Value Expression

P **value** *e*

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P **value** e	execute P then evaluate e but no state change

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P **value** *e* execute *P* then evaluate *e* but no state change

new *term*, *sum*: *rat* := 1·

for *i*:= 1;..15 **do** *term*:= *term*/*i*. *sum*:= *sum*+*term* **od**

value *sum*

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and don't substitute in (*P* **value** *e*)

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⊤ **value** axiom

= *x*:= *x*+1. (*x*:= *x*+1 **value** *x*)=*x*

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	⊤	**value** axiom
=	*x*:= *x*+1. (*x*:= *x*+1 **value** *x*)=*x*	Substitution Law but ...
=	(*x*:= *x*+1 **value** *x*) = *x*+1	

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don't substitute in (*P* **value** *e*)

	$y := (x := x+1 \textbf{ value } x)$	assignment
=	$y' = (x := x+1 \textbf{ value } x) \ \wedge \ x'=x$	previous calculation
=	$y' = x+1 \ \wedge \ x'=x$	assignment
=	$y := x+1$	

Value Expression

P **value** e	execute P then evaluate e but no state change

implementation

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Replace each nonlocal variable within *P* and *e* that is assigned within *P* by a fresh local variable initialized to the value of the nonlocal variable.

Then execute *P* and evaluate *e* .

Value Expression

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implementation

Replace each nonlocal variable within *P* and *e* that is assigned within *P* by a fresh local variable initialized to the value of the nonlocal variable. Then execute *P* and evaluate *e* .

but some language implementations don't introduce local variables

so expression evaluation can cause state change

Side Effects

Side Effects

$x = x$?

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not if there are side-effects !

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for reasoning

x:= (P **value** e)	becomes	P. x:= e
x:= (P **value** e) + y	becomes	(**new** z:= y· P. x:= e+z)

Side Effects

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for reasoning

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x:= (P **value** e) + y	becomes	(**new** z:= y· P. x:= e+z)

Don't neglect the time for expression evaluation.

Function

```
int bexp (int n)
{  int r = 1;
   int i;
   for (i=0; i<n; i++) r = r*2;
   return r; }
```

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\+ parameters

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\+ value expression

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```

bexp = ⟨ *n*: *int*·
new *r*: *int* := 1·
for *i*:= 0;..*n* **do** *r*:= *r*×2 **od**.
assert *r*: *int*
value *r* ⟩

C function = assertion about the result
+ name
+ parameters
+ scope control
+ value expression

Function

```
int bexp (int n)
{  int r = 1;
   int i;
   for (i=0; i<n; i++) r = r*2;
   return r; }
```

```
⟨ n: int·
  new r: int := 1·
  for i:= 0;..n do r:= r×2 od.
  assert r: int
  value r  ⟩
```

C function = assertion about the result
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C function = assertion about the result
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Procedure

procedure	=	name of procedure
	+	parameters
	+	scope control

Procedure

procedure = name of procedure

\+ parameters

\+ scope control

$P = \langle x\colon int \cdot\ a' < x < b' \rangle$

Procedure

procedure = name of procedure

\+ parameters

\+ scope control

$P = \langle x\colon int \cdot a' < x < b' \rangle$

$P\ 3 = a' < 3 < b'$

Procedure

procedure = name of procedure

\+ parameters

\+ scope control

$P = \langle x\colon int \cdot a' < x < b' \rangle$

$P\ 3 = a' < 3 < b'$

$P\ (a+1) = a' < a+1 < b'$

Procedure

procedure = name of procedure

\+ parameters

\+ scope control

$P = \langle x{:}\ int \cdot\ a' < x < b' \rangle$

$P\ 3 = a' < 3 < b'$

$P\ (a{+}1) = a' < a{+}1 < b'$

$a' < x < b' \Leftarrow a := x{-}1.\ b := x{+}1$

Procedure

procedure = name of procedure

\+ parameters

\+ scope control

$P = \langle x{:}\ int{\cdot}\ a' < x < b' \rangle$

$P\ 3 = a' < 3 < b'$

$P\ (a{+}1) = a' < a{+}1 < b'$

$a' < x < b' \Leftarrow a := x{-}1.\ b := x{+}1$

$\langle p{:}\ D{\cdot}\ B \rangle\ a = (\textbf{new}\ p{:}\ D := a{\cdot}\ B)$ if B doesn't use p' or $p:=$

Procedure

variable parameter reference parameter, var parameter

Procedure

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$\langle$**new** x: $int\cdot$ a:= 3. b:= 4. x:= 5$\rangle$

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= a:= 3. b:= 4. a:= 5

Procedure

variable parameter reference parameter, var parameter

$\langle$**new** x: $int\cdot$ a:= 3. b:= 4. x:= 5$\rangle$ a

= a:= 3. b:= 4. a:= 5

= a'=5 $\wedge$ b'=4

Procedure

variable parameter reference parameter, var parameter

$\langle$**new** x: $int\cdot$ a:= 3. b:= 4. x:= 5$\rangle$ a

= a:= 3. b:= 4. a:= 5

= a'=5 $\wedge$ b'=4

$\langle$**new** x: $int\cdot$ a'=3 $\wedge$ b'=4 $\wedge$ x'=5$\rangle$ a = ?

