1. Use the Pumping Lemma to prove that the following language is not regular:

$$L = \{0^i : i \text{ is a prime}\}$$

Solution

Assume for a contradiction that L is regular. Then L must satisfy the Pumping Lemma. That is, there exists $p \in \mathbb{N}$, $p > 0$ such that for every $w \in L$ with $|w| \geq p$, conditions (1) to (4) the in the Pumping Lemma are satisfied.

Let $w = 0^m$ where $m \geq p + 2$ is prime. Then $w \in L$ and $|w| \geq p$. Assume $w = xyz$. We show that for all x, y, z, as least one of the conditions (2)–(4) of the Pumping Lemma is not satisfied.
Assume that Conditions (2) and (3) are satisfied; that is, $|y| \geq 1$ and $|xy| \leq p$.
Note that since $w = xyz = 0^m$ (where $m \geq p + 2$) and $|xy| \leq p$, $|z|$ must be strictly greater than 1, and therefore $|xz| > 1$.

Now choose $i = |xz|$. Then, $|xy^iz| = |xz| + |y||xz| = (1 + |y|)|xz|$.
Since $(1 + |y|)$ and $|xz|$ are each greater than 1, the product must be a composite number. That is, $|xy^iz|$ is a composite number and so xy^iz is not a member of L; hence Condition (4) of the Pumping Lemma fails.

This is, however, a contradiction! So we can conclude that the original assumption (that L is regular) is false, and L is non-regular.