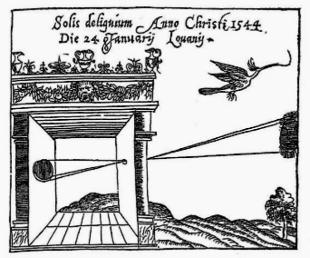

Gemma Frisius, 1558

- Basic principle known to Mozi (470-390 BC), Aristotle (384-322 BC)
- Drawing aid for artists: described by Leonardo da Vinci (1452-1519)

[Source: A. Efros]

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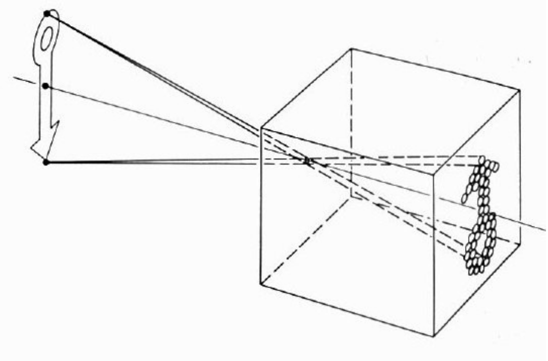


[Source: N. Snavely]

Home made pinhole camera

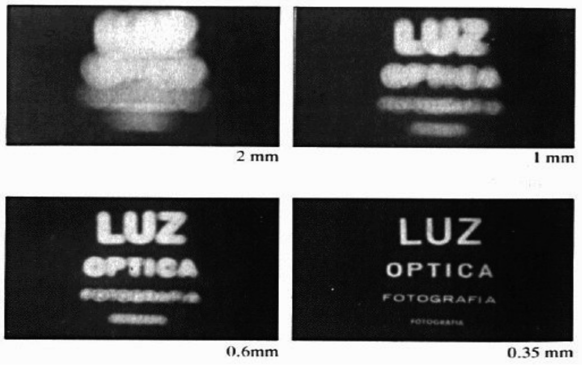
Why so blurry?

Slide by A. Efros

http://www.debevec.org/Pinhole/

[Source: N. Snavely]

Shrinking the aperture



Why not make the aperture as small as possible?

- Less light gets through
- Diffraction effects...

[Source: N. Snavely]

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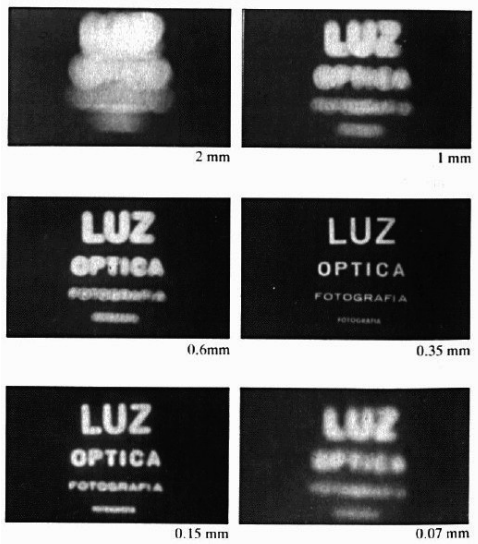

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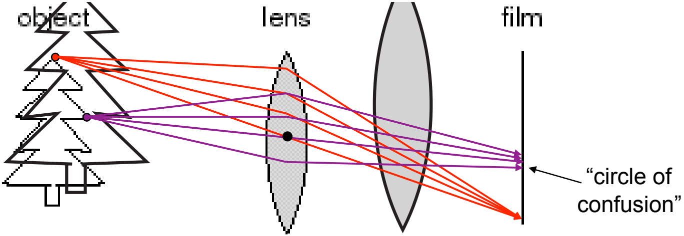
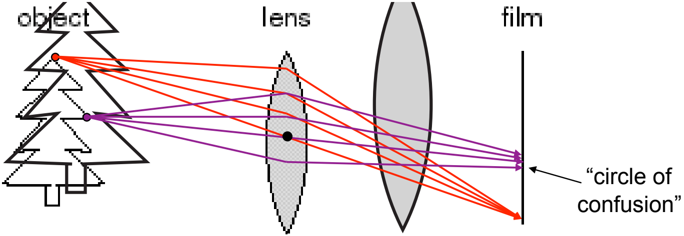
Adding a lens



