Monotonicity and Antimonotonicity

Monotonicity and Antimonotonicity

covariance and contravariance

Monotonicity and Antimonotonicity

covariance	**and**	**contravariance**
varies directly as	**and**	**varies inversely as**

Monotonicity and Antimonotonicity

covariance	**and**	**contravariance**
varies directly as	**and**	**varies inversely as**
nondecreasing	**and**	**nonincreasing**

Monotonicity and Antimonotonicity

covariance	**and**	**contravariance**
varies directly as	**and**	**varies inversely as**
nondecreasing	**and**	**nonincreasing**
sorted	**and**	**sorted backwards**

Monotonicity and Antimonotonicity

covariance	**and**	**contravariance**
varies directly as	**and**	**varies inversely as**
nondecreasing	**and**	**nonincreasing**
sorted	**and**	**sorted backwards**

$x \le y \;\Rightarrow\; f\,x \le f\,y$ $\qquad\qquad$ $x \le y \;\Rightarrow\; f\,x \ge f\,y$

Monotonicity and Antimonotonicity

covariance	**and**	**contravariance**
varies directly as	**and**	**varies inversely as**
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$x \le y \;\Rightarrow\; f\,x \le f\,y$

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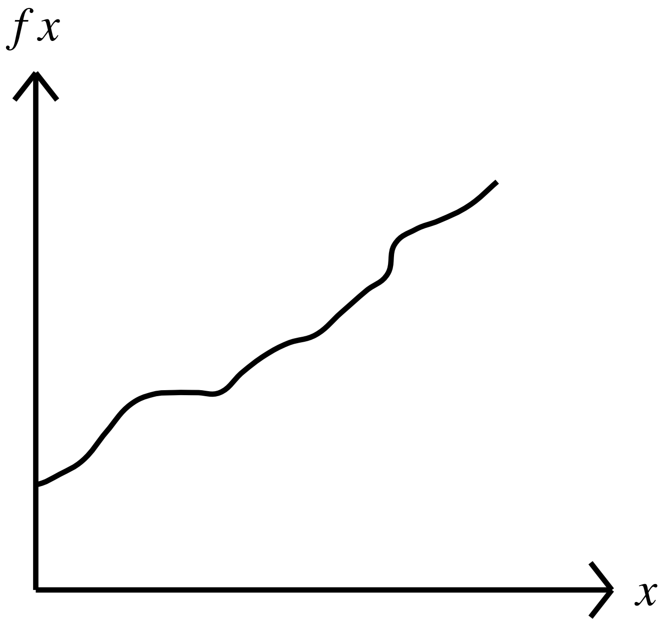

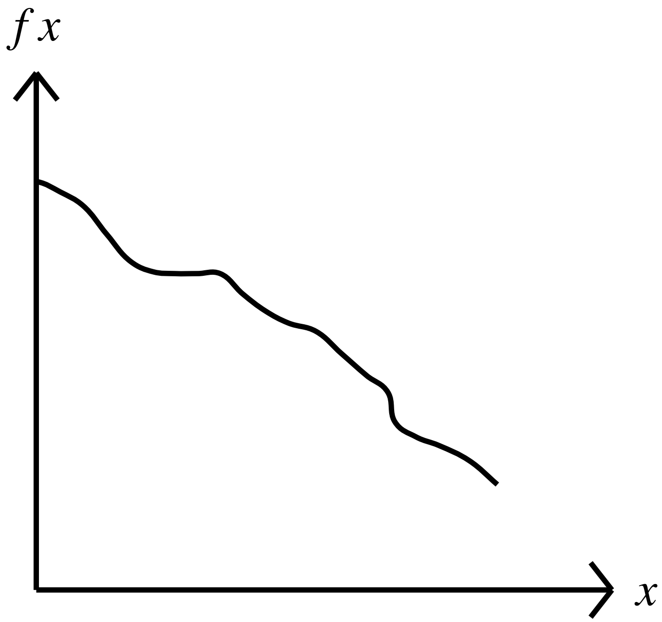

Monotonicity and Antimonotonicity

number:

binary:

Monotonicity and Antimonotonicity

number: $x \le y$

binary: $x \Rightarrow y$

Monotonicity and Antimonotonicity

number: $x \le y$ — x is less than or equal to y

binary: $x \Rightarrow y$ — x implies y

Monotonicity and Antimonotonicity

number: $x \le y$ — x is less than or equal to y

binary: $x \Rightarrow y$ — x implies y — x is falser than or equal to y

Monotonicity and Antimonotonicity

number: $x \le y$ x is less than or equal to y

binary: $x \Rightarrow y$ x implies y x is stronger than or equal to y

Monotonicity and Antimonotonicity

number:	$x \le y$	x is less than or equal to y	
	$-\infty \le +\infty$ $0 \le 1$	smaller $\le$ larger	
binary:	$x \Rightarrow y$	x implies y	x is stronger than or equal to y
	$\bot \Rightarrow \top$	stronger $\Rightarrow$ weaker	

