Computer Vision: Panorama

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What did we see in class last week?

Image Alignment Algorithm

Given images A and B

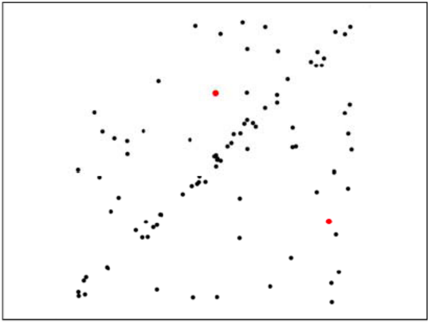
1. Compute image features for A and B
2. Match features between A and B
3. Compute homography between A and B using least squares on set of matches

Is there a problem with this?

[Source: N. Snavely]

RANSAC for line fitting example

1. Randomly select minimal subset of points
2. Hypothesize a model



[Source: R. Raguram]

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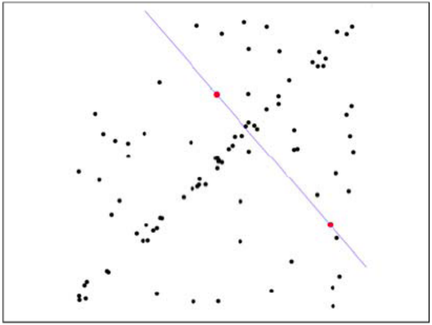

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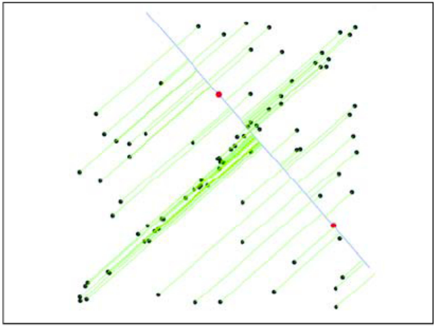

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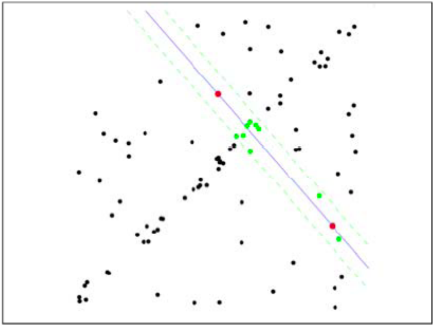

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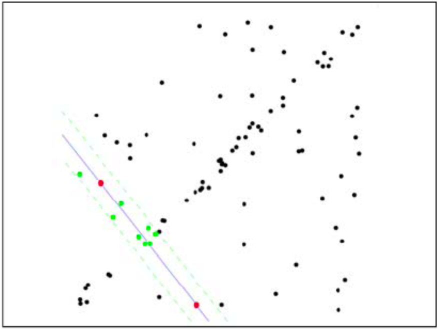

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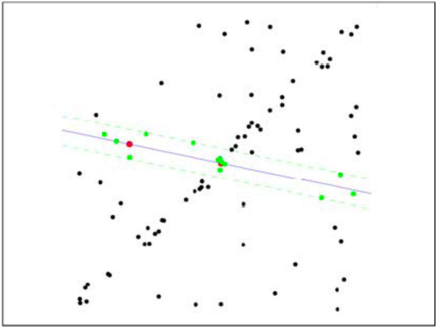

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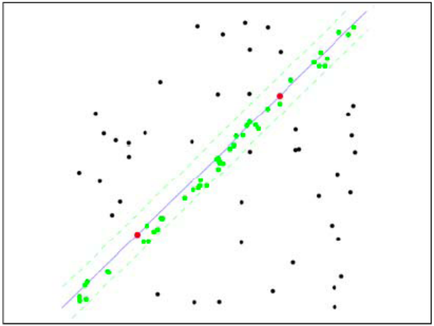
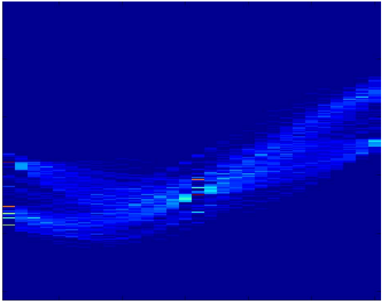


[Source: R. Raguram]

Hough Transform Algorithm

With the parameterization $x \cos\theta + y \sin\theta = r$

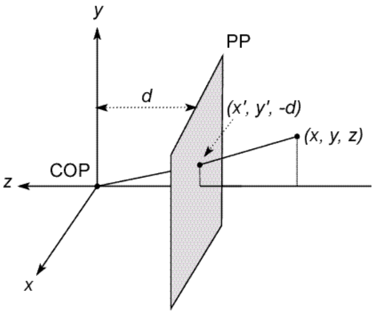
- Let $r \in [-R, R]$ and $\theta \in [0, \pi)$
- For each edge point (x_i, y_i), calculate: $\hat{r} = x_i \cos\hat{\theta} + y_i \sin\hat{\theta} \quad \forall\hat{\theta} \in [0, \pi)$
- Increase accumulator $A(\hat{r}, \hat{\theta}) = A(\hat{r}, \hat{\theta}) + 1$



- Threshold the accumulator values to get parameters for detected lines

[Source: M. Kazhdan]

Modeling projection



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- We will use the pinhole model as an approximation
- Put the **optical center** (Center Of Projection) at the origin

