Variable Declaration

new x: T· P

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new x: int· $x := 2.\ y := x+z$

Variable Declaration

$\textbf{new}\ x{:}\ T\cdot\ P$ declare local state variable x with type T and scope P

$=\quad \exists x, x'{:}\ T\cdot\ P$

$\textbf{new}\ x{:}\ int\cdot\ \ x{:=}\ 2.\ \ y{:=}\ x{+}z$

$=\quad \exists x, x'{:}\ int\cdot\ \ x'{=}2\ \land\ y' = 2{+}z\ \land\ z'{=}z$

Variable Declaration

	new x: T· P	declare local state variable x with type T and scope P
=	$\exists x, x'$: T· P	

	new x: int· x:= 2. y:= x+z	
=	$\exists x, x'$: int· x'=2 ∧ y' = 2+z ∧ z'=z	one-point for x' and idempotent for x
=	y' = 2+z ∧ z'=z	

Variable Declaration

new x: T· P declare local state variable x with type T and scope P

$=$ $\exists x, x'$: T· P

new x: int· $x{:=}\ 2.\ \ y{:=}\ x{+}z$

$=$ $\exists x, x'$: int· $x'{=}2 \ \land\ y' = 2{+}z \ \land\ z'{=}z$ one-point for x' and idempotent for x

$=$ $y' = 2{+}z \ \land\ z'{=}z$

new x: int· $y{:=}\ x$

Variable Declaration

new x: T· P declare local state variable x with type T and scope P

$=$ $\exists x, x'$: T· P

new x: int· x:= 2. y:= x+z

$=$ $\exists x, x'$: int· x'=2 $\land$ y' = 2+z $\land$ z'=z one-point for x' and idempotent for x

$=$ y' = 2+z $\land$ z'=z

new x: int· y:= x

$=$ $\exists x, x'$: int· x'=x $\land$ y'=x $\land$ z'=z

Variable Declaration

	new x: T· P	declare local state variable x with type T and scope P
$=$	$\exists x, x'$: T· P	
	new x: int· x:= 2. y:= x+z	
$=$	$\exists x, x'$: int· x'=2 ∧ y' = 2+z ∧ z'=z	one-point for x' and idempotent for x
$=$	y' = 2+z ∧ z'=z	
	new x: int· y:= x	
$=$	$\exists x, x'$: int· x'=x ∧ y'=x ∧ z'=z	one-point for x and x'
$=$	z'=z	

Variable Declaration

	new x: T· P	declare local state variable x with type T and scope P
=	$\exists x, x'$: T· P	

	new x: int· $x{:=}\,2.\ \ y{:=}\,x{+}z$	
=	$\exists x, x'$: int· $x'{=}2 \ \land\ y' = 2{+}z \ \land\ z'{=}z$	one-point for x' and idempotent for x
=	$y' = 2{+}z \ \land\ z'{=}z$	

	new x: int· $y{:=}\,x$	
=	$\exists x, x'$: int· $x'{=}x \land y'{=}x \land z'{=}z$	one-point for x and x'
=	$z'{=}z$	

new x: int· $y{:=}\,x{-}x$

Variable Declaration

$\textbf{new}\ x{:}\ T\cdot\ P$ declare local state variable x with type T and scope P

$=\quad \exists x, x'{:}\ T\cdot\ P$

$\textbf{new}\ x{:}\ int\cdot\ \ x{:=}\ 2.\ \ y{:=}\ x{+}z$

$=\quad \exists x, x'{:}\ int\cdot\ \ x'{=}2\ \land\ y' = 2{+}z\ \land\ z'{=}z$ one-point for x' and idempotent for x

$=\quad y' = 2{+}z\ \land\ z'{=}z$

$\textbf{new}\ x{:}\ int\cdot\ \ y{:=}\ x$

$=\quad \exists x, x'{:}\ int\cdot\ x'{=}x \land y'{=}x \land z'{=}z$ one-point for x and x'

$=\quad z'{=}z$

$\textbf{new}\ x{:}\ int\cdot\ \ y{:=}\ x{-}x$

$=\quad y'{=}0 \land z'{=}z$

Variable Declaration

new x: $T\cdot$ P

Variable Declaration

$$\textbf{new } x{:}\ T\cdot\ P$$

$$= \quad \exists x{:}\ \mathit{undefined}\cdot\ \exists x'{:}\ T, \mathit{undefined}\cdot\ P$$

Variable Declaration

$\textbf{new}\ x\text{: } T\cdot\ P$

$=\quad \exists x\text{: } undefined\cdot\ \exists x'\text{: } T, undefined\cdot\ P$

$\textbf{new}\ x\text{: } T := e\cdot\ \ P$

Variable Declaration

$\textbf{new } x\text{: } T\cdot\ P$

$= \quad \exists x\text{: } \mathit{undefined}\cdot\ \exists x'\text{: } T, \mathit{undefined}\cdot\ P$

$\textbf{new } x\text{: } T := e\cdot\ P$

$= \quad \exists x\text{: } e\cdot\ \exists x'\text{: } T\cdot\ P$

Variable Suspension

Suppose the state consists of variables w , x , y , and z .

frame $w, x \cdot P$

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	frame $w, x \cdot P$	within P , y and z are constants (no y' and z')
$=$	$P \land y'=y \land z'=z$	