Procedure

variable parameter reference parameter, var parameter

$\langle$**new** x: int· a:= 3. b:= 4. x:= 5$\rangle$ a

$=$ a:= 3. b:= 4. a:= 5

$=$ a'=5 $\wedge$ b'=4

$\langle$**new** x: int· x:= 5. b:= 4. a:= 3$\rangle$ a

$=$ a:= 5. b:= 4. a:= 3

$=$ a'=3 $\wedge$ b'=4

$\langle$**new** x: int· a'=3 $\wedge$ b'=4 $\wedge$ x'=5$\rangle$ a $=$?

Procedure

variable parameter reference parameter, var parameter

$\langle$**new** x: *int*$\cdot$ $a{:=}3.\ b{:=}4.\ x{:=}5\rangle\ a$

$= \quad a{:=}3.\ b{:=}4.\ a{:=}5$

$= \quad a'{=}5 \wedge b'{=}4$

$\langle$**new** x: *int*$\cdot$ $x{:=}5.\ b{:=}4.\ a{:=}3\rangle\ a$

$= \quad a{:=}5.\ b{:=}4.\ a{:=}3$

$= \quad a'{=}3 \wedge b'{=}4$

$\langle$**new** x: *int*$\cdot$ $a'{=}3 \wedge b'{=}4 \wedge x'{=}5\rangle\ a \quad = \quad ?$

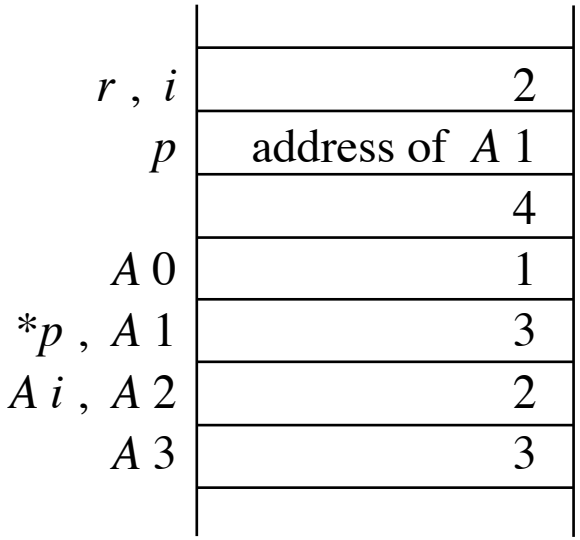
warning Use only for programs, not for arbitrary specifications.

Do not manipulate the procedure body.

Substitute arguments for parameters before any other manipulations.

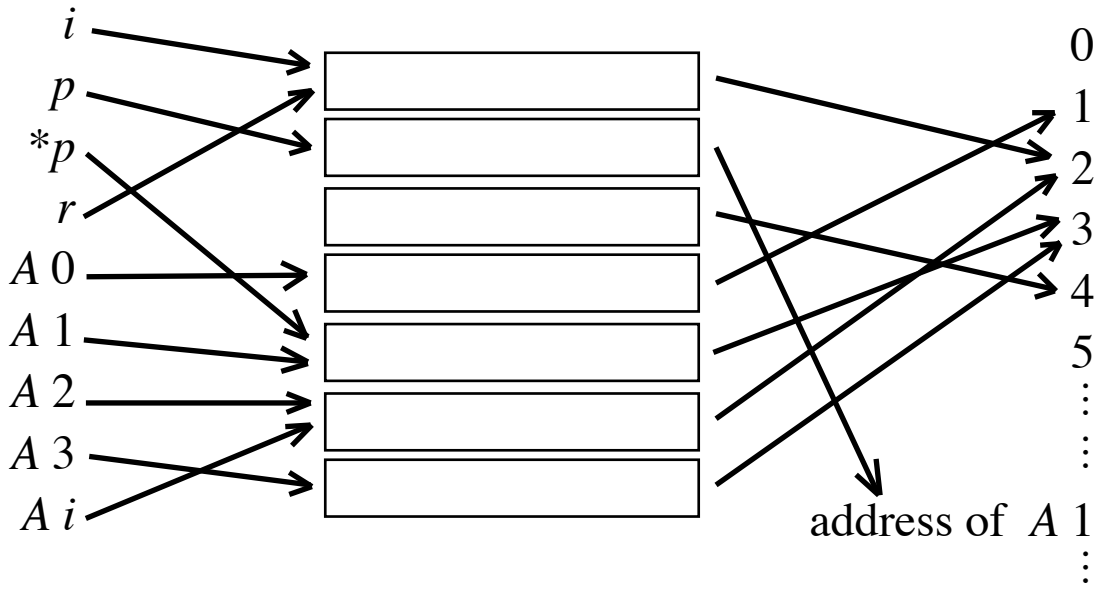
Apply programming theory separately for each call.