Monotonicity and Antimonotonicity

number: $x \le y$ — x is less than or equal to y

$-\infty \le +\infty \quad 0 \le 1$ — smaller $\le$ larger

$x \le y \Rightarrow f\,x \le f\,y$ — f is monotonic

binary: $x \Rightarrow y$ — x implies y — x is stronger than or equal to y

$\bot \Rightarrow \top$ — stronger $\Rightarrow$ weaker

$x \Rightarrow y \;\Longrightarrow\; f\,x \Rightarrow f\,y$ — f is monotonic

Monotonicity and Antimonotonicity

number:	$x \le y$	x is less than or equal to y
	$-\infty \le +\infty$ $\quad 0 \le 1$	smaller $\le$ larger
	$x \le y \Rightarrow f\,x \le f\,y$	f is monotonic
		as x gets larger, $f\,x$ gets larger (or equal)

binary:	$x \Rightarrow y$	x implies y $\qquad$ x is stronger than or equal to y
	$\bot \Rightarrow \top$	stronger $\Rightarrow$ weaker
	$x \Rightarrow y \implies f\,x \Rightarrow f\,y$	f is monotonic
		as x gets weaker, $f\,x$ gets weaker (or equal)

Monotonicity and Antimonotonicity

number:	$x \le y$	x is less than or equal to y
	$-\infty \le +\infty$ $\quad 0 \le 1$	smaller $\le$ larger
	$x \le y \Rightarrow f\,x \le f\,y$	f is monotonic
		as x gets larger, $f\,x$ gets larger (or equal)
	$x \le y \Rightarrow f\,x \ge f\,y$	f is antimonotonic
		as x gets larger, $f\,x$ gets smaller (or equal)
binary:	$x \Rightarrow y$	x implies y $\qquad$ x is stronger than or equal to y
	$\bot \Rightarrow \top$	stronger $\Rightarrow$ weaker
	$x \Rightarrow y \implies f\,x \Rightarrow f\,y$	f is monotonic
		as x gets weaker, $f\,x$ gets weaker (or equal)
	$x \Rightarrow y \implies f\,x \Leftarrow f\,y$	f is antimonotonic
		as x gets weaker, $f\,x$ gets stronger (or equal)

Monotonicity and Antimonotonicity

$\neg a$	antimonotonic in a

Monotonicity and Antimonotonicity

$\neg a$	antimonotonic in a	
$a \wedge b$	monotonic in a	monotonic in b

Monotonicity and Antimonotonicity

$\neg a$	antimonotonic in a	
$a \wedge b$	monotonic in a	monotonic in b
$a \vee b$	monotonic in a	monotonic in b

Monotonicity and Antimonotonicity

$\neg a$	antimonotonic in a	
$a \wedge b$	monotonic in a	monotonic in b
$a \vee b$	monotonic in a	monotonic in b
$a \Rightarrow b$	antimonotonic in a	monotonic in b

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Monotonicity and Antimonotonicity

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$a \Rightarrow b$	antimonotonic in a	monotonic in b
$a \Leftarrow b$	monotonic in a	antimonotonic in b
if a **then** b **else** c **fi**	monotonic in b	monotonic in c

Monotonicity and Antimonotonicity

$\neg a$	antimonotonic in a	
$a \wedge b$	monotonic in a	monotonic in b
$a \vee b$	monotonic in a	monotonic in b
$a \Rightarrow b$	antimonotonic in a	monotonic in b
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if a **then** b **else** c **fi**	monotonic in b	monotonic in c

$\neg(a \wedge \neg(a \vee b))$

Monotonicity and Antimonotonicity

$\neg a$	antimonotonic in a	
$a \land b$	monotonic in a	monotonic in b
$a \lor b$	monotonic in a	monotonic in b
$a \Rightarrow b$	antimonotonic in a	monotonic in b
$a \Leftarrow b$	monotonic in a	antimonotonic in b
if a **then** b **else** c **fi**	monotonic in b	monotonic in c

$$\neg(a \land \neg(\underline{a \lor b}))$$

$$\Uparrow$$

$$\neg(a \land \neg\ \underline{a}\ )$$

use the Law of Generalization $a \Rightarrow a \lor b$

Monotonicity and Antimonotonicity

$\neg a$	antimonotonic in a	
$a \wedge b$	monotonic in a	monotonic in b
$a \vee b$	monotonic in a	monotonic in b
$a \Rightarrow b$	antimonotonic in a	monotonic in b
$a \Leftarrow b$	monotonic in a	antimonotonic in b
if a **then** b **else** c **fi**	monotonic in b	monotonic in c

