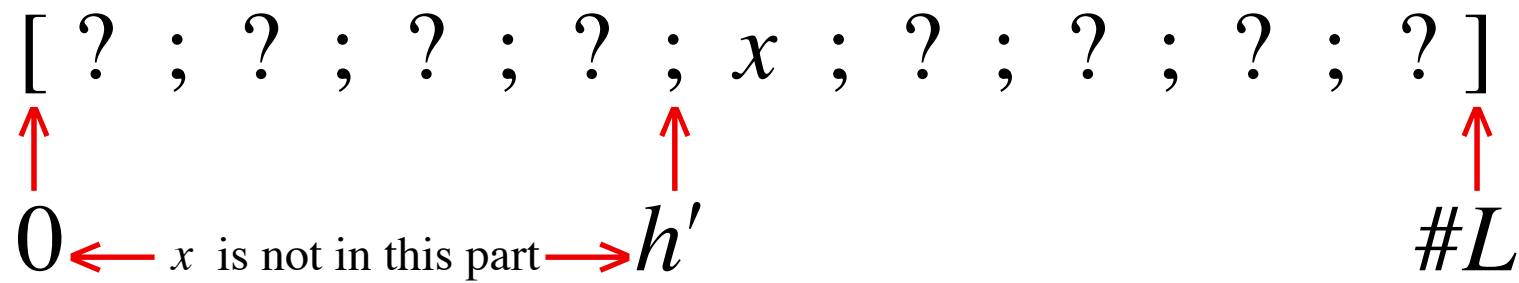


Linear Search

Find the first occurrence of item x in list L . The execution time must be linear in $\#L$.

Linear Search

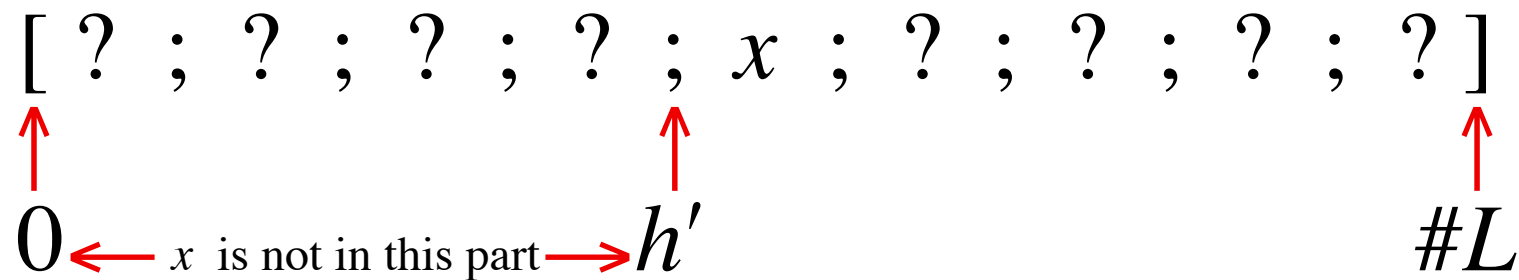
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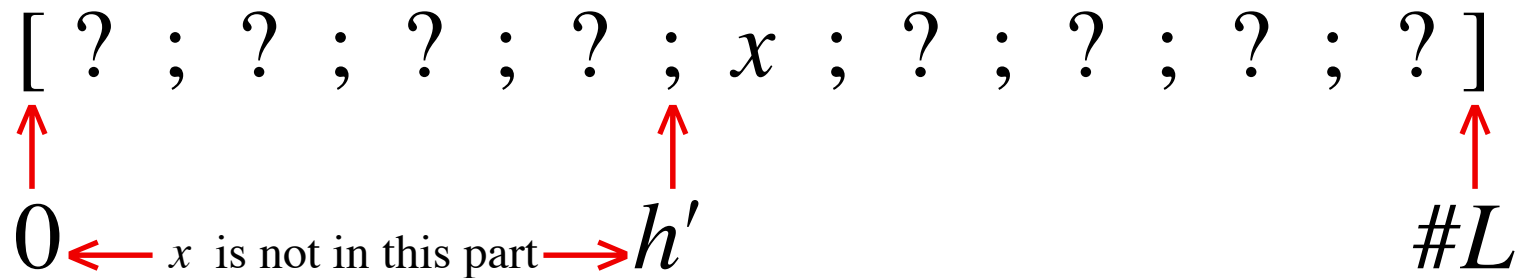
$$\neg x: L(0, ..h') \wedge (L h'=x \vee h'=\#L)$$



