- 1. Give a NFA and a regular expression for each language below.
  - (a)  $L_1 = \{s \in \{0, 1\}^* : s \text{ contains at least 2 characters and } s's \text{ second character is a 1} \}$ .

SOLUTION ELEMENTS

The first letter in a string  $s \in L_1$  can be either 0 or 1, the second letter must be 1, and then there can be any combination of 0's and 1's:

$$(0+1)1(0+1)^*$$



(b)  $L_2 = \{s \in \{0, 1\}^* : s \text{ contains at least 2 characters and does not end with 10}\}.$ 

SOLUTION ELEMENTS

If s (where  $s \in L_4$ ) does not end in 10, then it must end in one of 11 or 01 or 00 and begin with any combination of 0s and 1s:

$$(0+1)^*(11+01+00)$$



(c)  $L_3 = \{s \in \{a, b\}^* : \text{ every } a \text{ in } s \text{ is eventually followed by } b\}.$ 

## SOLUTION ELEMENTS

The strings in  $L_2$  are either empty, or end by *b*:

$$\epsilon + (a+b)^*b$$



(d)  $L_4 = \{s \in \{a, b\}^* : s \text{ contains some substring of length 4 whose first and last characters are the same}\}$ .

## SOLUTION ELEMENTS

Let  $s \in L_3$ . Assume x is a substring of s with length 4 whose first and last characters are the same. The second and third characters of x can be either a or b. Both first and last letters in x are either a, (i.e., a(a+b)(a+b)a), or b (i.e., b(a+b)(a+b)b). s can begin and end with any combination of a's and b's:

$$(a+b)^{*}(a(a+b)(a+b)a+b(a+b)(a+b)b)(a+b)^{*}$$



- 2. Give an NFA for each language below.
  - (a)  $L_5 = \{s \in \{0, 1, 2\}^*$ : the final digit has not appeared before}. Use at most five states.

SOLUTION ELEMENTS See the following diagram.



Note that a DFA is an NFA.

(b)  $L_6 = \{s \in \{0, 1, 2\}^*$ : the final digit has appeared before}.

SOLUTION ELEMENTS See the following diagram.