- A lens focuses light onto the film
- There is a specific distance at which objects are **in focus**

Adding a lens



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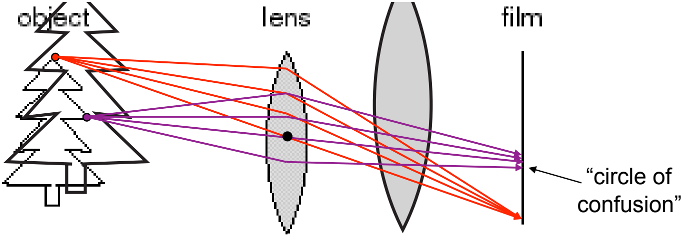

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[Source: N. Snavely]

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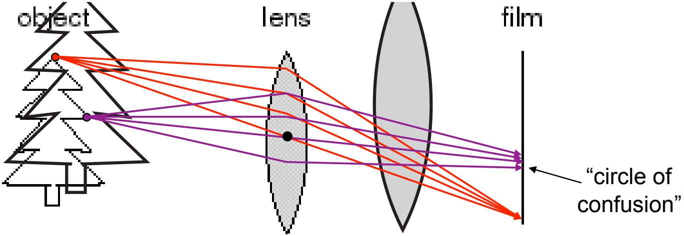

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[Source: N. Snavely]

Projection

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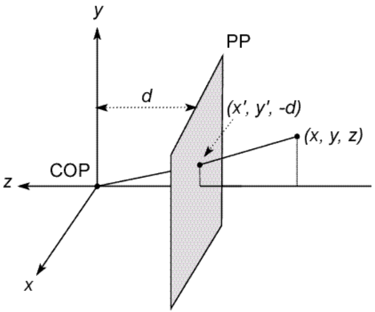

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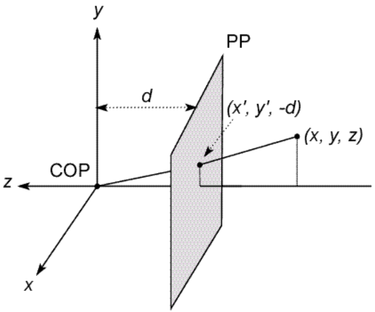

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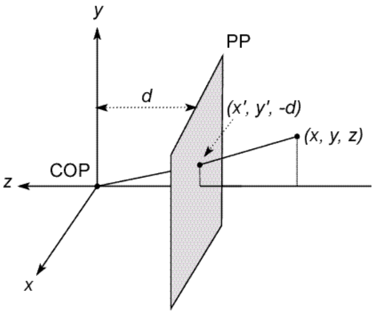

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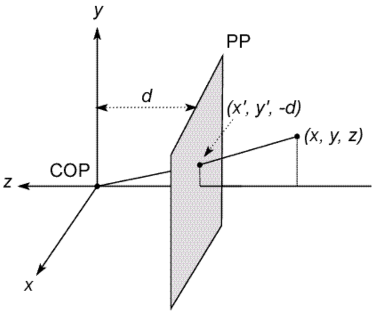

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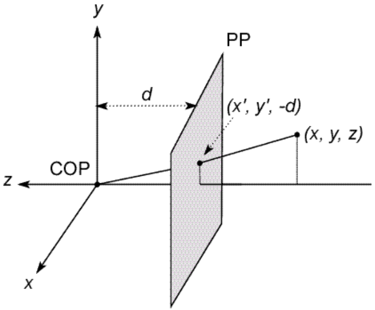