$$\neg(a \wedge \underline{\neg(a \vee b)})$$

$$\Downarrow$$

$$\neg(a \wedge \overline{\neg \; a} \;)$$

use the Law of Generalization $a \Rightarrow a \vee b$

Monotonicity and Antimonotonicity

$\neg a$	antimonotonic in a	
$a \wedge b$	monotonic in a	monotonic in b
$a \vee b$	monotonic in a	monotonic in b
$a \Rightarrow b$	antimonotonic in a	monotonic in b
$a \Leftarrow b$	monotonic in a	antimonotonic in b
if a **then** b **else** c **fi**	monotonic in b	monotonic in c

$$\neg(\underline{a \wedge \neg(a \vee b)})$$
$$\Downarrow$$
$$\neg(\overline{a \wedge \neg \quad a \quad})$$

use the Law of Generalization $a \Rightarrow a \vee b$

Monotonicity and Antimonotonicity

$\neg a$	antimonotonic in a	
$a \wedge b$	monotonic in a	monotonic in b
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$a \Rightarrow b$	antimonotonic in a	monotonic in b
$a \Leftarrow b$	monotonic in a	antimonotonic in b
if a **then** b **else** c **fi**	monotonic in b	monotonic in c

$$\frac{\neg(a \wedge \neg(a \vee b))}{\neg(a \wedge \neg \quad a \quad)} \Uparrow$$

use the Law of Generalization $a \Rightarrow a \vee b$

Monotonicity and Antimonotonicity

$\neg a$	antimonotonic in a	
$a \wedge b$	monotonic in a	monotonic in b
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$a \Leftarrow b$	monotonic in a	antimonotonic in b
if a **then** b **else** c **fi**	monotonic in b	monotonic in c

$\neg(a \wedge \neg(a \vee b))$ use the Law of Generalization $a \Rightarrow a \vee b$

$\Leftarrow \quad \neg(a \wedge \neg \quad a \quad)$

Monotonicity and Antimonotonicity

$\neg a$	antimonotonic in a	
$a \wedge b$	monotonic in a	monotonic in b
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$a \Rightarrow b$	antimonotonic in a	monotonic in b
$a \Leftarrow b$	monotonic in a	antimonotonic in b
if a **then** b **else** c **fi**	monotonic in b	monotonic in c

$\neg(a \wedge \neg(a \vee b))$ — use the Law of Generalization $a \Rightarrow a \vee b$

$\Leftarrow \quad \neg(a \wedge \neg \quad a \quad)$ — now use the Law of Noncontradiction

$= \quad \top$

Context

In $a \wedge b$, when changing a , we can assume b .

Context

In $a \wedge b$, when changing a , we can assume b .

$$
\begin{array}{ll}
 & a \wedge b \\
 & \downarrow \\
= & c \wedge b
\end{array}
$$

Context

In $a \land b$, when changing a , we can assume b .

$$
\begin{aligned}
& a \land b \\
= \quad & \textcolor{red}{\downarrow} \\
& c \land b
\end{aligned}
$$

If b is $\top$, we have assumed correctly.

Context

In $a \wedge b$, when changing a , we can assume b .

$$
\begin{array}{ll}
 & a \wedge b \\
 & \downarrow \\
= & c \wedge b
\end{array}
$$

If b is $\top$, we have assumed correctly.

If b is $\bot$, then $a \wedge b$ and $c \wedge b$ are both $\bot$, so the equation is $\top$ anyway.

Context

In $a \wedge b$, when changing a , we can assume b .

Context

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In $a \wedge b$, when changing b , we can assume a .

Context

In $a \wedge b$, when changing a , we can assume b .

In $a \wedge b$, when changing b , we can assume a .

$$\neg(a \wedge \neg(a \vee b))$$

Context

In $a \wedge b$, when changing a , we can assume b .

In $a \wedge b$, when changing b , we can assume a .

$\neg(a \wedge \underline{\neg(a \vee b)})$ assume a to simplify $\neg(a \vee b)$

Context

In $a \wedge b$, when changing a , we can assume b .

In $a \wedge b$, when changing b , we can assume a .

	$\neg(a \wedge \underline{\neg(a \vee b)})$	assume a to simplify $\neg(a \vee b)$
$=$	$\neg(a \wedge \neg(\top \vee b))$	

Context

In $a \wedge b$, when changing a , we can assume b .

In $a \wedge b$, when changing b , we can assume a .

	$\neg(a \wedge \underline{\neg(a \vee b)})$	assume a to simplify $\neg(a \vee b)$
$=$	$\neg(a \wedge \neg(\top \vee b))$	Symmetry Law and Base Law for $\vee$
$=$	$\neg(a \wedge \neg\top)$	

Context

In $a \wedge b$, when changing a , we can assume b .

In $a \wedge b$, when changing b , we can assume a .