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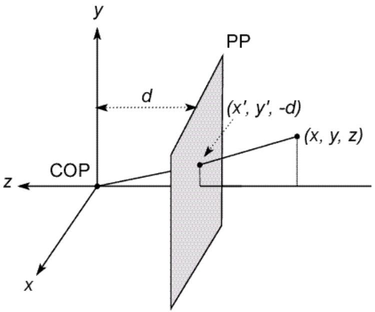

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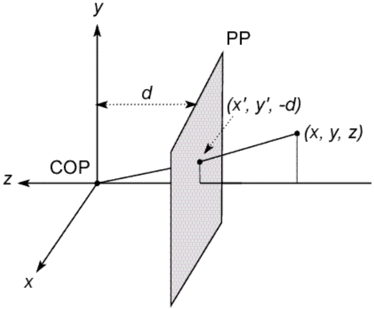

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[Source: N. Snavely]

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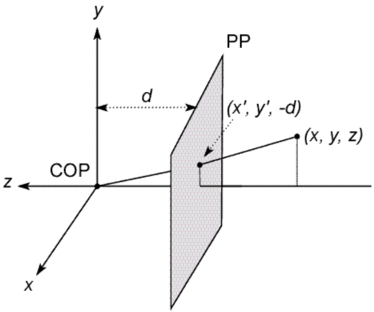

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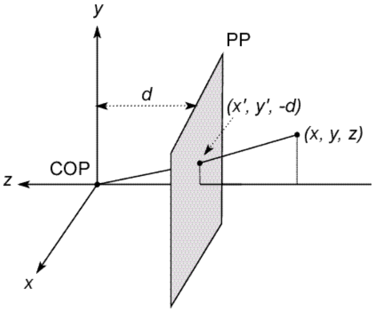

Projection Equations

- Compute intersection with PP of ray from (x,y,z) to COP. How?
- Derived using similar triangles

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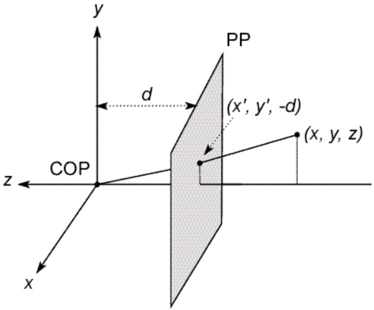

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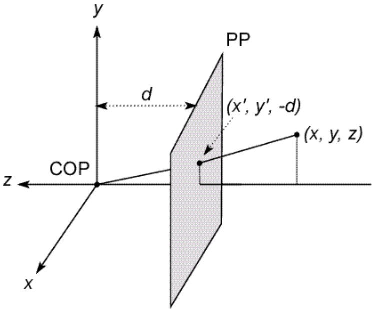

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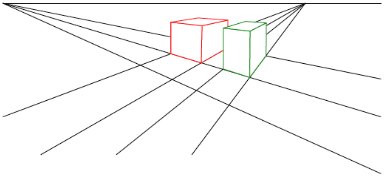
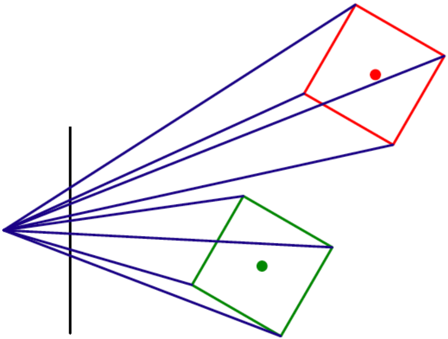
Perspective

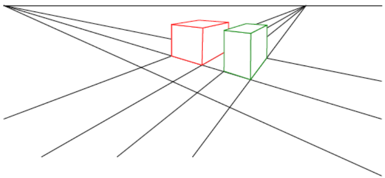


3D World



Perspective Projection

Variants of Orthographic



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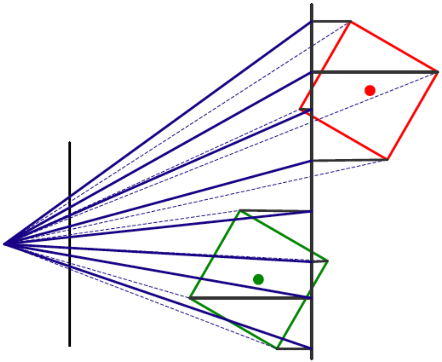

Orthographic Projection

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- Points $\rightarrow$ points

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How many numbers do we need to describe a camera?

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Projection Equations

- The projection matrix models the cumulative effect of all intrinsic and extrinsic parameters

$$\mathbf{X} = \begin{bmatrix} ax \\ ay \\ a \end{bmatrix} = \mathbf{P} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

- It can be computed as

$$\mathbf{P} = \underbrace{\begin{bmatrix} -f \cdot s_x & 0 & x'_c \\ 0 & -f \cdot s_y & y'_c \\ 0 & 0 & 1 \end{bmatrix}}_{\text{intrinsics}} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{projection}} \underbrace{\begin{bmatrix} \mathbf{R}_{3\times3} & 0_{3\times1} \\ 0_{1\times3} & 1 \end{bmatrix}}_{\text{rotation}} \underbrace{\begin{bmatrix} \mathbf{I}_{3\times3} & \mathbf{T}_{3\times1} \\ 0_{1\times3} & 1 \end{bmatrix}}_{\text{translation}}$$