Projection Equations

- Compute intersection with PP of ray from (x,y,z) to COP. How?
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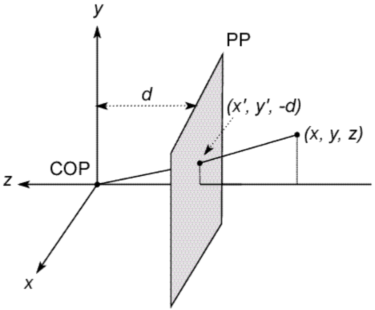

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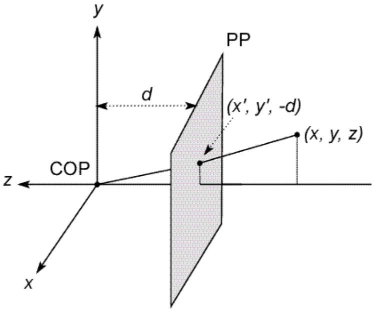

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Modeling Projection

- This is NOT a linear transformation as a division by z is non-linear

Homogeneous coordinates to the rescue!

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \qquad (x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous image coordinates

homogeneous scene coordinates

Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w) \qquad \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

[Source: N. Snavely]

Perspective Projection

- Projection is a matrix multiply using homogeneous coordinates

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ -z/d \end{bmatrix} \Rightarrow (-d\frac{x}{z}, \quad -d\frac{y}{z})$$

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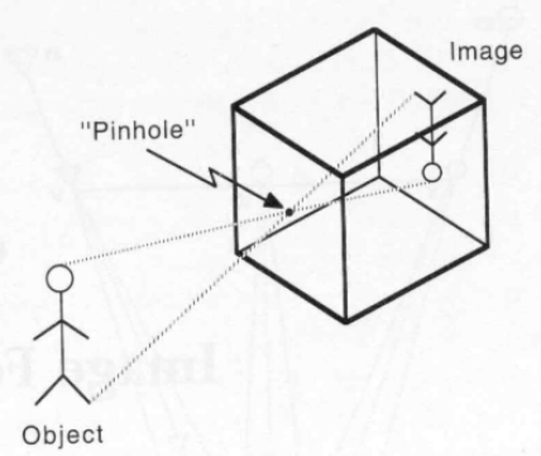
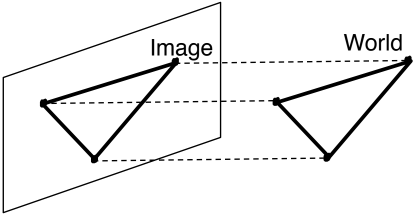


Perspective Projection

[Source: S. Seitz]

Orthographic Projection

- Requires no division and simply drops the z coordinate.
- Special case of perspective projection where the distance from the COP to the PP is infinity



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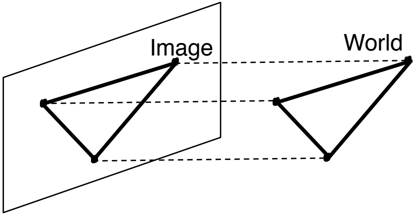

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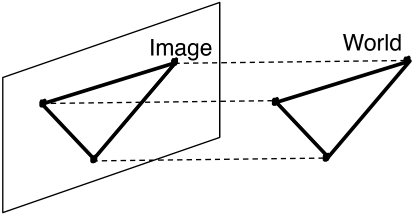

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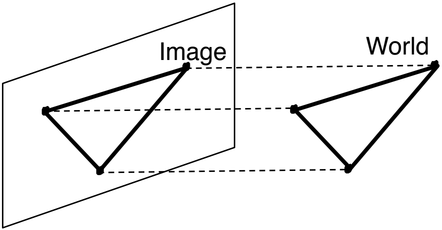

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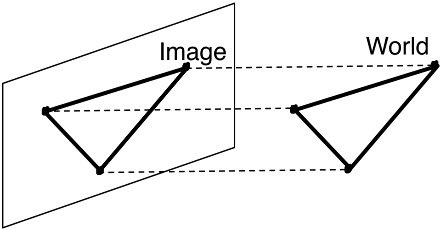