Variable Suspension

Suppose the state consists of variables w , x , y , and z .

$$\textbf{frame } w, x\cdot\ P \qquad \text{within } P,\ y \text{ and } z \text{ are constants (no } y' \text{ and } z'\text{)}$$

$$= \qquad P \land y'=y \land z'=z$$

$$x:= e \quad = \quad \textbf{frame } x\cdot\ x' = e$$

Variable Suspension

Suppose the state consists of variables w , x , y , and z .

$$\textbf{frame}\ w, x\cdot\ P \qquad \text{within } P\text{ , } y \text{ and } z \text{ are constants (no } y' \text{ and } z' \text{)}$$

$$= \qquad P \wedge y'=y \wedge z'=z$$

$$x:= e \quad = \quad \textbf{frame}\ x\cdot\ x' = e$$

$$ok \quad = \quad \textbf{frame}\cdot\ \top$$

Variable Suspension

Suppose the state consists of variables w , x , y , and z .

frame $w, x\cdot P$ within P , y and z are constants (no y' and z')

$=$ $P \land y'=y \land z'=z$

$x:= e \quad = \quad$ **frame** $x\cdot x' = e$

$ok \quad = \quad$ **frame**$\cdot \top$

$s:= \Sigma L$

Variable Suspension

Suppose the state consists of variables w , x , y , and z .

frame $w, x\cdot\ P$ within P , y and z are constants (no y' and z')

$=$ $P \land y'=y \land z'=z$

$x:= e \;=\;$ **frame** $x\cdot\ x' = e$

$ok \;=\;$ **frame**$\cdot\ \top$

$s:= \Sigma L \;\Leftarrow\;$ **frame** $s\cdot$

Variable Suspension

Suppose the state consists of variables w , x , y , and z .

$\textbf{frame}\ w, x\cdot\ P$ within P , y and z are constants (no y' and z')

$=$ $P \wedge y'{=}y \wedge z'{=}z$

$x{:=}\ e \ \ = \ \ \textbf{frame}\ x\cdot\ x' = e$

$ok \ \ = \ \ \textbf{frame}\cdot\ \top$

$s{:=}\ \Sigma L \ \ \Leftarrow \ \ \textbf{frame}\ s\cdot\ \ \textbf{new}\ n{:}\ nat\cdot$

Variable Suspension

Suppose the state consists of variables w , x , y , and z .

$$\textbf{frame}\ w, x\cdot\ P \qquad \text{within } P\text{, } y \text{ and } z \text{ are constants (no } y' \text{ and } z'\text{)}$$

$$= \qquad P \wedge y'{=}y \wedge z'{=}z$$

$$x{:=}\ e \quad = \quad \textbf{frame}\ x\cdot\ x' = e$$

$$ok \quad = \quad \textbf{frame}\cdot\ \top$$

$$s{:=}\ \Sigma L \quad \Leftarrow \quad \textbf{frame}\ s\cdot\ \ \textbf{new}\ n{:}\ nat\cdot\ \ s' = \Sigma L$$

Variable Suspension

Suppose the state consists of variables w , x , y , and z .

$$\textbf{frame}\ w, x\cdot\ P \qquad \text{within } P,\ y \text{ and } z \text{ are constants (no } y' \text{ and } z' \text{)}$$

$$=\qquad P \wedge y'{=}y \wedge z'{=}z$$

$$x{:=}\ e \quad = \quad \textbf{frame}\ x\cdot\ x' = e$$

$$ok \quad = \quad \textbf{frame}\cdot\ \top$$

$$s{:=}\ \Sigma L \quad \Leftarrow \quad \textbf{frame}\ s\cdot\ \ \textbf{new}\ n{:}\ nat\cdot\ \ s' = \Sigma L$$

$$s' = \Sigma L \quad \Leftarrow$$

Array

$A(i) := e$

Array

$A[i] := e$

Array

$A\ i := e$

Array

$$A\ i := e \quad = \quad A'i{=}e \land (\forall j\cdot\ j{\ne}i \Rightarrow A'j{=}A\ j) \land x'{=}x \land y'{=}y \land \ldots$$

Array

$$A\ i := e \quad = \quad \underline{A'i{=}e} \land (\forall j\cdot\ j{\ne}i \Rightarrow A'j{=}A\ j) \land x'{=}x \land y'{=}y \land \ldots$$

Array

$A\ i := e \quad = \quad A'i{=}e \land (\forall j\cdot\ j{\ne}i \Rightarrow A'j{=}A\ j) \land x'{=}x \land y'{=}y \land \ldots$

Array

$$A\ i := e \quad = \quad A'i{=}e \land (\forall j\cdot\ j{\ne}i \Rightarrow A'j{=}A\ j) \underline{\land\ x'{=}x \land y'{=}y \land}\ \ldots$$

Array

$$A\ i := e \quad = \quad A'i=e \land (\forall j\cdot\ j\ne i \Rightarrow A'j=A\ j) \land x'=x \land y'=y \land \ldots$$

Array

$$A\ i := e \quad = \quad A'i{=}e \land (\forall j\cdot\ j{\ne}i \Rightarrow A'j{=}A\ j) \land x'{=}x \land y'{=}y \land \ldots$$

$$A\ 2{:=}\ 3.\ \ i{:=}\ 2.\ \ A\ i{:=}\ 4.\ \ A\ i = A\ 2$$

Array

$A\ i := e \quad = \quad A'i{=}e \wedge (\forall j\cdot\ j{\neq}i \Rightarrow A'j{=}A\ j) \wedge x'{=}x \wedge y'{=}y \wedge \ldots$