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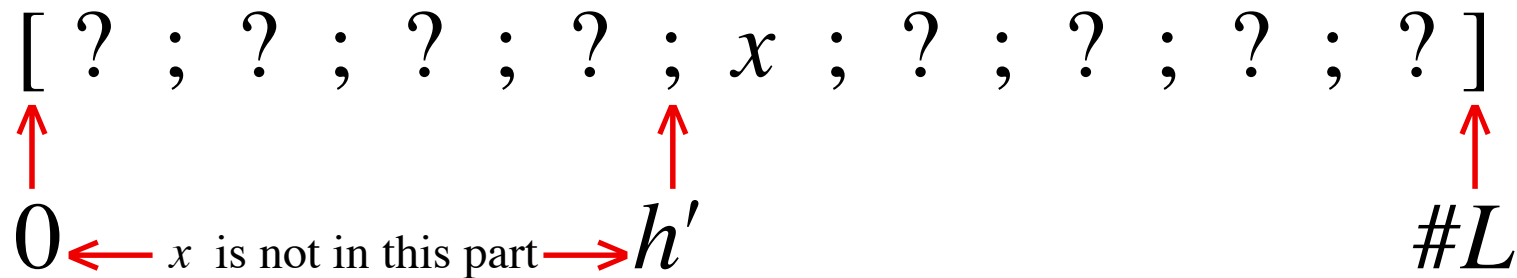
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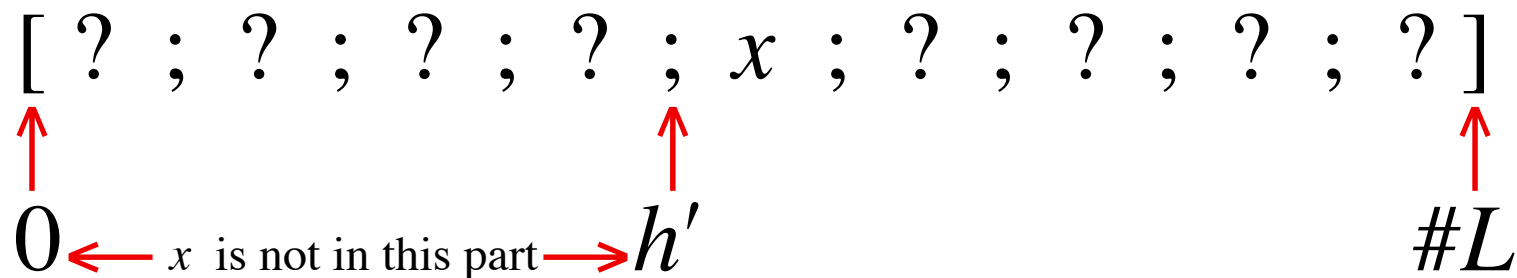
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$$\neg x: L(0, ..h') \wedge (L h'=x \vee h'=\#L) \Leftarrow$$

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$\neg x: L(0, ..h') \wedge (L h'=x \vee h'=\#L) \Leftarrow h:=0.$