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Extrinsics

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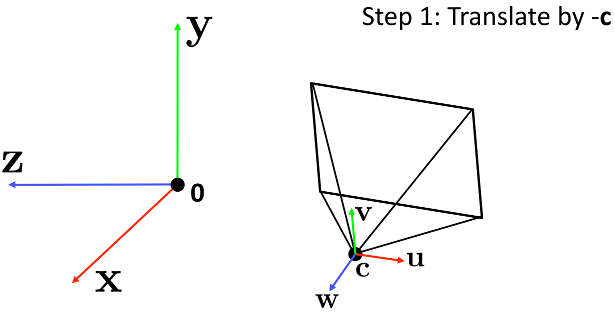

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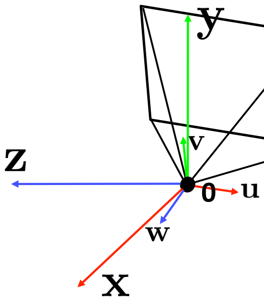

Step 1: Translate by -**c**

How do we represent translation as a matrix multiplication?

$$\mathbf{T} = \begin{bmatrix} \mathbf{I}_{3\times 3} & -\mathbf{c} \\ 0 \quad 0 \quad 0 & 1 \end{bmatrix}$$

[Source: N. Snavely]

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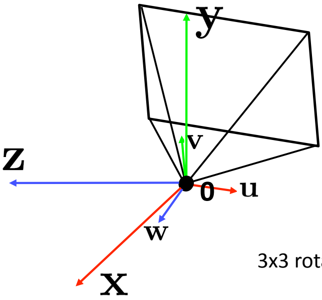

Step 1: Translate by -**c**

Step 2: Rotate by **R**

$$\mathbf{R} = \begin{bmatrix} \mathbf{u}^T \\ \mathbf{v}^T \\ \mathbf{w}^T \end{bmatrix}$$

3x3 rotation matrix

[Source: N. Snavely]

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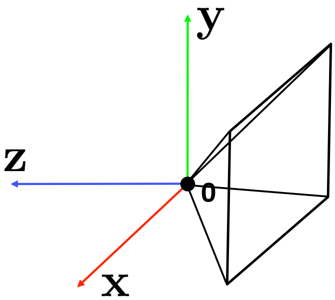

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Perspective Projection

$$\underbrace{\begin{bmatrix} -f & 0 & 0 \\ 0 & -f & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{K}} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(intrinsics)

(converts from 3D rays in camera coordinate system to pixel coordinates)

in general, $\mathbf{K} = \begin{bmatrix} -f & s & c_x \\ 0 & -\alpha f & c_y \\ 0 & 0 & 1 \end{bmatrix}$ (upper triangular matrix)

α : **aspect ratio** (1 unless pixels are not square)

s : **skew** (0 unless pixels are shaped like rhombi/parallelograms)

: **principal point** ((0,0) unless optical axis doesn't intersect projection plane at origin)

- Simplifications used in practice

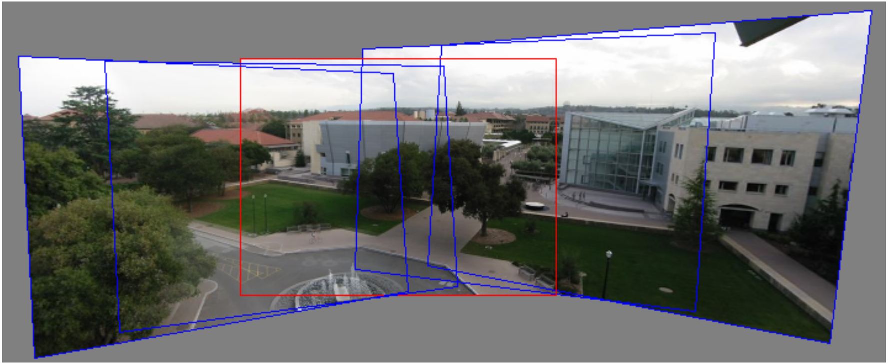
[Source: N. Snavely]

Today's Readings

- Chapter 9 of Szeliski's book

Let's look at panoramas again

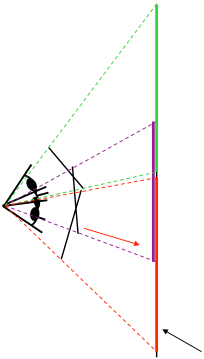
Can we use homography to create a 360 panorama?

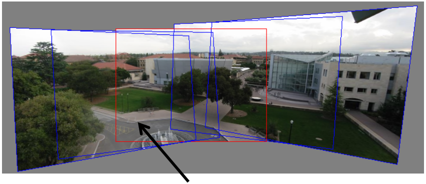


[Source: N Snavely]

Can we use homography to create a 360 panorama?

- Idea: projecting images onto a common plane





each image is warped with a homography $\mathbf{H}$

We'll see what this homograph means later.

First -- Can't create a 360 panorama this way…

[Source: N Snavely]

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(a) translation [2 dof] (b) affine [6 dof] (c) perspective [8 dof] (d) 3D rotation [3+ dof]

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Simple Motion Model

- Consists of 2D rotation and translation
- In a **panography**, images are translated, rotated and scaled.
- We saw the case of linear transformations, where we used least squares
- To be more robust we employed RANSAC or Hough transform

Estimating the Motion

- Consider, the problem of estimating a **rigid Euclidean 2D transformation** (translation plus rotation) between two sets of points.
- If we parameterize this transformation by the translation $(t_x; t_y)$ and the rotation angle θ, the Jacobian of this transformation, depends on the current value of θ.
- Is this problematic?