$A\ 2{:=}\ 3.\ \ i{:=}\ 2.\ \ \underline{A\ i{:=}\ 4.\ \ A\ i = A\ 2}$ Substitution Law

Array

$$A\ i := e \quad = \quad A'i=e \land (\forall j\cdot\ j\ne i \Rightarrow A'j=A\ j) \land x'=x \land y'=y \land \ldots$$

$A\ 2:= 3.\ \ i:= 2.\ \ \underline{A\ i:= 4.\ \ A\ i = A\ 2}$ ✗ Substitution Law

Array

$A\,i := e \quad = \quad A'i{=}e \land (\forall j\cdot\ j{\ne}i \Rightarrow A'j{=}A\,j) \land x'{=}x \land y'{=}y \land ...$

$A\,2{:=}\,3.\ \ i{:=}\,2.\ \ \underline{A\,i{:=}\,4.\ \ A\,i = A\,2}$ ✗ Substitution Law

$=\quad A\,2{:=}\,3.\ \ i{:=}\,2.\ \ 4 = A\,2$

Array

$A\ i := e \quad = \quad A'i=e \land (\forall j\cdot\ j \neq i \Rightarrow A'j=A\ j) \land x'=x \land y'=y \land ...$

$A\ 2:= 3.\ \ i:= 2.\ \ A\ i:= 4.\ \ A\ i = A\ 2$ ✗ Substitution Law

$=\quad A\ 2:= 3.\ \ i:= 2.\ \ 4 = A\ 2$ ✔ Substitution Law

Array

$A\ i := e \quad = \quad A'i=e \wedge (\forall j \cdot j \neq i \Rightarrow A'j=A\ j) \wedge x'=x \wedge y'=y \wedge ...$

$A\ 2:= 3.\ \ i:= 2.\ \ A\ i:= 4.\ \ A\ i = A\ 2$ ✗ Substitution Law

$=\quad A\ 2:= 3.\ \ i:= 2.\ \ 4 = A\ 2$ ✔ Substitution Law

$=\quad A\ 2:= 3.\ \ 4 = A\ 2$

Array

$A\ i := e \quad = \quad A'i=e \wedge (\forall j\cdot\ j \neq i \Rightarrow A'j=A\ j) \wedge x'=x \wedge y'=y \wedge \ldots$

	$A\ 2:= 3.\ \ i:= 2.\ \ A\ i:= 4.\ \ A\ i = A\ 2$	✗ Substitution Law
$=$	$A\ 2:= 3.\ \ i:= 2.\ \ 4 = A\ 2$	✔ Substitution Law
$=$	$A\ 2:= 3.\ \ 4 = A\ 2$	✗ Substitution Law

Array

$A\ i := e \quad = \quad A'i{=}e \land (\forall j\cdot\ j{\neq}i \Rightarrow A'j{=}A\ j) \land x'{=}x \land y'{=}y \land \ldots$

	$A\ 2{:=}\ 3.\ \ i{:=}\ 2.\ \ A\ i{:=}\ 4.\ \ A\ i = A\ 2$	✗ Substitution Law
=	$A\ 2{:=}\ 3.\ \ i{:=}\ 2.\ \ 4 = A\ 2$	✔ Substitution Law
=	$A\ 2{:=}\ 3.\ \ 4 = A\ 2$	✗ Substitution Law
=	$4{=}3$	

Array

$A\ i := e \quad = \quad A'i{=}e \wedge (\forall j\cdot\ j{\neq}i \Rightarrow A'j{=}A\ j) \wedge x'{=}x \wedge y'{=}y \wedge \ldots$

	$A\ 2{:=}\ 3.\ \ i{:=}\ 2.\ \ A\ i{:=}\ 4.\ \ A\ i = A\ 2$	✗ Substitution Law
$=$	$A\ 2{:=}\ 3.\ \ i{:=}\ 2.\ \ 4 = A\ 2$	✔ Substitution Law
$=$	$A\ 2{:=}\ 3.\ \ 4 = A\ 2$	✗ Substitution Law
$=$	$4{=}3$	
$=$	$\bot$	

Array

$A\ i := e \quad = \quad A'i{=}e \wedge (\forall j \cdot\ j{\ne}i \Rightarrow A'j{=}A\ j) \wedge x'{=}x \wedge y'{=}y \wedge \ldots$

	$A\ 2{:=}\ 3.\ \ i{:=}\ 2.\ \ A\ i{:=}\ 4.\ \ A\ i = A\ 2$	✗ Substitution Law
$=$	$A\ 2{:=}\ 3.\ \ i{:=}\ 2.\ \ 4 = A\ 2$	✔ Substitution Law
$=$	$A\ 2{:=}\ 3.\ \ 4 = A\ 2$	✗ Substitution Law
$=$	$4{=}3$	
$=$	$\bot$ ✗	

Array

$A\ i := e \quad = \quad A'i{=}e \land (\forall j\cdot\ j{\ne}i \Rightarrow A'j{=}A\ j) \land x'{=}x \land y'{=}y \land \ldots$