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Linear Search

timing

$$t' \leq t + \#L \iff h := 0. h \leq \#L \Rightarrow t' \leq t + \#L - h$$

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substitution law

$$= h := h + 1. h \leq \#L \Rightarrow t' \leq t + 1 + \#L - h$$

Linear Search

timing

$$t' \leq t + \#L \iff h := 0. h \leq \#L \Rightarrow t' \leq t + \#L - h$$

$$h \leq \#L \Rightarrow t' \leq t + \#L - h \iff \mathbf{if\ } h = \#L \mathbf{\ then\ } ok \mathbf{\ else\ } h < \#L \Rightarrow t' \leq t + \#L - h \mathbf{\ fi}$$

$$h < \#L \Rightarrow t' \leq t + \#L - h \iff \mathbf{if\ } L\ h = x \mathbf{\ then\ } ok \mathbf{\ else\ } h := h + 1. t := t + 1. h \leq \#L \Rightarrow t' \leq t + \#L - h \mathbf{\ fi}$$

$$h := h + 1. t := t + 1. h \leq \#L \Rightarrow t' \leq t + \#L - h$$

substitution law

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simplify

$$= h < \#L \Rightarrow t' \leq t + \#L - h$$

Linear Search

Find the first occurrence of item x in list L . The execution time must be linear in $\#L$.

$$\neg x: L(0, ..h') \wedge (L h'=x \vee h'=\#L) \iff h:=0. h \leq \#L \Rightarrow \neg x: L(h, ..h') \wedge (L h'=x \vee h'=\#L)$$