Transform	Matrix	Parameters p	Jacobian J
translation	$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \end{bmatrix}$	(t_x, t_y)	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
Euclidean	$\begin{bmatrix} c_\theta & -s_\theta & t_x \\ s_\theta & c_\theta & t_y \end{bmatrix}$	(t_x, t_y, θ)	$\begin{bmatrix} 1 & 0 & -s_\theta x - c_\theta y \\ 0 & 1 & c_\theta x - s_\theta y \end{bmatrix}$
similarity	$\begin{bmatrix} 1+a & -b & t_x \\ b & 1+a & t_y \end{bmatrix}$	(t_x, t_y, a, b)	$\begin{bmatrix} 1 & 0 & x & -y \\ 0 & 1 & y & x \end{bmatrix}$
affine	$\begin{bmatrix} 1+a_{00} & a_{01} & t_x \\ a_{10} & 1+a_{11} & t_y \end{bmatrix}$	$(t_x, t_y, a_{00}, a_{01}, a_{10}, a_{11})$	$\begin{bmatrix} 1 & 0 & x & y & 0 & 0 \\ 0 & 1 & 0 & 0 & x & y \end{bmatrix}$

Minimizing the non-linear least-squares

- **Iteratively** update $\Delta \mathbf{p}$ to the current parameter estimate $\Delta \mathbf{p}$ by minimizing

$$E_{NLS}(\Delta \mathbf{p}) = \sum_i ||f(x_i; \mathbf{p} + \Delta \mathbf{p}) - x_i'||_2^2$$

- We can approximate this by

$$E_{NLS}(\Delta \mathbf{p}) \approx \sum_i ||\mathbf{J}(x_i; \mathbf{p})\Delta \mathbf{p} - r_i'||_2^2$$

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$$E_{NLS}(\Delta \mathbf{p}) \approx \Delta_{\mathbf{p}}^T \mathbf{A} \Delta \mathbf{p} - 2\Delta \mathbf{p}^T \mathbf{b} + c$$

with $\mathbf{A} = \sum_i \mathbf{J}^T \mathbf{J}$ the Hessian and

$$\mathbf{b} = \sum_i \mathbf{J}^T(x_i)\mathbf{r}_i$$

is a Jacobian-weighted sum of residual vectors

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Minimizing the non-linear least-squares

- The parameters are pulled in the direction of the prediction error with strength proportional to the Jacobian
- Once $\mathbf{A}$ and $\mathbf{b}$ are computed, one solves for $\Delta\mathbf{p}$ by solving

$$(\mathbf{A} + \lambda \text{diag}(\mathbf{A}))\Delta\mathbf{p} = \mathbf{b}$$

  with λ a damping parameter

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$$(\mathbf{A} + \lambda \text{diag}(\mathbf{A}))\Delta\mathbf{p} = \mathbf{b}$$

  with λ a damping parameter
- Thus the algorithm looks like

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Planar Perspective Motion

- The mapping between two camera viewing a common plane can be described with a 3×3 homography.
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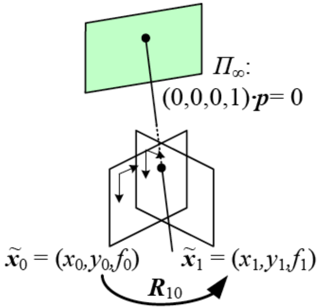

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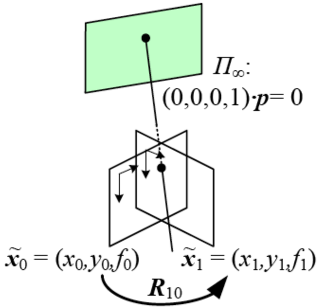

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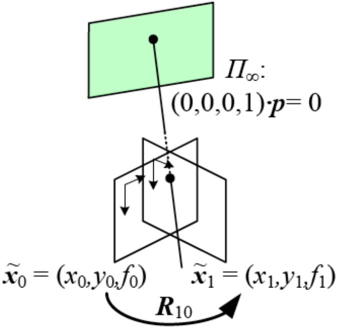

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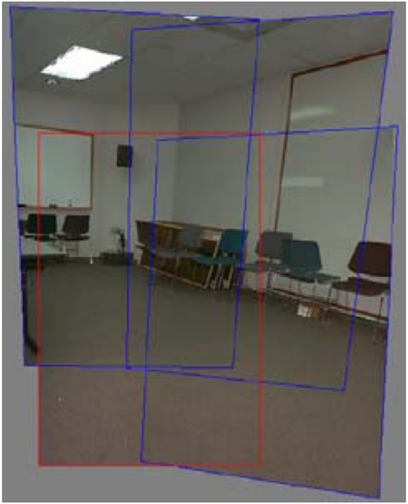
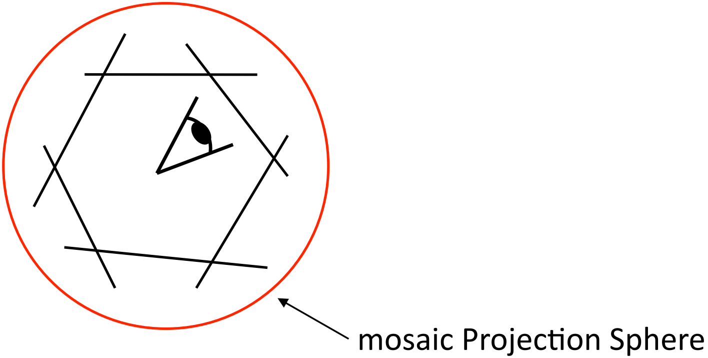


Figure: Four images taken with a hand-held camera registered using a 3D rotation motion model (Szeliski and Shum 1997)

Panorama

- What if you want a 360 field of view?