$A\ 2{:=}\ 3.\ \ i{:=}\ 2.\ \ A\ i{:=}\ 4.\ \ A\ i = A\ 2$ ✗ Substitution Law

$=\quad A\ 2{:=}\ 3.\ \ i{:=}\ 2.\ \ 4 = A\ 2$ ✔ Substitution Law

$=\quad A\ 2{:=}\ 3.\ \ 4 = A\ 2$ ✗ Substitution Law

$=\quad 4{=}3$

$=\quad \bot$ ✗

$A\ 2{:=}\ 2.\ \ A(A\ 2){:=}\ 3.\ \ A\ 2 = 2$

Array

$A\ i := e \quad = \quad A'i{=}e \wedge (\forall j\cdot\ j{\ne}i \Rightarrow A'j{=}A\ j) \wedge x'{=}x \wedge y'{=}y \wedge \ldots$

$A\ 2{:=}\ 3.\ \ i{:=}\ 2.\ \ A\ i{:=}\ 4.\ \ A\ i = A\ 2$ ✗ Substitution Law

$=\quad A\ 2{:=}\ 3.\ \ i{:=}\ 2.\ \ 4 = A\ 2$ ✔ Substitution Law

$=\quad A\ 2{:=}\ 3.\ \ 4 = A\ 2$ ✗ Substitution Law

$=\quad 4{=}3$

$=\quad \bot$ ✗

$A\ 2{:=}\ 2.\ \ \underline{A(A\ 2){:=}\ 3.\ \ A\ 2 = 2}$ ✗ Substitution Law

Array

$A\ i := e \quad = \quad A'i=e \wedge (\forall j\cdot\ j \neq i \Rightarrow A'j=A\ j) \wedge x'=x \wedge y'=y \wedge \ldots$

	$A\ 2:= 3.\ \ i:= 2.\ \ A\ i:= 4.\ \ A\ i = A\ 2$	✗ Substitution Law
$=$	$A\ 2:= 3.\ \ i:= 2.\ \ 4 = A\ 2$	✔ Substitution Law
$=$	$A\ 2:= 3.\ \ 4 = A\ 2$	✗ Substitution Law
$=$	$4=3$	
$=$	$\bot$ ✗	

	$A\ 2:= 2.\ \ A(A\ 2):= 3.\ \ A\ 2 = 2$	✗ Substitution Law
$=$	$A\ 2:= 2.\ \ A\ 2 = 2$	

Array

$A\ i := e \quad = \quad A'i{=}e \land (\forall j\cdot\ j{\neq}i \Rightarrow A'j{=}A\ j) \land x'{=}x \land y'{=}y \land \ldots$

	$A\ 2{:=}\ 3.\ \ i{:=}\ 2.\ \ A\ i{:=}\ 4.\ \ A\ i = A\ 2$	✗ Substitution Law
$=$	$A\ 2{:=}\ 3.\ \ i{:=}\ 2.\ \ 4 = A\ 2$	✔ Substitution Law
$=$	$A\ 2{:=}\ 3.\ \ 4 = A\ 2$	✗ Substitution Law
$=$	$4{=}3$	
$=$	$\bot$ ✗	

	$A\ 2{:=}\ 2.\ \ A(A\ 2){:=}\ 3.\ \ A\ 2 = 2$	✗ Substitution Law
$=$	$A\ 2{:=}\ 2.\ \ A\ 2 = 2$	✗ Substitution Law
$=$	$2{=}2$	

Array

$A\ i := e \quad = \quad A'i=e \land (\forall j\cdot\ j \neq i \Rightarrow A'j=A\ j) \land x'=x \land y'=y \land ...$

	$A\ 2:= 3.\ \ i:= 2.\ \ A\ i:= 4.\ \ A\ i = A\ 2$	✗	Substitution Law
$=$	$A\ 2:= 3.\ \ i:= 2.\ \ 4 = A\ 2$	✔	Substitution Law
$=$	$A\ 2:= 3.\ \ 4 = A\ 2$	✗	Substitution Law
$=$	$4=3$		
$=$	$\bot$ ✗		

	$A\ 2:= 2.\ \ A(A\ 2):= 3.\ \ A\ 2 = 2$	✗	Substitution Law
$=$	$A\ 2:= 2.\ \ A\ 2 = 2$	✗	Substitution Law
$=$	$2=2$		
$=$	$\top$		

Array

$A\,i := e \quad = \quad A'i{=}e \land (\forall j\cdot\ j{\ne}i \Rightarrow A'j{=}A\,j) \land x'{=}x \land y'{=}y \land \ldots$

	$A\,2{:=}\,3.\ \ i{:=}\,2.\ \ A\,i{:=}\,4.\ \ A\,i = A\,2$	✗ Substitution Law
$=$	$A\,2{:=}\,3.\ \ i{:=}\,2.\ \ 4 = A\,2$	✔ Substitution Law
$=$	$A\,2{:=}\,3.\ \ 4 = A\,2$	✗ Substitution Law
$=$	$4{=}3$	
$=$	$\bot$ ✗	

	$A\,2{:=}\,2.\ \ A(A\,2){:=}\,3.\ \ A\,2 = 2$	✗ Substitution Law
$=$	$A\,2{:=}\,2.\ \ A\,2 = 2$	✗ Substitution Law
$=$	$2{=}2$	
$=$	$\top$ ✗	