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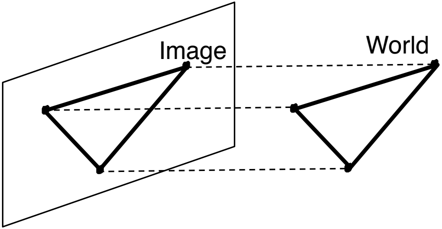

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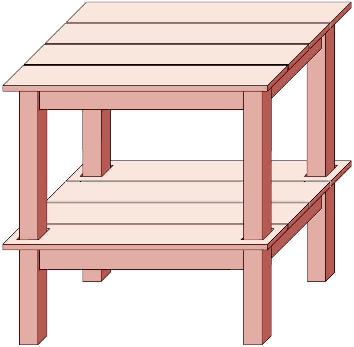

[Source: N. Snavely]

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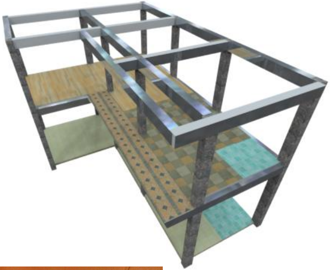

[Source: N. Snavely]

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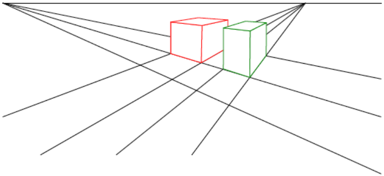
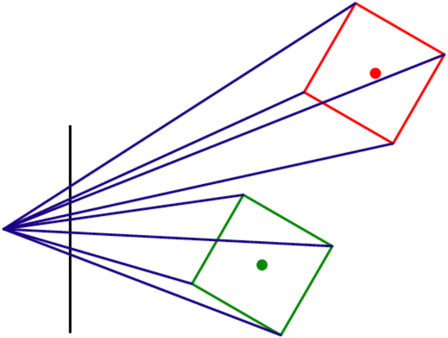


3D World



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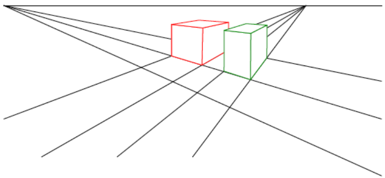

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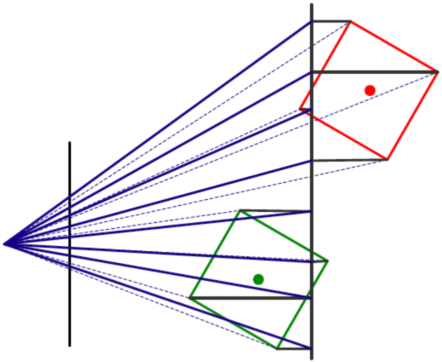

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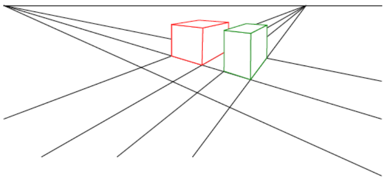

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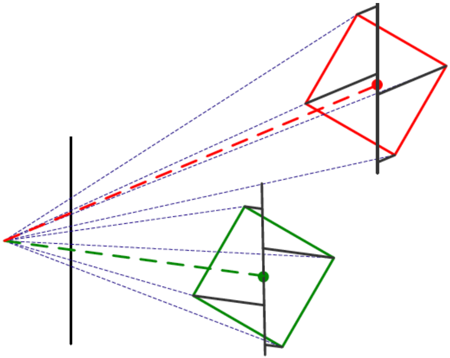
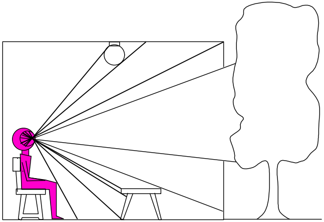


Paraperspective

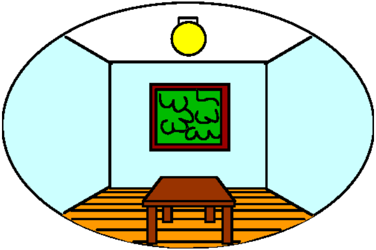
Dimensionality Reduction Machine (3D to 2D)

3D world



Point of observation

2D image



What have we lost?

