Problem Set 2

Graphical Models

May 6, 2011

1 Problems

1. Chapter 9, Exercise 9.2

2. Chapter 9, Exercise 9.11

3. Chapter 10, Exercise 10.12

4. Chapter 10, Exercise 10.17

2 Programming Assignment

1. **Sum-product on Trees** Consider an undirected tree $T = (V, E)$, where V are nodes and E are edges. For each node i, the associated variable x_i $\{1, 2, 3, ..., m\}$ and the potentials for each $\theta_i(x_i)$ and $\theta_{i,j}(x_i, x_j)$ are drawn from uniform distribution $U[-1, 1]$.

$$P(x_1, x_2, ..., x_N) \propto \exp(\sum_i \theta_i(x_i) + \sum_{(i,j)\in E} \theta_{i,j}(x_i, x_j))$$

Please write the sum-product algorithm for this tree to yield both node marginals $p_i(x_i)$ and edge marginals $p_{i,j}(x_i, x_j)$. What's the running time of this algorithm?

2. **Loopy belief propagation (sum-product) algorithm for grid graphs** In this problem, we consider a 8×8 grid graph (V, E). For each node $i \in \{(1, 1), (1, 2), ..., (8, 8)\}$, the associated variable $x_i \in \{0, 1\}$, and node potentials $\theta_i(x_i)$ are set to be 0.05 if $x_i = 1$ and -0.05 if $x_i = 0$. For each edge $(i, j) \in E$, we draw a $\hat{\theta}_{(i,j)}$ from uniform distribution $U[0, \mu]$, where μ is a parameter controlling the interaction strength. If $x_i = x_j$, we set the edge potential $\theta_{(i,j)}(x_i, x_j) = \hat{\theta}_{(i,j)}$. If $x_i \neq x_j$, we set the edge potential $\theta_{(i,j)}(x_i, x_j) = -\hat{\theta}_{(i,j)}$. Write a loopy sum-product algorithm for this grid graphical model. For each $\mu \in \{0, 0.2, 0.4, ..., 2\}$, generate 100 samples of the 8×8 grid graph, run your LBP algorithm and compute the averaged L_1 error of all node marginals $MAE = \frac{1}{100} \sum_i |p_i^{true}(x_i = 1) - p_i^{LBP}(x_i = 1)|$. Show the trend of MAE as μ increases. (Hint: the true node marginal probability can be calculated by the junction-tree algorithm.)