Array

$A\ i := e \quad = \quad A'i{=}e \wedge (\forall j\cdot\ j{\neq}i \Rightarrow A'j{=}A\ j) \wedge x'{=}x \wedge y'{=}y \wedge \ldots$

Array

$$A\ i := e \quad = \quad A'i=e \land (\forall j\cdot\ j\ne i \Rightarrow A'j=A\ j) \land x'=x \land y'=y \land \ldots$$

$$= \quad A' = i\rightarrow e \mid A \ \land\ x'=x \ \land\ y'=y \ \land \ldots$$

Array

$$A\ i := e \quad = \quad A'i=e \land (\forall j\cdot\ j \neq i \Rightarrow A'j=A\ j) \land x'=x \land y'=y \land ...$$

$$= \quad A' = i \rightarrow e \mid A \ \land \ x'=x \ \land \ y'=y \ \land ...$$

$$= \quad A := i \rightarrow e \mid A$$

Array

$$A\ i := e \quad = \quad A'i{=}e \land (\forall j\cdot\ j{\neq}i \Rightarrow A'j{=}A\ j) \land x'{=}x \land y'{=}y \land \ldots$$

$$= \quad A' = i{\rightarrow}e \mid A \ \land\ x'{=}x \ \land\ y'{=}y \ \land \ldots$$

$$= \quad A{:=}\ i{\rightarrow}e \mid A$$

$A\ 2{:=}\ 3.\ \ i{:=}\ 2.\ \ A\ i{:=}\ 4.\ \ A\ i = A\ 2$

Array

$$A\,i := e \quad = \quad A'i{=}e \wedge (\forall j\cdot\ j{\neq}i \Rightarrow A'j{=}A\,j) \wedge x'{=}x \wedge y'{=}y \wedge \ldots$$

$$= \quad A' = i{\to}e \mid A \;\wedge\; x'{=}x \;\wedge\; y'{=}y \;\wedge\; \ldots$$

$$= \quad A{:=}\ i{\to}e \mid A$$

$$A\,2{:=}\ 3.\;\; i{:=}\ 2.\;\; A\,i{:=}\ 4.\;\; A\,i = A\,2$$

$$= \quad A{:=}\ 2{\to}3 \mid A.\;\; i{:=}\ 2.\;\; A{:=}\ i{\to}4 \mid A.\;\; A\,i = A\,2$$

Array

$A\ i := e \quad = \quad A'i{=}e \wedge (\forall j\cdot\ j{\ne}i \Rightarrow A'j{=}A\ j) \wedge x'{=}x \wedge y'{=}y \wedge \ldots$

$= \quad A' = i{\to}e \mid A \ \wedge\ x'{=}x \ \wedge\ y'{=}y \ \wedge \ldots$

$= \quad A{:=}\ i{\to}e \mid A$

$A\ 2{:=}\ 3.\ \ i{:=}\ 2.\ \ A\ i{:=}\ 4.\ \ A\ i = A\ 2$

$= \quad A{:=}\ 2{\to}3 \mid A.\ \ i{:=}\ 2.\ \ \underline{A{:=}\ i{\to}4 \mid A.\ \ A\ i = A\ 2}$ Substitution Law

Array

$A\ i := e \quad = \quad A'i=e \land (\forall j\cdot\ j \neq i \Rightarrow A'j=A\ j) \land x'=x \land y'=y \land \ldots$

$\quad = \quad A' = i \rightarrow e \mid A \ \land \ x'=x \ \land \ y'=y \ \land \ldots$

$\quad = \quad A := i \rightarrow e \mid A$

$A\ 2 := 3.\ \ i := 2.\ \ A\ i := 4.\ \ A\ i = A\ 2$

$= \quad A := 2 \rightarrow 3 \mid A.\ \ i := 2.\ \ A := i \rightarrow 4 \mid A.\ \ A\ i = A\ 2$ Substitution Law

$= \quad A := 2 \rightarrow 3 \mid A.\ \ i := 2.\ \ (i \rightarrow 4 \mid A)i = (i \rightarrow 4 \mid A)2$

Array

$A\ i := e \quad = \quad A'i{=}e \wedge (\forall j\cdot\ j{\ne}i \Rightarrow A'j{=}A\ j) \wedge x'{=}x \wedge y'{=}y \wedge \ldots$

$\quad = \quad A' = i{\to}e \mid A \;\wedge\; x'{=}x \;\wedge\; y'{=}y \;\wedge\; \ldots$

$\quad = \quad A{:=}\ i{\to}e \mid A$

$A\ 2{:=}\ 3.\ \ i{:=}\ 2.\ \ A\ i{:=}\ 4.\ \ A\ i = A\ 2$

$=\quad A{:=}\ 2{\to}3 \mid A.\ \ i{:=}\ 2.\ \ A{:=}\ i{\to}4 \mid A.\ \ A\ i = A\ 2$ — Substitution Law

$=\quad A{:=}\ 2{\to}3 \mid A.\ \ i{:=}\ 2.\ \ (i{\to}4 \mid A)i = (i{\to}4 \mid A)2$ — Substitution Law

Array

$A\ i := e \quad = \quad A'i{=}e \land (\forall j\cdot\ j{\ne}i \Rightarrow A'j{=}A\ j) \land x'{=}x \land y'{=}y \land \ldots$