- Angles
- Distances (lengths)

Slide by A. Efros

Projection properties

- **Many-to-one**: any points along same ray map to same point in image
- Points → points

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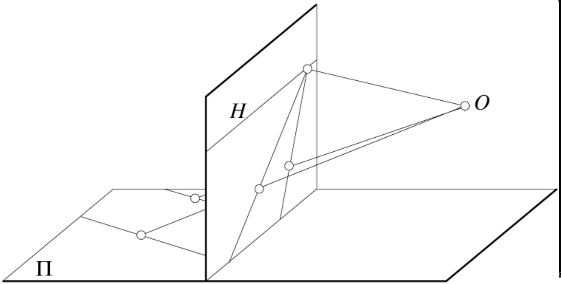
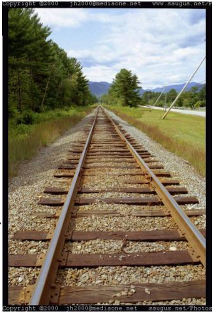
Projection properties

Parallel lines converge at a vanishing point

- Each direction in space has its own vanishing point
- But parallels parallel to the image plane remain parallel





[Source: N. Snavely]

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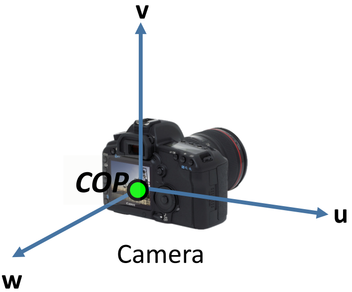
Which coordinate system to use?

Two important coordinate systems:

- World coordinate system
- Camera coordinate system



[Source: N. Snavely]

Camera parameters

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- We then project into the image plane
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[Source: N. Snavely]

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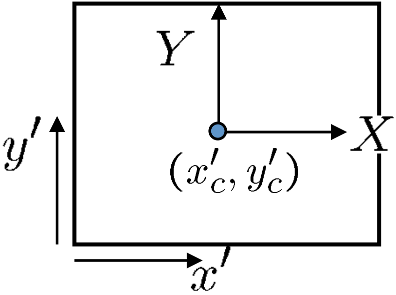
[Source: N. Snavely]

More on camera parameters

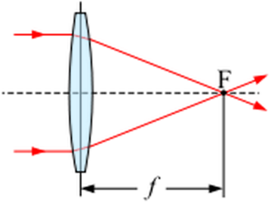
A camera is described by several parameters

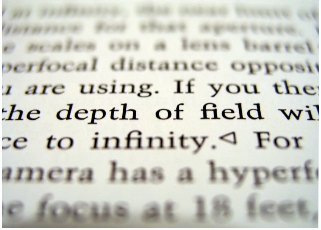
- Translation T of the optical center from the origin of world coords
- Rotation R of the image plane
- Focal length f, principle point (x'_c, y'_c), pixel size (s_x, s_y)
- Which parameters are **extrinsics** and which **intrinsics**?



Focal Length

- Distance over which initially collimated rays (i.e., parallel) are brought to a focus.





Focal Length

- Can be thought of as **zoom**
- Related to the **field of view**

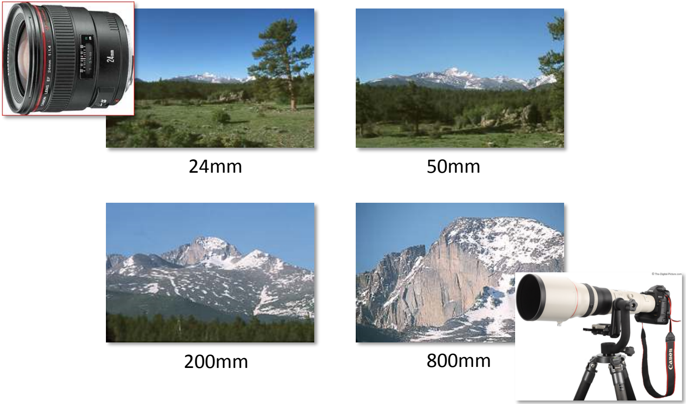


Figure: Image from N. Snavely

Projection Equations

- The projection matrix models the cumulative effect of all intrinsic and extrinsic parameters

$$\mathbf{X} = \begin{bmatrix} ax \\ ay \\ a \end{bmatrix} = \mathbf{P} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