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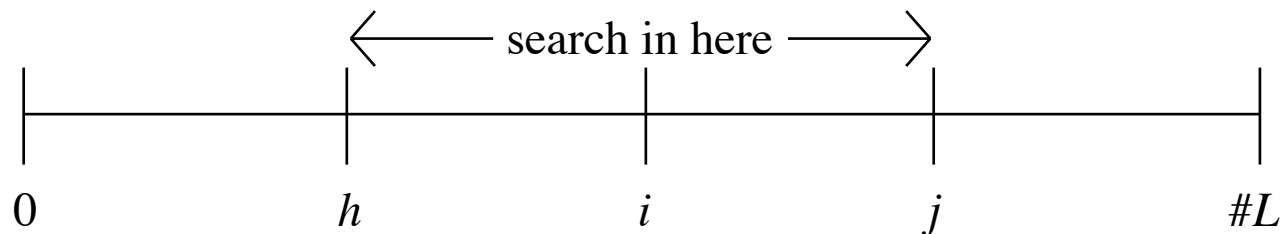
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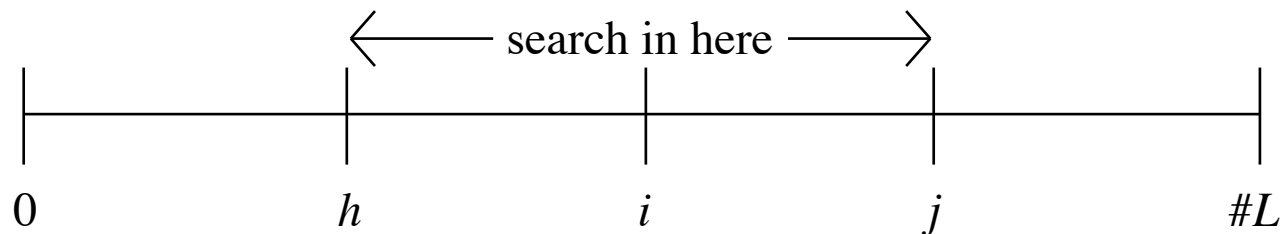
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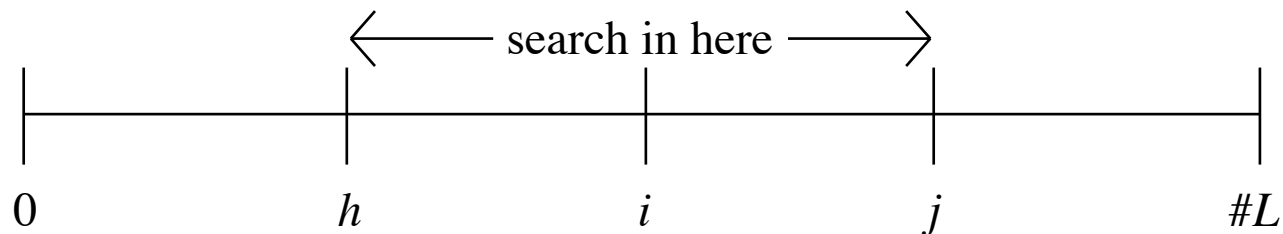
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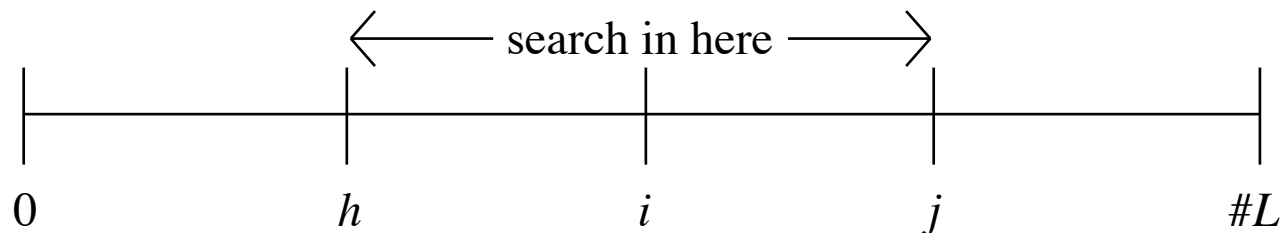
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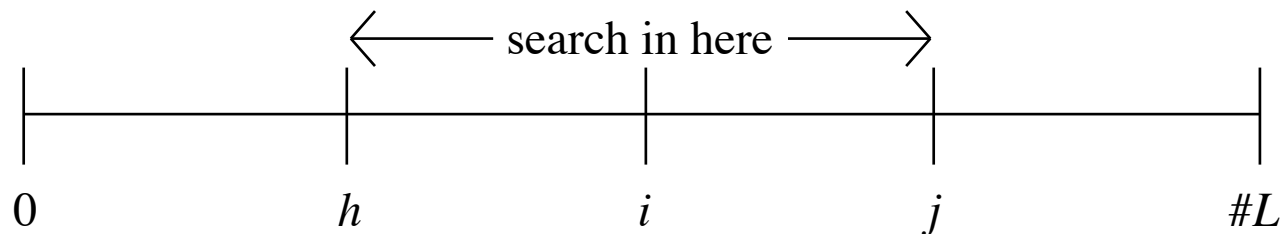
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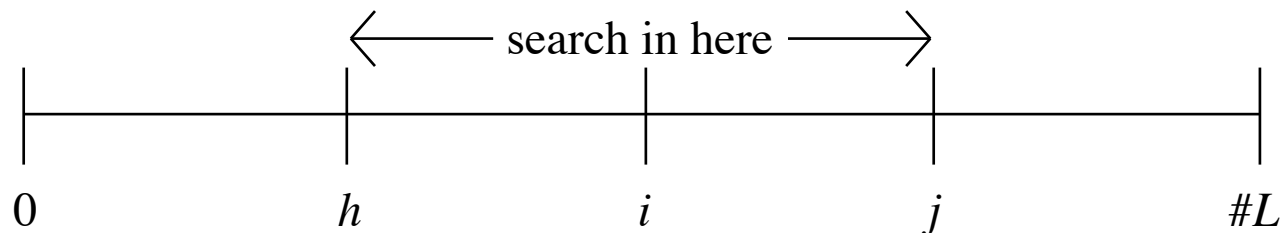
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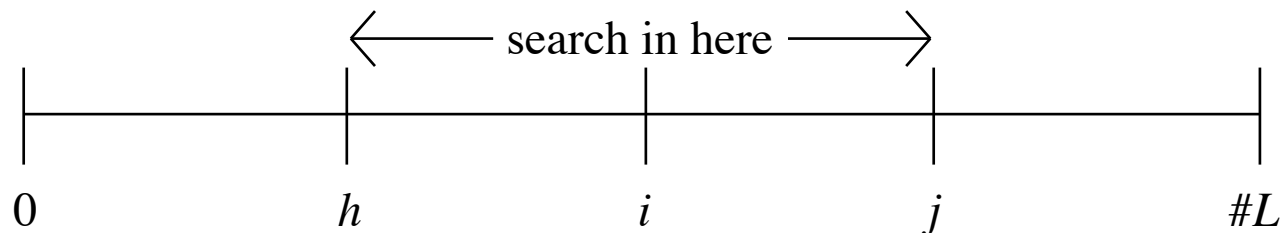
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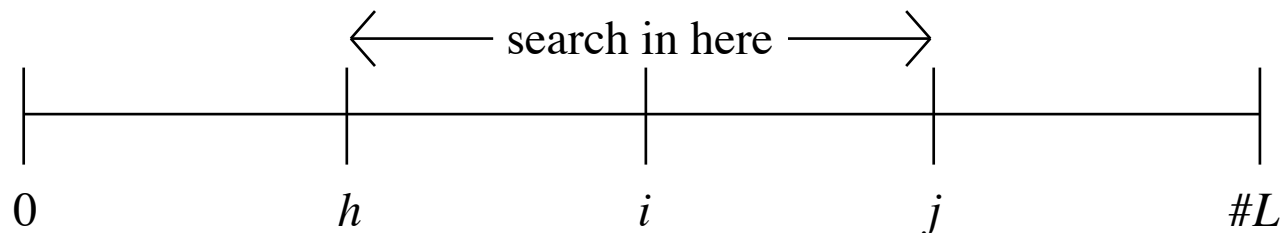