$= \quad A' = i{\to}e \mid A \ \land\ x'{=}x \ \land\ y'{=}y \ \land \ldots$

$= \quad A{:=}\ i{\to}e \mid A$

$A\ 2{:=}\ 3.\ \ i{:=}\ 2.\ \ A\ i{:=}\ 4.\ \ A\ i = A\ 2$

$= \quad A{:=}\ 2{\to}3 \mid A.\ \ i{:=}\ 2.\ \ A{:=}\ i{\to}4 \mid A.\ \ A\ i = A\ 2$ Substitution Law

$= \quad A{:=}\ 2{\to}3 \mid A.\ \ i{:=}\ 2.\ \ (i{\to}4 \mid A)i = (i{\to}4 \mid A)2$ Substitution Law

$= \quad A{:=}\ 2{\to}3 \mid A.\ \ (2{\to}4 \mid A)2 = (2{\to}4 \mid A)2$

Array

$A\ i := e \quad = \quad A'i{=}e \wedge (\forall j\cdot\ j{\neq}i \Rightarrow A'j{=}A\ j) \wedge x'{=}x \wedge y'{=}y \wedge \ldots$

$= \quad A' = i{\rightarrow}e \mid A \ \wedge\ x'{=}x \ \wedge\ y'{=}y \ \wedge \ldots$

$= \quad A{:=}\ i{\rightarrow}e \mid A$

$A\ 2{:=}\ 3.\ \ i{:=}\ 2.\ \ A\ i{:=}\ 4.\ \ A\ i = A\ 2$

$= \quad A{:=}\ 2{\rightarrow}3 \mid A.\ \ i{:=}\ 2.\ \ A{:=}\ i{\rightarrow}4 \mid A.\ \ A\ i = A\ 2$ — Substitution Law

$= \quad A{:=}\ 2{\rightarrow}3 \mid A.\ \ i{:=}\ 2.\ \ (i{\rightarrow}4 \mid A)i = (i{\rightarrow}4 \mid A)2$ — Substitution Law

$= \quad A{:=}\ 2{\rightarrow}3 \mid A.\ \ (2{\rightarrow}4 \mid A)2 = (2{\rightarrow}4 \mid A)2$ — $=$ is reflexive

$= \quad A{:=}\ 2{\rightarrow}3 \mid A.\ \ \top$

Array

$$A\ i := e \quad = \quad A'i{=}e \land (\forall j \cdot j{\ne}i \Rightarrow A'j{=}A\ j) \land x'{=}x \land y'{=}y \land \ldots$$

$$= \quad A' = i{\rightarrow}e \mid A \ \land\ x'{=}x \ \land\ y'{=}y \ \land \ldots$$

$$= \quad A{:=}\ i{\rightarrow}e \mid A$$

$$A\ 2{:=}\ 3.\ \ i{:=}\ 2.\ \ A\ i{:=}\ 4.\ \ A\ i = A\ 2$$

$= \quad A{:=}\ 2{\rightarrow}3 \mid A.\ \ i{:=}\ 2.\ \ A{:=}\ i{\rightarrow}4 \mid A.\ \ A\ i = A\ 2$ — Substitution Law

$= \quad A{:=}\ 2{\rightarrow}3 \mid A.\ \ i{:=}\ 2.\ \ (i{\rightarrow}4 \mid A)i = (i{\rightarrow}4 \mid A)2$ — Substitution Law

$= \quad A{:=}\ 2{\rightarrow}3 \mid A.\ \ (2{\rightarrow}4 \mid A)2 = (2{\rightarrow}4 \mid A)2$ — $=$ is reflexive

$= \quad A{:=}\ 2{\rightarrow}3 \mid A.\ \ \top$ — Substitution Law

$= \quad \top$

Array

$A\ i := e \quad = \quad A'i{=}e \land (\forall j\cdot\ j{\ne}i \Rightarrow A'j{=}A\ j) \land x'{=}x \land y'{=}y \land \ldots$

$\quad = \quad A' = i{\to}e \mid A \ \land\ x'{=}x \ \land\ y'{=}y \ \land \ldots$

$\quad = \quad A{:=}\ i{\to}e \mid A$

$A\ 2{:=}\ 3.\ \ i{:=}\ 2.\ \ A\ i{:=}\ 4.\ \ A\ i = A\ 2$

$=\quad A{:=}\ 2{\to}3 \mid A.\ \ i{:=}\ 2.\ \ A{:=}\ i{\to}4 \mid A.\ \ A\ i = A\ 2$ — Substitution Law

$=\quad A{:=}\ 2{\to}3 \mid A.\ \ i{:=}\ 2.\ \ (i{\to}4 \mid A)i = (i{\to}4 \mid A)2$ — Substitution Law

$=\quad A{:=}\ 2{\to}3 \mid A.\ \ (2{\to}4 \mid A)2 = (2{\to}4 \mid A)2$ — $=$ is reflexive

