Week 10: Finish Equivalence Proof and Limitations

CSC 236:Introduction to the Theory of Computation

Summer 2024

Instructor: Lily

Announcement

- Final Exam Time is out! July 19th in EX 200 (7:00~10:00pm)
  - **BRING YOUR TCARD!** (or other valid forms of ID as listed in the Exam Toolkit). No ID = no entry
  - Students who arrive more than 30 minutes *after* the start of the exam ***will not* be able to take the exam**
  - You **must score** (40 + 10*(# missed midterms))% to pass
  - True or False question marking update: +1 correct answer, -0.5 incorrect answer (used to be -1 incorrect answer)
- Tutorials this week: proving a language is not regular, Turing Machines, and decidability

Consider the following languages

For each of the following languages determine if it regular or not and try to explain why or why not

1. $L_1 = \{0^m 1^n : m, n \in \mathbb{N}\}$.
2. For some $k \in \mathbb{N}$, $L_2 = \{0^n 1^n : n \leq k\}$.
3. $L_3 = \{0^n 1^n : n \in \mathbb{N}\}$.

1. $L_1 = \{0^m 1^n : m, n \in \mathbb{N}\}$.

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Proving Non-Regularity

Pumping Lemma

If $L \subset \Sigma^*$ is regular, then there exists a pumping length $p \in \mathbb{N}$ such that for every $w \in L$ of length *greater than or equal to* p, $w = xyz$ for $x, y, z \in \Sigma^*$ such that:

1. $|xy| \leq p$,
2. $|y| \geq 1$, and
3. $xy^k z \in L$ for every $k \in \mathbb{N}$.

Note: the Pumping Lemma is only useful for infinite languages (infinite sets of strings). What about finite languages?

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Proof.

If $L \subset \Sigma^*$ is regular, then there exists a pumping length $p \in \mathbb{N}$ such that for every $w \in L$ of length *greater than or equal to* p, $w = xyz$ for $x, y, z \in \Sigma^*$ such that:

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Proof.

Example.

Show that $L = \{vv : v \in \{0,1\}^*\}$ is not regular.

Now you try!

Show that the following languages are *not* regular.

1. $L_1 = \{0^n 1^n : n \in \mathbb{N}\}$.
2. $L_2 = \{0^n : n \text{ is a prime}\}$.

Prove that $L_1 = \{0^n 1^n : n \in \mathbb{N}\}$ is not regular.

Prove that $L_2 = \{0^n : n \text{ is a prime}\}$ is not regular.

Consider $L = \{0^n : n \text{ is even}\}$.

General problem format

Given a language L, you will be asked: is L regular?

1. Make a decision: regular or not regular.
2. If regular: proof of regularity.
   - Produce a finite automaton or regular expression which accept the given language
   - Prove correctness of finite automaton (if asked)
3. If not regular: proof of non-regularity.
   - Assume L is regular and apply pumping lemma

NFA to DFA

Given a NFA $M_N = (Q_N, \Sigma, \delta_N, s_N, F_N)$, construct a DFA $M_D = (Q_D, \Sigma, \delta_D, s_D, F_D)$ such that $\boldsymbol{\mathcal{L}}(M_N) = \boldsymbol{\mathcal{L}}(M_D)$.

NFA to DFA Example

Consider $L = \mathcal{L}\left(\left(0(0+10+1)\right)^*\right)$ and its corresponding NFA.

NFA to DFA

Given a NFA $M_N = (Q_N, \Sigma, \delta_N, s_N, F_N)$, construct a DFA $M_D = (Q_D, \Sigma, \delta_D, s_D, F_D)$ such that $\mathcal{L}(M_N) = \mathcal{L}(M_D)$.

NFA to DFA

Recap

- DFAs, NFAs, and regular expressions have the same expressive power (we showed the first two are equivalent, you will show the last is equivalent to the first two)
- Pumping lemma: used to show that a language is not regular
- Regular languages cannot count

Next time… review!