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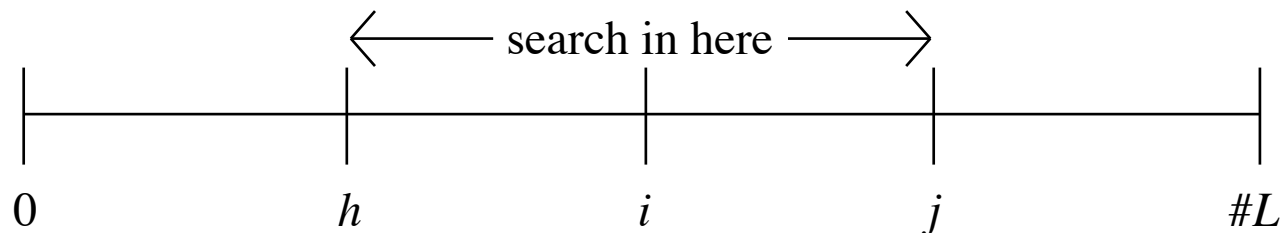
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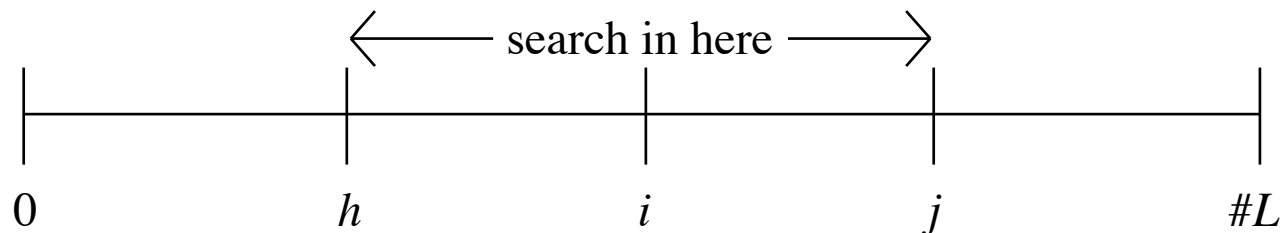
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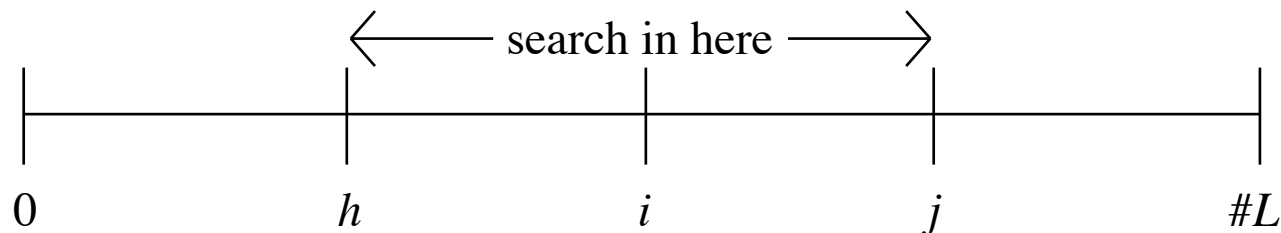
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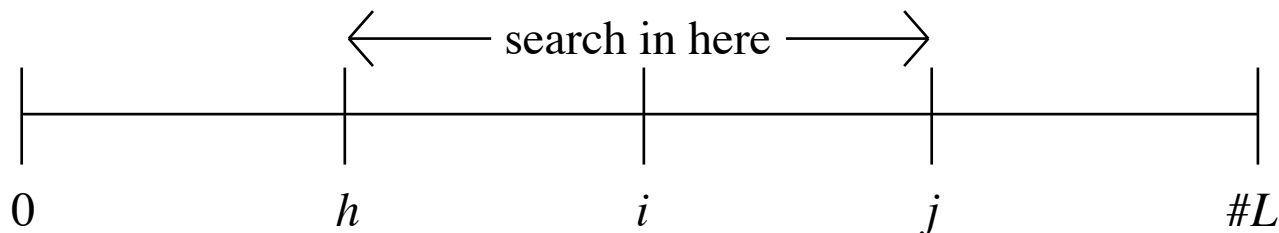
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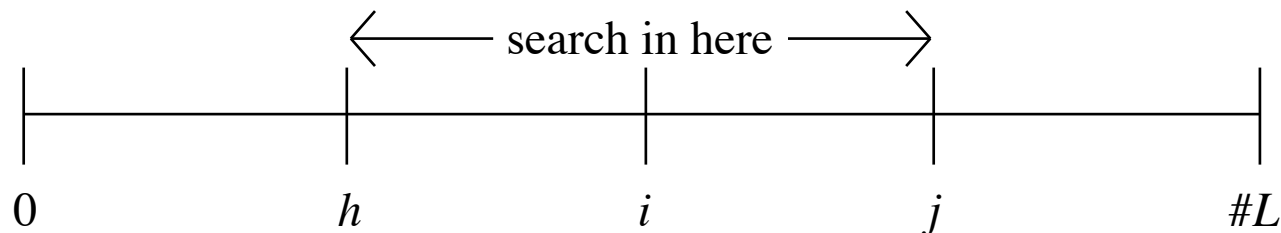
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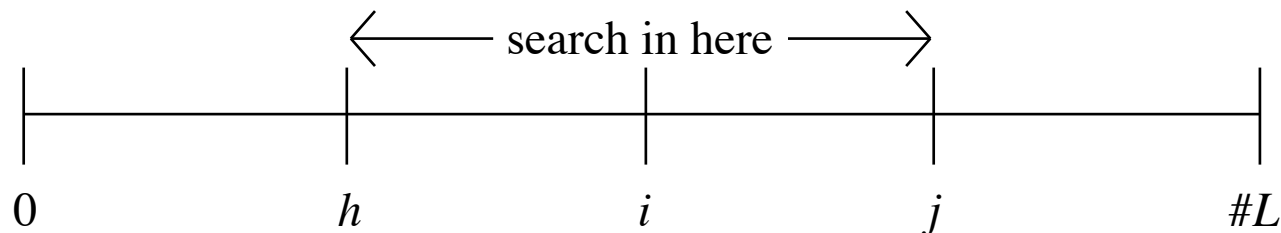
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$j-h \geq 2 \Rightarrow h'=h < i' < j=j' \Leftarrow i:=h+1$