$=\quad A{:=}\ 2{\to}3 \mid A.\ \ \top$ — Substitution Law

$=\quad \top$ ✔

Array

$$A\,i := e \quad = \quad A'i=e \land (\forall j\cdot\ j\ne i \Rightarrow A'j=A\,j) \land x'=x \land y'=y \land \ldots$$

$$= \quad A' = i\to e \mid A \ \land\ x'=x \ \land\ y'=y \ \land \ldots$$

$$= \quad A := i\to e \mid A$$

Array

$$A\ i := e \quad = \quad A'i{=}e \wedge (\forall j\cdot\ j{\neq}i \Rightarrow A'j{=}A\ j) \wedge x'{=}x \wedge y'{=}y \wedge \ldots$$

$$= \quad A' = i{\rightarrow}e \mid A \ \wedge\ x'{=}x \ \wedge\ y'{=}y \ \wedge\ \ldots$$

$$= \quad A{:=}\ i{\rightarrow}e \mid A$$

$A\ 2{:=}\ 2.\quad A(A\ 2){:=}\ 3.\quad A\ 2 = 2$

Array

$A\ i := e \quad = \quad A'i{=}e \wedge (\forall j\cdot\ j{\ne}i \Rightarrow A'j{=}A\ j) \wedge x'{=}x \wedge y'{=}y \wedge \ldots$

$= \quad A' = i{\rightarrow}e \mid A \ \wedge\ x'{=}x \ \wedge\ y'{=}y \ \wedge \ldots$

$= \quad A{:=}\ i{\rightarrow}e \mid A$

$A\ 2{:=}\ 2.\quad A(A\ 2){:=}\ 3.\quad A\ 2 = 2$

$= \quad A{:=}\ 2{\rightarrow}2 \mid A.\quad A{:=}\ A\ 2 \rightarrow 3 \mid A.\quad A\ 2 = 2$

Array

$A\ i := e \quad = \quad A'i=e \land (\forall j\cdot\ j \neq i \Rightarrow A'j=A\ j) \land x'=x \land y'=y \land \ldots$

$\quad = \quad A' = i \rightarrow e \mid A \ \land\ x'=x \ \land\ y'=y \ \land \ldots$

$\quad = \quad A := i \rightarrow e \mid A$

$A\ 2 := 2.\quad A(A\ 2) := 3.\quad A\ 2 = 2$

$= \quad A := 2 \rightarrow 2 \mid A.\quad A := A\ 2 \rightarrow 3 \mid A.\quad A\ 2 = 2$ Substitution Law

$= \quad A := 2 \rightarrow 2 \mid A.\quad (A\ 2 \rightarrow 3 \mid A)2 = 2$

Array

$$A\ i := e \quad = \quad A'i{=}e \wedge (\forall j\cdot\ j{\ne}i \Rightarrow A'j{=}A\ j) \wedge x'{=}x \wedge y'{=}y \wedge \ldots$$

$$= \quad A' = i{\rightarrow}e \mid A \ \wedge\ x'{=}x \ \wedge\ y'{=}y \ \wedge \ldots$$

$$= \quad A{:=}\ i{\rightarrow}e \mid A$$

$$A\ 2{:=}\ 2.\quad A(A\ 2){:=}\ 3.\quad A\ 2 = 2$$

$$= \quad A{:=}\ 2{\rightarrow}2 \mid A.\quad A{:=}\ A\ 2 \rightarrow 3 \mid A.\quad A\ 2 = 2 \qquad \text{Substitution Law}$$

$$= \quad A{:=}\ 2{\rightarrow}2 \mid A.\quad (A\ 2 \rightarrow 3 \mid A)2 = 2 \qquad \text{Substitution Law}$$

$$= \quad ((2{\rightarrow}2 \mid A)2{\rightarrow}3 \mid 2{\rightarrow}2 \mid A)2 = 2$$

Array

$A\ i := e \quad = \quad A'i{=}e \wedge (\forall j\cdot\ j{\neq}i \Rightarrow A'j{=}A\ j) \wedge x'{=}x \wedge y'{=}y \wedge \ldots$

$= \quad A' = i{\to}e \mid A \ \wedge\ x'{=}x \ \wedge\ y'{=}y \ \wedge \ldots$

$= \quad A{:=}\ i{\to}e \mid A$

$A\ 2{:=}\ 2.\quad A(A\ 2){:=}\ 3.\quad A\ 2 = 2$

$= \quad A{:=}\ 2{\to}2 \mid A.\quad A{:=}\ A\ 2 \to 3 \mid A.\quad A\ 2 = 2$ Substitution Law

$= \quad A{:=}\ 2{\to}2 \mid A.\quad (A\ 2 \to 3 \mid A)2 = 2$ Substitution Law

$= \quad ((2{\to}2 \mid A)2{\to}3 \mid 2{\to}2 \mid A)2 = 2$

Array

$A\ i := e \quad = \quad A'i{=}e \wedge (\forall j\cdot\ j{\neq}i \Rightarrow A'j{=}A\ j) \wedge x'{=}x \wedge y'{=}y \wedge \ldots$

$= \quad A' = i{\rightarrow}e \mid A \ \wedge\ x'{=}x \ \wedge\ y'{=}y \ \wedge \ldots$

$= \quad A{:=}\ i{\rightarrow}e \mid A$

$A\ 2{:=}\ 2.\quad A(A\ 2){:=}\ 3.\quad A\ 2 = 2$

$= \quad A{:=}\ 2{\rightarrow}2 \mid A.\quad A{:=}\ A\ 2 \rightarrow 3 \mid A.\quad A\ 2 = 2$ Substitution Law

