Week 3: Combinatorics and Graph Theory

CSC 236:Introduction to the Theory of Computation

Summer 2024

Instructor: Lily

Announcements

- Deferred students: **email us!**
  - Gives access to syllabus and quizzes (marks don’t count)
- Tutorial location changed:
  - Section 5101 + 5104: BA 1130 (Logan)
  - Section 5102 + 5103: BA 2135 (Lawrence)
- Peer review procedure

Discrete Probability: Definitions

- Ω: probability space, $w \in \Omega$ is an event
- $\mathbb{P}: \Omega \to [0,1]$ Probability distribution
  - $\mathbb{P}[w] \geq 0$
  - $\sum_{w \in \Omega} \mathbb{P}[w] = 1$
  - For disjoint $S_1, \ldots, S_n \subseteq \Omega$, $\mathbb{P}[\cup_{i=1}^{n} S_i] = \sum_{i=1}^{n} \mathbb{P}[S_i]$
- Random Variable:
- Expectation:

- Variance:

Tossing n coins and counting number of heads.

Poker Hands

Imagine being dealt a hand of five cards from a standard 52 card deck (4 suits, 13 values)

1. How many hands are two pairs (not full house)

2. How many hands are flushes (not straight flushes)?

Pigeonhole Principle (PHP)

Flock of 17 pigeons flies into 16 roosts:

Pigeonhole Principle (PHP)

Flock of n pigeons flies into m roosts ($n > m$):

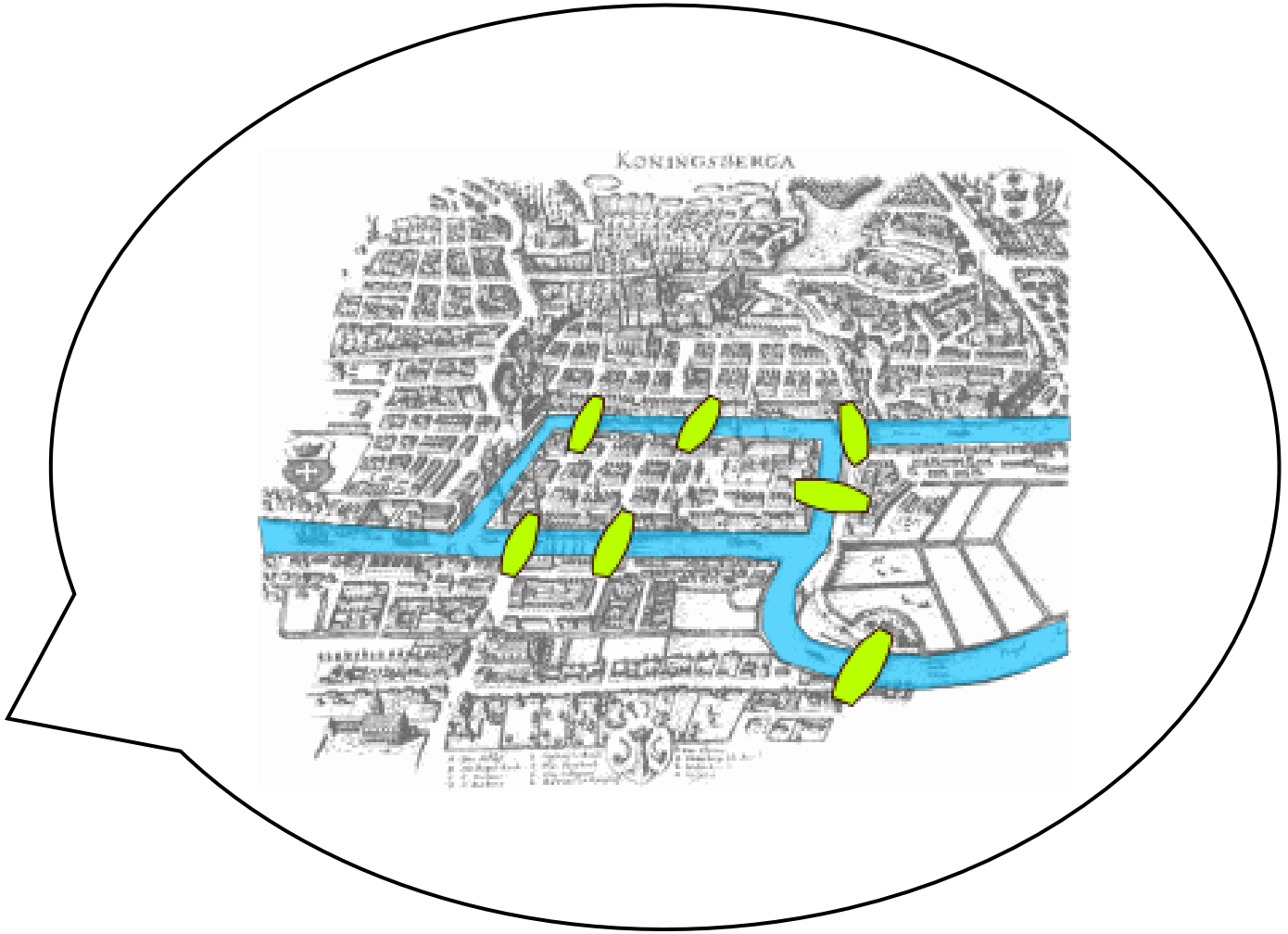
There is at least one roost with more than one pigeon.

Pigeonhole Principle (PHP):

For $m \in \mathbb{Z}_+$, if there are $m + 1$ or more objects and m boxes to put them in, then at least one box contains two or more objects.

Example. Among $n+1$ positive integers $a_0, \ldots, a_n$ where $a_i \leq 2n$, there must be one integer which divides another.

Graph Theory Fundamentals



https://en.wikipedia.org/wiki/Graph_theory#/media/File:Konigsberg_bridges.png

Now You Try!

1. What's the probability that a hand of five cards contains a four of a kind? What about a full house?
2. Every integer n has a multiple that has only 0s and 1s.

3. **Bridges of Königsberg:** Does there exist a Euler Tour for the bridge configuration?

Q1. What’s the probability that a hand of five cards contains a four of a kind? What about a full house?

Q2. Every integer n has a multiple that has only 0s and 1s.

Graph Terminology

- Walk: List of vertex, edge, vertex, …, vertex
- Trail: *walk* where no edge is repeated
- Path: *trail* with no repeated *internal* vertices
  - u, v-Path: *path with* endpoints u and v
- Cycle: u, u-Path
- Length of walk/trail/path/cycle: number of edges

Graph Terminology

- Connected: between every pair of vertices there exists a path
- Tree: Connected but no cycles
- Degree: # of neighbours

Every u, v-walk contains a u, v-path.

Every odd closed walk contains an odd cycle.

Recap

- Discrete Probability
  - Probability distributions
  - Expectation
  - Variance
- Pigeonhole Principle + Examples
- Graph Theory
  - Motivation: Bridges of Königsberg
  - Definitions: walk, trail, path, cycle
  - Basic properties (proof by induction)

Next time… more graph theory and structural induction.