- It can be computed as

$$\mathbf{P} = \underbrace{\begin{bmatrix} -f \cdot s_x & 0 & x'_c \\ 0 & -f \cdot s_y & y'_c \\ 0 & 0 & 1 \end{bmatrix}}_{\text{intrinsics}} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{projection}} \underbrace{\begin{bmatrix} \mathbf{R}_{3\times 3} & 0_{3\times 1} \\ 0_{1\times 3} & 1 \end{bmatrix}}_{\text{rotation}} \underbrace{\begin{bmatrix} \mathbf{I}_{3\times 3} & \mathbf{T}_{3\times 1} \\ 0_{1\times 3} & 1 \end{bmatrix}}_{\text{translation}}$$

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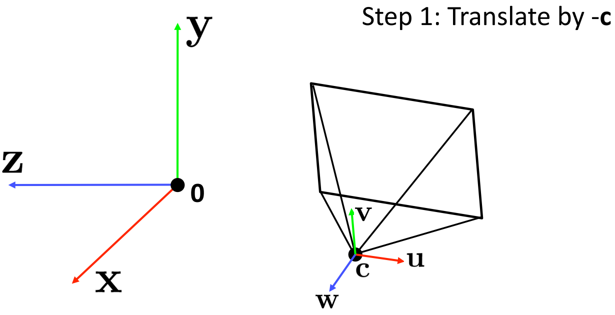
Extrinsics

How do we get the camera to canonical form?



[Source: N. Snavely]

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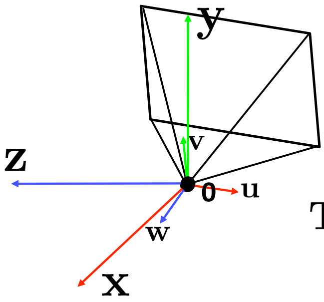

Step 1: Translate by -**c**

How do we represent translation as a matrix multiplication?

$$\mathbf{T} = \begin{bmatrix} \mathbf{I}_{3\times 3} & -\mathbf{c} \\ 0 \quad 0 \quad 0 & 1 \end{bmatrix}$$

[Source: N. Snavely]

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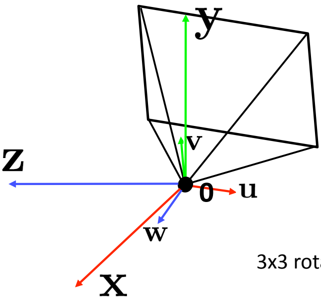

Step 1: Translate by -**c**

Step 2: Rotate by **R**

$$\mathbf{R} = \begin{bmatrix} \mathbf{u}^T \\ \mathbf{v}^T \\ \mathbf{w}^T \end{bmatrix}$$

3x3 rotation matrix

[Source: N. Snavely]

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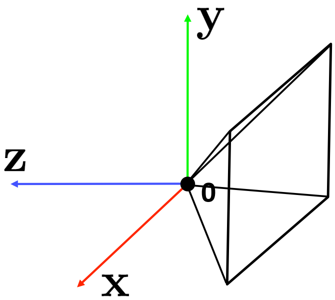

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[Source: N. Snavely]

Perspective Projection

$$\underbrace{\begin{bmatrix} -f & 0 & 0 \\ 0 & -f & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{K}} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(intrinsics) (converts from 3D rays in camera coordinate system to pixel coordinates)

in general, $\mathbf{K} = \begin{bmatrix} -f & s & c_x \\ 0 & -\alpha f & c_y \\ 0 & 0 & 1 \end{bmatrix}$ (upper triangular matrix)