$= \quad A{:=}\ 2{\rightarrow}2 \mid A.\quad (A\ 2 \rightarrow 3 \mid A)2 = 2$ Substitution Law

$= \quad ((2{\rightarrow}2 \mid A)2{\rightarrow}3 \mid 2{\rightarrow}2 \mid A)2 = 2$

$= \quad (2{\rightarrow}3 \mid 2{\rightarrow}2 \mid A)2 = 2$

Array

$$A\ i := e \quad = \quad A'i{=}e \land (\forall j\cdot\ j{\ne}i \Rightarrow A'j{=}A\ j) \land x'{=}x \land y'{=}y \land \ldots$$

$$= \quad A' = i{\to}e \mid A \ \land\ x'{=}x \ \land\ y'{=}y \ \land \ldots$$

$$= \quad A{:=}\ i{\to}e \mid A$$

$$A\ 2{:=}\ 2.\quad A(A\ 2){:=}\ 3.\quad A\ 2 = 2$$

$= \quad A{:=}\ 2{\to}2 \mid A.\quad A{:=}\ A\ 2 \to 3 \mid A.\quad A\ 2 = 2$ Substitution Law

$= \quad A{:=}\ 2{\to}2 \mid A.\quad (A\ 2 \to 3 \mid A)2 = 2$ Substitution Law

$$= \quad ((2{\to}2 \mid A)2{\to}3 \mid 2{\to}2 \mid A)2 = 2$$

$$= \quad (2{\to}3 \mid 2{\to}2 \mid A)2 = 2$$

$$= \quad 3{=}2$$

Array

$A\ i := e \quad = \quad A'i=e \wedge (\forall j\cdot\ j \neq i \Rightarrow A'j=A\ j) \wedge x'=x \wedge y'=y \wedge \ldots$

$= \quad A' = i \rightarrow e \mid A \ \wedge\ x'=x \ \wedge\ y'=y \ \wedge \ldots$

$= \quad A := i \rightarrow e \mid A$

$A\ 2 := 2.\ \ A(A\ 2) := 3.\ \ A\ 2 = 2$

$= \quad A := 2 \rightarrow 2 \mid A.\ \ A := A\ 2 \rightarrow 3 \mid A.\ \ A\ 2 = 2$ — Substitution Law

$= \quad A := 2 \rightarrow 2 \mid A.\ \ (A\ 2 \rightarrow 3 \mid A)2 = 2$ — Substitution Law

$= \quad ((2 \rightarrow 2 \mid A)2 \rightarrow 3 \mid 2 \rightarrow 2 \mid A)2 = 2$

$= \quad (2 \rightarrow 3 \mid 2 \rightarrow 2 \mid A)2 = 2$

$= \quad 3=2$

$= \quad \bot$

Array

$A\ i := e \quad = \quad A'i{=}e \wedge (\forall j\cdot\ j{\neq}i \Rightarrow A'j{=}A\ j) \wedge x'{=}x \wedge y'{=}y \wedge \ldots$

$= \quad A' = i{\to}e \mid A \ \wedge\ x'{=}x \ \wedge\ y'{=}y \ \wedge \ldots$

$= \quad A{:=}\ i{\to}e \mid A$

$A\ 2{:=}\ 2.\quad A(A\ 2){:=}\ 3.\quad A\ 2 = 2$

$= \quad A{:=}\ 2{\to}2 \mid A.\quad A{:=}\ A\ 2 \to 3 \mid A.\quad A\ 2 = 2$ Substitution Law

$= \quad A{:=}\ 2{\to}2 \mid A.\quad (A\ 2 \to 3 \mid A)2 = 2$ Substitution Law

$= \quad ((2{\to}2 \mid A)2{\to}3 \mid 2{\to}2 \mid A)2 = 2$

$= \quad (2{\to}3 \mid 2{\to}2 \mid A)2 = 2$

$= \quad 3{=}2$

$= \quad \bot$ ✔

Array

Array

remember

$A\ i{:=}\ e$ becomes $A{:=}\ i{\to}e \mid A$

Array

remember

$A\ i:= e$ becomes $A:= i \rightarrow e \mid A$

$A\ i\ j:= e$ becomes $A:= (i;j) \rightarrow e \mid A$

Record

Record

$$
\begin{aligned}
\mathit{person} \;=\; & \quad \text{"name"} \rightarrow \mathit{text} \\
\mid & \quad \text{"age"} \rightarrow \mathit{nat}
\end{aligned}
$$

Record

$$person = \text{“name”} \to text \;\mid\; \text{“age”} \to nat$$

new *p*: *person*

Record

$$person = \text{"name"} \rightarrow text \mid \text{"age"} \rightarrow nat$$

new *p*: *person*

$$p := \text{"name"} \rightarrow \text{"Josh"} \mid \text{"age"} \rightarrow 17$$

Record

$$person \;=\; \text{“name”} \rightarrow text \;\mid\; \text{“age”} \rightarrow nat$$

new *p*: *person*

p:= “name” → “Josh” | “age” → 17

p “age”:= 18

Record

$$person \;=\; \text{“name”} \rightarrow text$$
$$\quad\quad | \;\; \text{“age”} \rightarrow nat$$

new p: $person$

p:= “name” → “Josh” | “age” → 17

p “age”:= 18

p:= “age” → 18 | p