[Source: N Snavely]

Cylindrical and Spherical Coordinates

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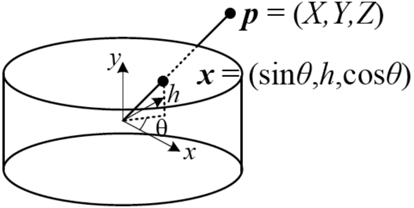
- Points on this surface are parameterized by an angle θ and a height h with the 3D cylindrical given by $(\sin\theta, h, \cos\theta) \propto (x, y, f)$



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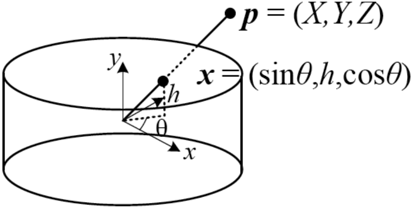

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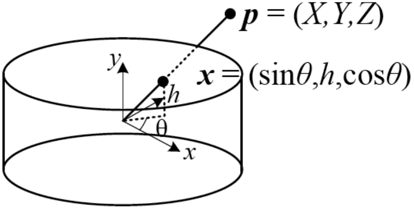

- We can compute the correspondence between **warped** and **mapped** coordinates

$$
\begin{aligned}
x' &= s\theta = s \tan^{-1} \frac{x}{f}, \\
y' &= sh = s\frac{y}{\sqrt{x^2 + f^2}},
\end{aligned}
$$

$$
\begin{aligned}
x &= f \tan\theta = f \tan \frac{x'}{s}, \\
y &= h\sqrt{x^2 + f^2} = \frac{y'}{s} f \sqrt{1 + \tan^2 x'/s} = f\frac{y'}{s} \sec \frac{x'}{s}
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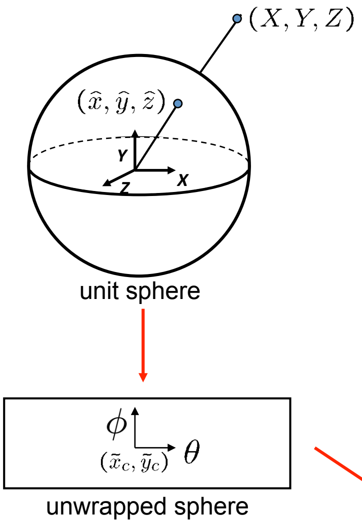
$$

Cylindrical Panorama

- Cylindrical is used if the camera is level and we have only rotation around its vertical axis
- Then we only need to estimate a translation

Figure: A cylindrical panorama (Szeliski and Shum 1997)

Spherical Projection



– Map 3D point (X,Y,Z) onto sphere

$$(\hat{x}, \hat{y}, \hat{z}) = \frac{1}{\sqrt{X^2+Y^2+Z^2}}(X, Y, Z)$$

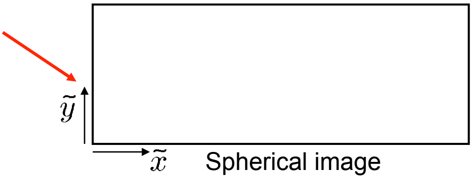
- Convert to spherical coordinates

$$(sin\theta cos\phi, sin\phi, cos\theta cos\phi) = (\hat{x}, \hat{y}, \hat{z})$$

- Convert to spherical image coordinates

$$(\tilde{x}, \tilde{y}) = (s\theta, s\phi) + (\tilde{x}_c, \tilde{y}_c)$$

  – s defines size of the final image
    » often convenient to set s = camera focal length in pixels



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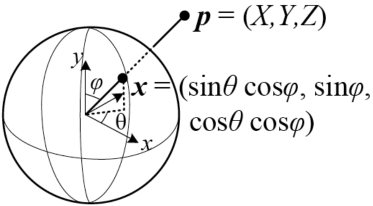

$$x' = s\theta = s\tan^{-1}\frac{x}{f},$$
$$y' = s\phi = s\tan^{-1}\frac{y}{\sqrt{x^2+f^2}},$$

while the inverse is given by

$$x = f\tan\theta = f\tan\frac{x'}{s},$$
$$y = \sqrt{x^2+f^2}\tan\phi = \tan\frac{y'}{s}f\sqrt{1+\tan^2 x'/s} = f\tan\frac{y'}{s}\sec\frac{x'}{s}$$

Spherical Re-Projection

input **f = 200 (pixels)** **f = 400** **f = 800**

- It is desirable if the global motion model is translation
- For a pure panning motion, if we convert two images to their cylindrical maps with known f, the relationship between them is a translation.
- Similarly, we can map an image to its longitude/latitude spherical coordinates as well if f is given