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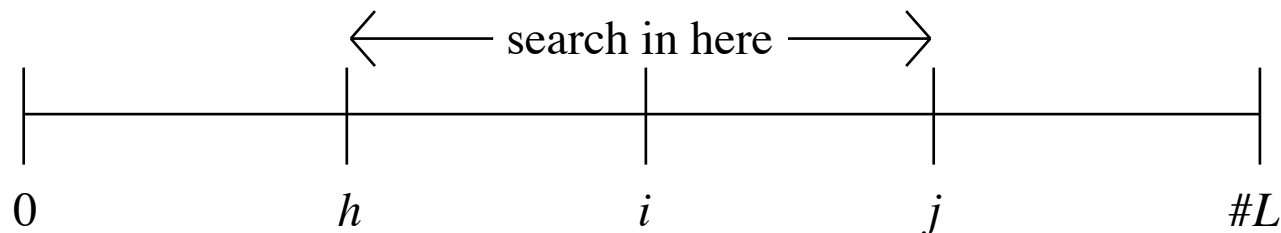
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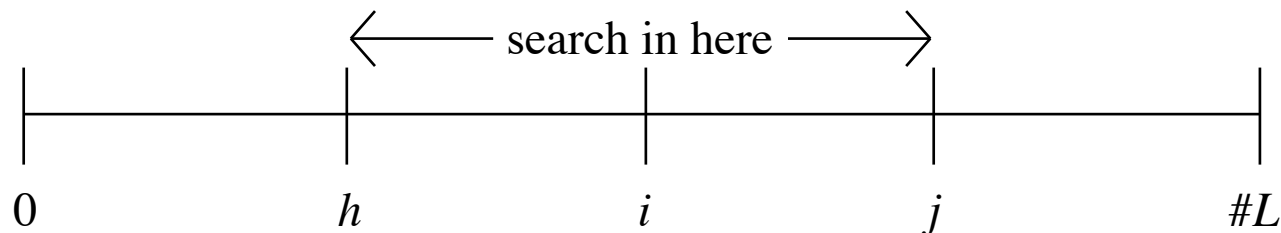
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$T \Leftarrow h:=0. j:=\#L. U$