α : **aspect ratio** (1 unless pixels are not square)

s : **skew** (0 unless pixels are shaped like rhombi/parallelograms)

: **principal point** ((0,0) unless optical axis doesn't intersect projection plane at origin)

- Simplifications used in practice

[Source: N. Snavely]

Camera matrix

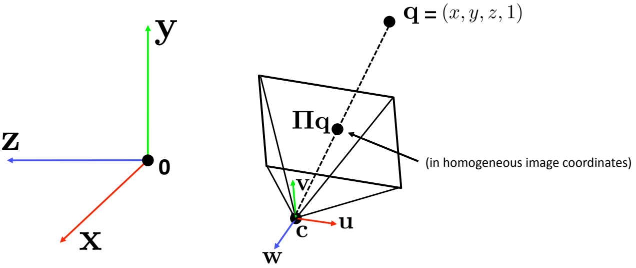
- The projection matrix is defined as

$$\mathbf{P} = \underbrace{\mathbf{K}}_{\text{intrinsics}} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{projection}} \underbrace{\underbrace{\begin{bmatrix} \mathbf{R}_{3\times3} & 0_{3\times1} \\ 0_{1\times3} & 1 \end{bmatrix}}_{\text{rotation}} \underbrace{\begin{bmatrix} \mathbf{I}_{3\times3} & \mathbf{T}_{3\times3} \\ 0_{1\times3} & 1 \end{bmatrix}}_{\text{translation}}}_{\begin{bmatrix} \mathbf{R} & -\mathbf{Rc} \end{bmatrix}}$$

- More compactly

$$\mathbf{P} = \mathbf{K} \begin{bmatrix} \mathbf{R} & -\mathbf{Rc} \end{bmatrix} = \mathbf{K} \begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix}$$

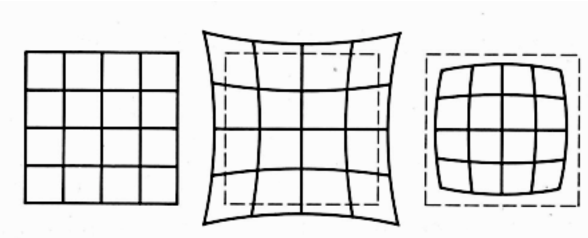
Camera matrix



[Source: N. Snavely]

Radial Distorsion

- Caused by imperfect lenses
- Deviations are most noticeable for rays that pass through the edge of the lens



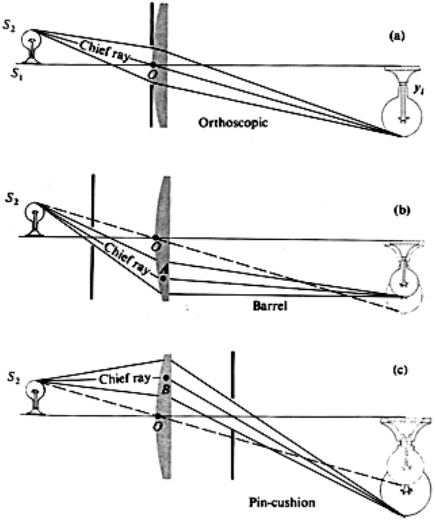
No distortion Pin cushion Barrel

[Source: N. Snavely]

Correcting Radial Distorsion

from Helmut Dersch

Distorsion



[Source: N. Snavely]

Modeling Distorsion

- Project point to normalized image coordinates

$$
\begin{aligned}
x_n &= \frac{x}{z} \\
y_n &= \frac{y}{z}
\end{aligned}
$$

- Apply radial distortion

$$
\begin{aligned}
r^2 &= x_n^2 + y_n^2 \\
x_d &= x_n(1 + \kappa_1 r^2 + \kappa_2 r^4) \\
y_d &= y_n(1 + \kappa_1 r^2 + \kappa_2 r^4)
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- Apply focal length and translate image center

$$\begin{aligned} x' &= f x_d + x_c \\ y' &= f y_d + y_c \end{aligned}$$

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Next class ... more on panoramas