Modeling Distorsion with Panoramas

- Project point to normalized image coordinates

$$x_n = \frac{x}{z}$$
$$y_n = \frac{y}{z}$$

- Apply radial distortion

$$r^2 = x_n^2 + y_n^2$$
$$x_d = x_n(1 + \kappa_1 r^2 + \kappa_2 r^4)$$
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- Apply focal length and translate image center

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Aligning spherical images

- Suppose we rotate the camera by θ about the vertical axis
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[Source: N. Snavely]

Aligning spherical images

- Suppose we rotate the camera by θ about the vertical axis
- How does this change the spherical image?
- This means that we can align spherical images by translation

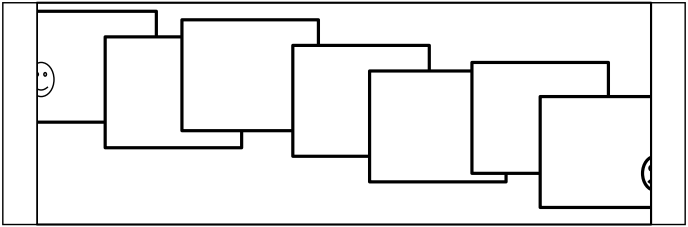
[Source: N. Snavely]

Assembling the panorama

- Stitch pairs together, blend, then crop

[Source: N. Snavely]

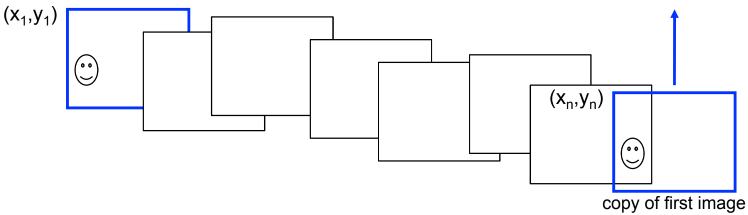
Problem: Drift



- Small errors accumulate over time

[Source: N. Snavely]

Solutions to Drift



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  - add displacement of $(y_1 - y_n)/(n-1)$ to each image after the first

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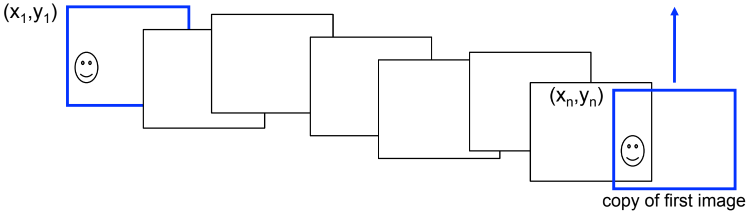

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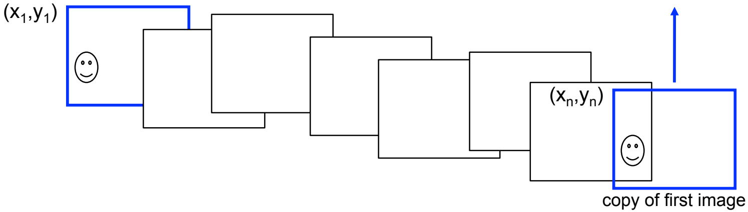

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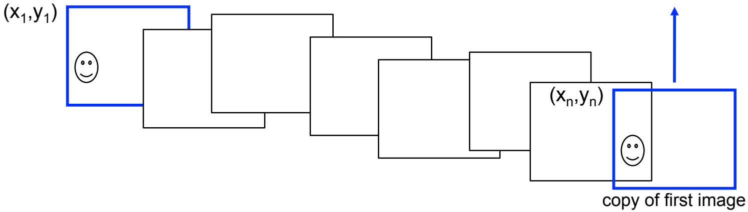

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Dealing with multiple images

- Extend the pairwise matching criteria to deal with multiple images
- Typical pipeline include
  - **Panorama recognition:** Decide which images to align
  - **Global alignment**
  - **Local adjustments**

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Problems

The multiview formulation

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has two potential disadvantages:

- Since a summation is taken over all pairs with corresponding features, features that are observed many times are overweighted in the final solution (a feature observed m times gets counted $\binom{m}{2}$ instead of m times).
- Second, the derivatives of $\tilde{\mathbf{x}}_{ij}$ with respect to $\{(\mathbf{R}_j, f_j)\}$ are a little cumbersome

Alternative Formulation

- Use **true bundle adjustment** solving for pose $\{\mathbf{R}_j, f_j\}$ and 3D positions $\{\mathbf{x}_i\}$

$$E_{BA-2D} = \sum_i \sum_j c_{ij} ||\tilde{\mathbf{x}}_{ij}(\mathbf{x}_i; \mathbf{R}_j, f_j) - \hat{\mathbf{x}}_{ij}||_2^2$$

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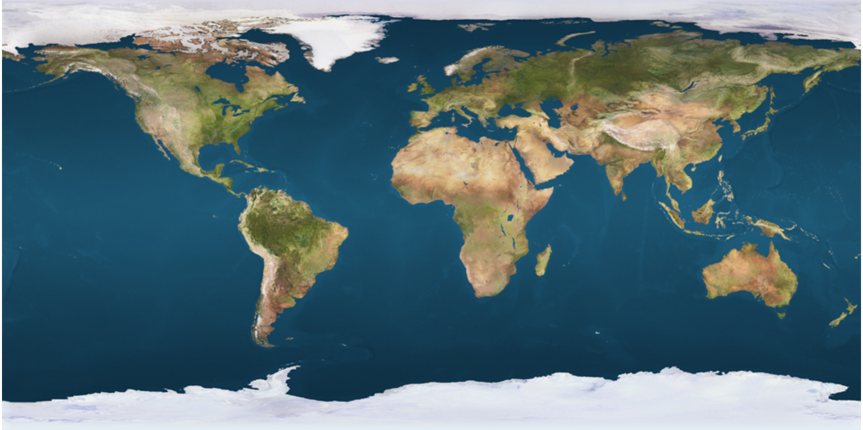
Unwrapping a sphere