$U \Leftarrow \mathbf{if } j-h = 1 \mathbf{ then } p:=L h = x \mathbf{ else } V \mathbf{ fi}$

$V \Leftarrow i:=\text{div}(h+j) 2.$

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$\#L = 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18$

$t'-t = 0 1 2 2 3 3 3 3 4 4 4 4 4 4 4 4 5 5$

$T \Leftarrow h:=0. j:=\#L. U$

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$\mathbf{if } L \ i \leq x \mathbf{ then } h:=i \mathbf{ else } j:=i \mathbf{ fi.}$

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$\#L = 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18$

$t'-t = 0 \ 1 \ 2 \ 2 \ 3 \ 3 \ 3 \ 3 \ 4 \ 4 \ 4 \ 4 \ 4 \ 4 \ 4 \ 4 \ 5 \ 5$

$T = t' \leq t + \text{ceil}(\log(\#L))$

$U = h < j \Rightarrow t' \leq t + \text{ceil}(\log(j-h))$

$V = j-h \geq 2 \Rightarrow t' \leq t + \text{ceil}(\log(j-h))$

Three Levels of Care

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lowest

don't bother with specifications

don't bother with refinements

just write code

Three Levels of Care

middle

write all specifications

but don't prove the refinements (just argue them informally)

lowest

don't bother with specifications

don't bother with refinements

just write code

Three Levels of Care

highest

write all specifications

prove all refinements (an automated theorem prover can help)

middle

write all specifications

but don't prove the refinements (just argue them informally)

lowest

don't bother with specifications

don't bother with refinements

just write code