	$\neg(a \wedge \neg(a \vee b))$	assume a to simplify $\neg(a \vee b)$
$=$	$\neg(a \wedge \neg(\top \vee b))$	Symmetry Law and Base Law for $\vee$
$=$	$\neg(a \wedge \neg\top)$	Truth Table for $\neg$
$=$	$\neg(a \wedge \bot)$	

Context

In $a \wedge b$, when changing a , we can assume b .

In $a \wedge b$, when changing b , we can assume a .

	$\neg(a \wedge \underline{\neg(a \vee b)})$	assume a to simplify $\neg(a \vee b)$
$=$	$\neg(a \wedge \neg(\top \vee b))$	Symmetry Law and Base Law for $\vee$
$=$	$\neg(a \wedge \neg\top)$	Truth Table for $\neg$
$=$	$\neg(a \wedge \bot)$	Base Law for $\wedge$
$=$	$\neg\bot$	

Context

In $a \wedge b$, when changing a, we can assume b.

In $a \wedge b$, when changing b, we can assume a.

	$\neg(a \wedge \underline{\neg(a \vee b)})$	assume a to simplify $\neg(a \vee b)$
$=$	$\neg(a \wedge \neg(\top \vee b))$	Symmetry Law and Base Law for $\vee$
$=$	$\neg(a \wedge \neg\top)$	Truth Table for $\neg$
$=$	$\neg(a \wedge \bot)$	Base Law for $\wedge$
$=$	$\neg\bot$	Binary Axiom, or Truth Table for $\neg$
$=$	$\top$	

Context

In $a \wedge b$, when changing a , we can assume b .

In $a \wedge b$, when changing b , we can assume a .

Context

In $a \wedge b$, when changing a , we can assume b .

In $a \wedge b$, when changing b , we can assume a .

In $a \vee b$, when changing a , we can assume $\neg b$.

In $a \vee b$, when changing b , we can assume $\neg a$.

In $a \Rightarrow b$, when changing a , we can assume $\neg b$.

In $a \Rightarrow b$, when changing b , we can assume a .

In $a \Leftarrow b$, when changing a , we can assume b .

In $a \Leftarrow b$, when changing b , we can assume $\neg a$.

In **if** a **then** b **else** c **fi** , when changing a , we can assume $b \ne c$.

In **if** a **then** b **else** c **fi** , when changing b , we can assume a .

In **if** a **then** b **else** c **fi** , when changing c , we can assume $\neg a$.

Number Theory

number expressions represent quantity

Number Theory

number expressions represent quantity

number expressions

$0 \quad 1 \quad 2 \quad 597 \quad 1.2 \quad 1e10 \quad \infty$

$-x \quad x+y \quad x-y \quad x \times y \quad x/y \quad \frac{x}{y} \quad x^y \quad x \uparrow y \quad x \downarrow y$

if *a* **then** *x* **else** *y* **fi**

Number Theory

number expressions represent quantity

number expressions

$0 \quad 1 \quad 2 \quad 597 \quad 1.2 \quad 1e10 \quad \infty$

$\uparrow$

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Number Theory

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if *a* **then** *x* **else** *y* **fi**

binary expressions

$x=y \quad x \neq y \quad x<y \quad x>y \quad x \leq y \quad x \geq y$

Character Theory

“A”   “a”   “ ”   ‘‘‘‘‘‘’’   ‘‘’’’’’’

succ   *pred*   **if then else fi**

$= \quad \neq \quad < \quad > \quad \leq \quad \geq$

Character Theory

“A”   “a”   “ ”   “““”   “”””

↑

succ   *pred*   **if then else fi**

$= \quad \neq \quad < \quad > \quad \leq \quad \geq$

Character Theory

“A”　　“a”　　“ ”　　“““”　　“”””

↑

succ　　*pred*　　**if then else fi**

= ≠ < > ≤ ≥

Character Theory

“A”   “a”   “ ”   “““”   “”””

succ   *pred*   **if then else fi**

$= \quad \neq \quad < \quad > \quad \leq \quad \geq$

Character Theory

“A” “a” “ ” “““” “”””

↑ ↑

succ *pred* **if then else fi**

$=$ $\neq$ $<$ $>$ $\leq$ $\geq$

Character Theory

“A” “a” “ ” “““” “”””

succ *pred* **if then else fi**

↑ ↑

$= \quad \neq \quad < \quad > \quad \leq \quad \geq$

Character Theory

“A” “a” “ ” “““““” “””””””

succ *pred* **if then else fi**

$= \quad \neq \quad < \quad > \quad \leq \quad \geq$

Character Theory

“A” “a” “ ” “““” “”””

succ *pred* **if then else fi**

$= \quad \neq \quad < \quad > \quad \leq \quad \geq \quad \leftarrow$