Credit: JHT's Planetary Pixel Emporium



Spherical panoramas

Microsoft Lobby: `http://www.acm.org/pubs/citations/proceedings/graph/258734/p251-szeliski`

Different projections are possible

[Source: N. Snavely]

Blending

- We want to seamlessly blend them together

[Source: N. Snavely]

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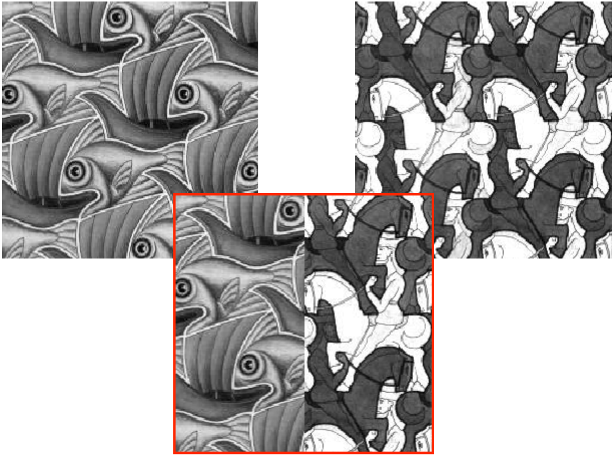
[Source: N. Snavely]

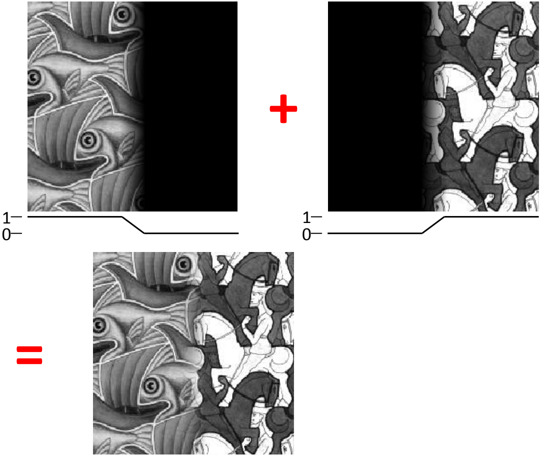
Image Blending



[Source: N. Snavely]

Feathering

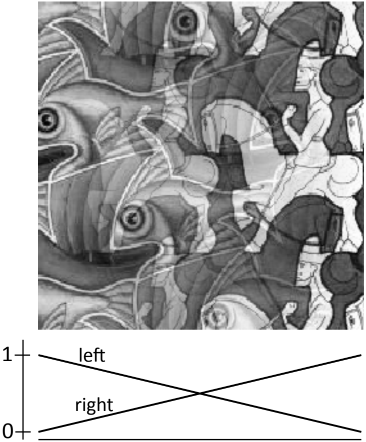
Take the average value at each pixel

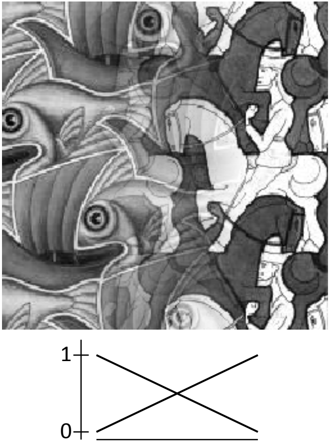


[Source: N. Snavely]

Effect of window size

Use window to do average





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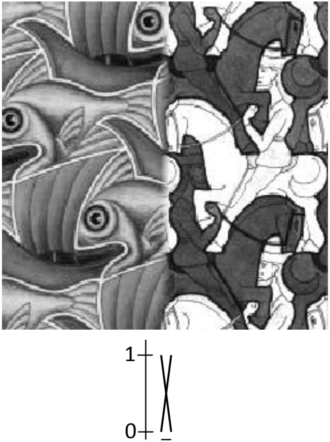
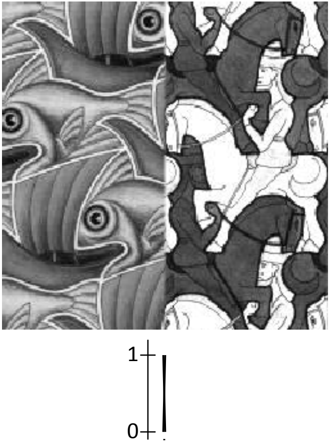
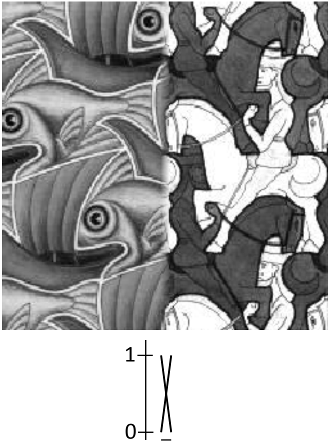
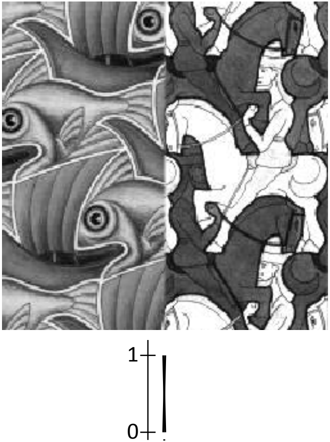