Alias

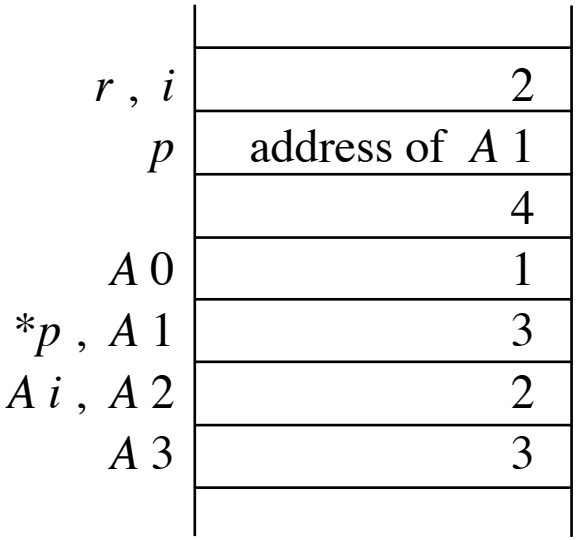


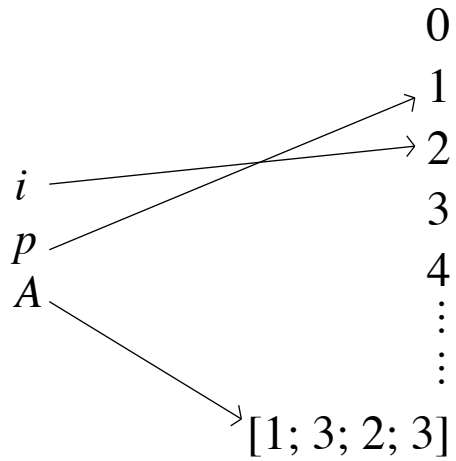
Alias

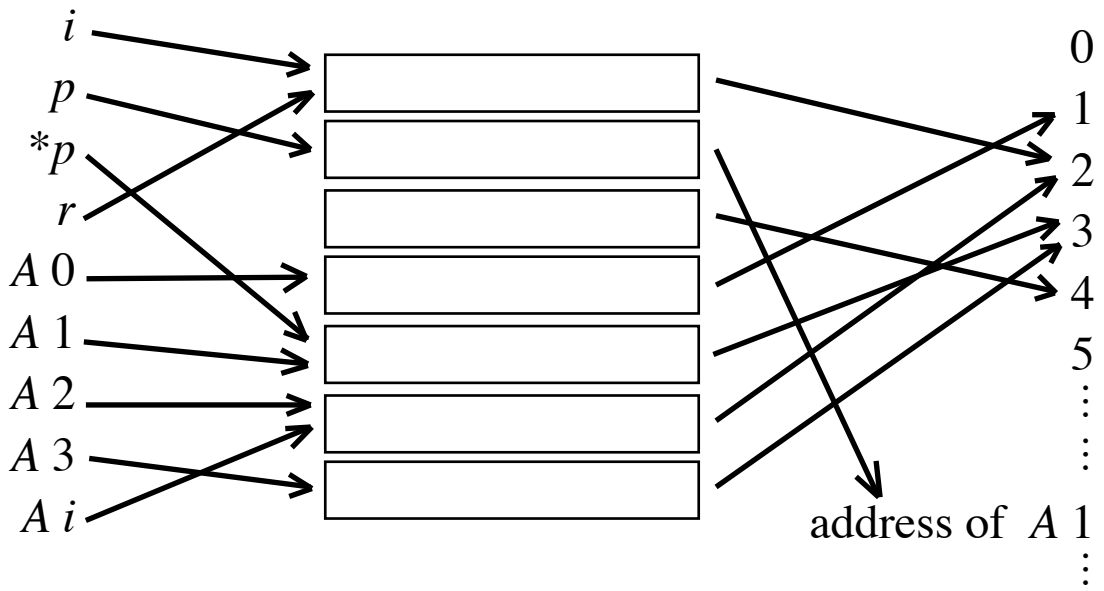
r , *i*	2
p	address of *A* 1
	4
A 0	1
**p* , *A* 1	3
A i , *A* 2	2
A 3	3



Alias


r , i
2
p
address of A 1
4
A 0
1
*p , A 1
3
A i , A 2
2
A 3
3


i
p
A
0
1
2
3
4
[1; 3; 2; 3]


i
p
*p
r
A 0
A 1
A 2
A 3
A i
0
1
2
3
4
5
address of A 1