[Source: N. Snavely]

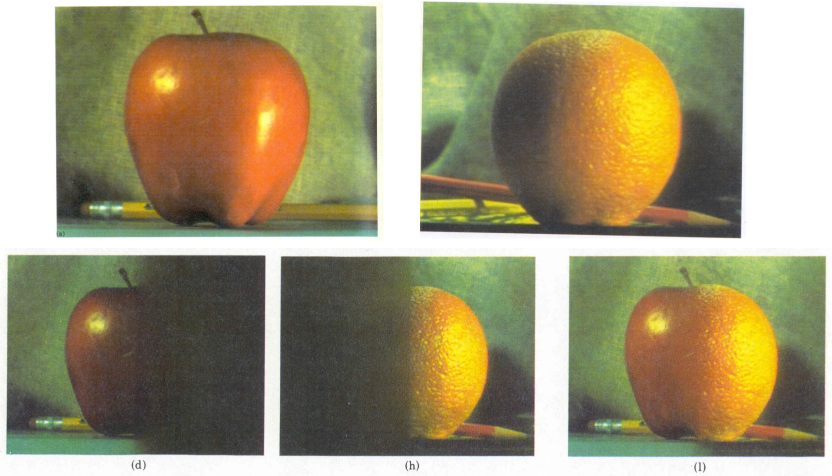
Good window size





- Optimal window: smooth but not ghosted
- It doesn't always work

[Source: N. Snavely]

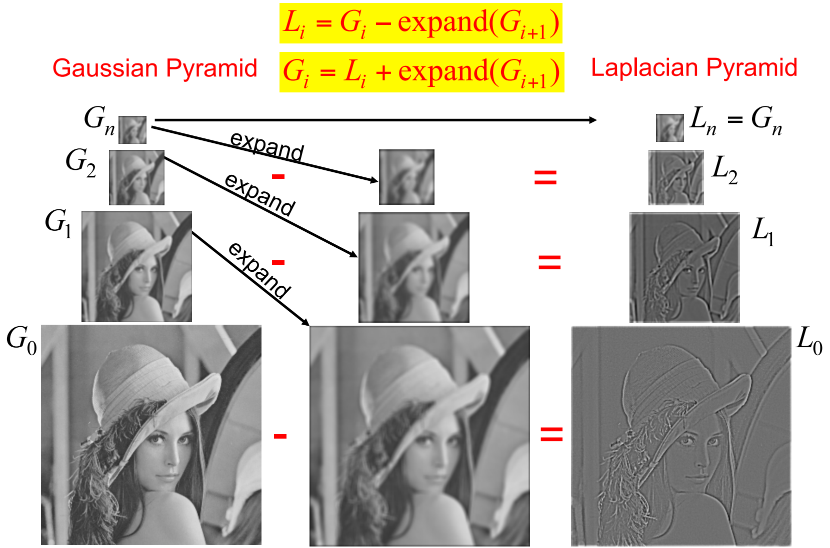


Create a Laplacian pyramid, blend each level

- Burt, P. J. and Adelson, E. H., A multiresolution spline with applications to image mosaics, ACM Transactions on Graphics, 42(4), October 1983, 217-236.

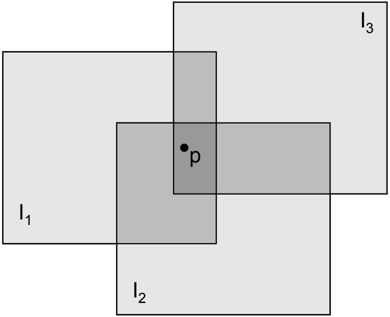
[Source: N. Snavely]

Laplacian Pyramid



[Source: N. Snavely]

Alpha Blending



Encoding blend weights: I(x,y) = (αR, αG, αB, α)

color at p = $$\frac{(\alpha_1 R_1,\ \alpha_1 G_1,\ \alpha_1 B_1) + (\alpha_2 R_2,\ \alpha_2 G_2,\ \alpha_2 B_2) + (\alpha_3 R_3,\ \alpha_3 G_3,\ \alpha_3 B_3)}{\alpha_1 + \alpha_2 + \alpha_3}$$

Implement this in two steps:

1. accumulate: add up the (α premultiplied) RGBα values at each pixel
2. normalize: divide each pixel's accumulated RGB by its α value

   Q: what if α = 0?

[Source: N. Snavely]

Poisson Image Editing

- Gradient domain reconstruction can be used to do object insertion in image editing applications

Figure: Perez et al. SIGGRAPH 2003

Panorama Examples

- Every image on Google Streetview

[Source: N. Snavely]

Figure: Uyttendaele et al. ICCV01

[Source: N. Snavely]

Ghost Removal

Figure: Uyttendaele et al. ICCV01

[Source: N. Snavely]

Other Types

- Can mosaic onto any surface if you know the geometry
- See NASAs Visible Earth project for some stunning earth mosaics

[Source: N. Snavely]