Limited Queue

user's variables: c: *bin* and x: X

old implementer's variables: Q: $[n*X]$ and p: $0,..n+1$

operations

$mkemptyq \;=\; p:= 0$

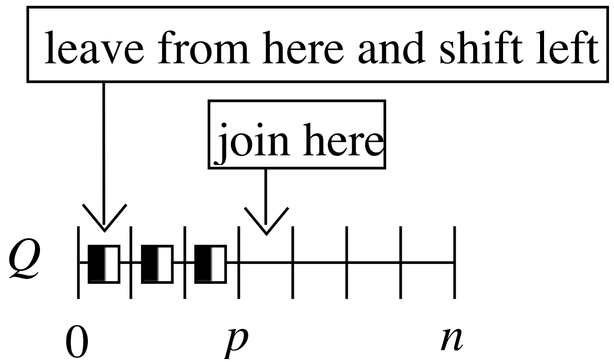
$isemptyq \;=\; c:= p=0$

$isfullq \;=\; c:= p=n$

$join \;=\; Q\,p:= x.\;\; p:= p+1$

$leave \;=\; \textbf{for } i:= 1;..p \textbf{ do } Q(i-1):= Q\,i \textbf{ od}.\;\; p:= p-1$

$front \;=\; x:= Q\,0$



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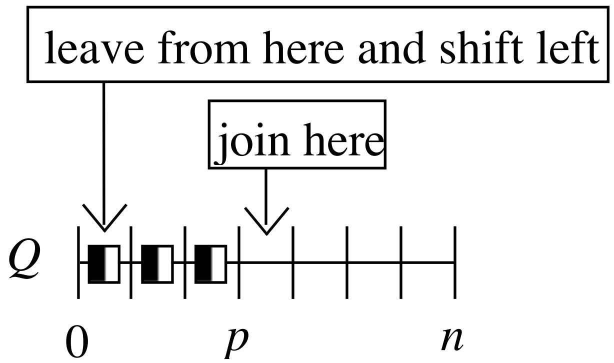

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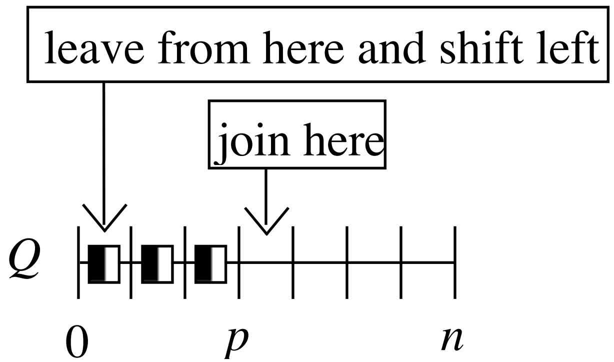

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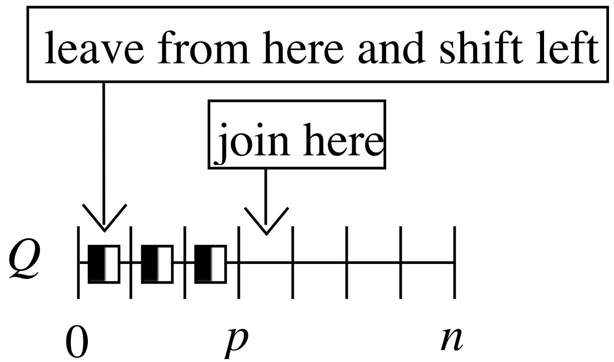

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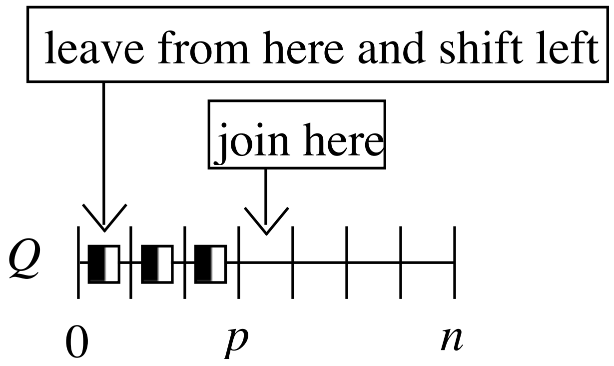

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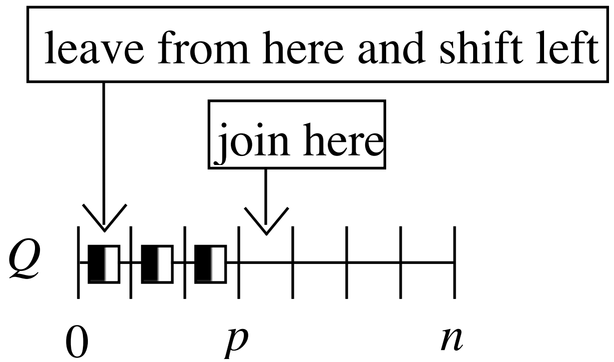

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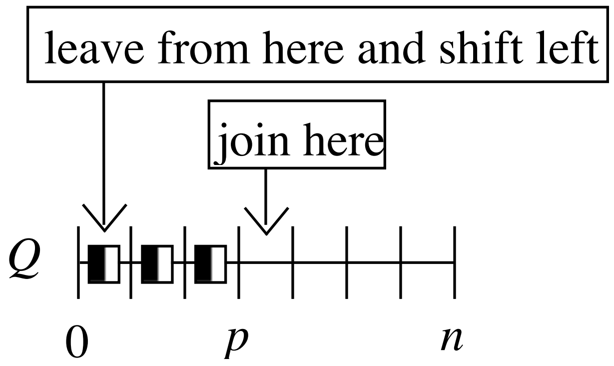

Limited Queue

new implementer's variables: R: $[n{*}X]$ and f, b: $0,..n+1$

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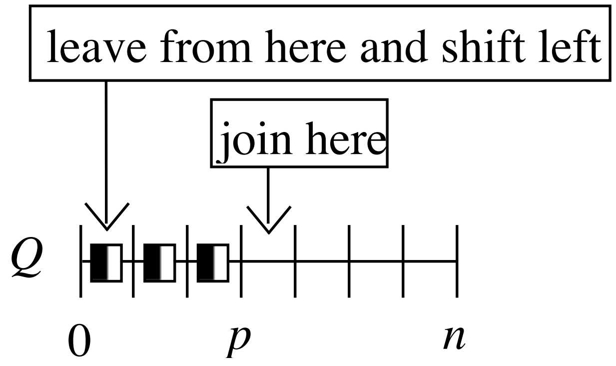

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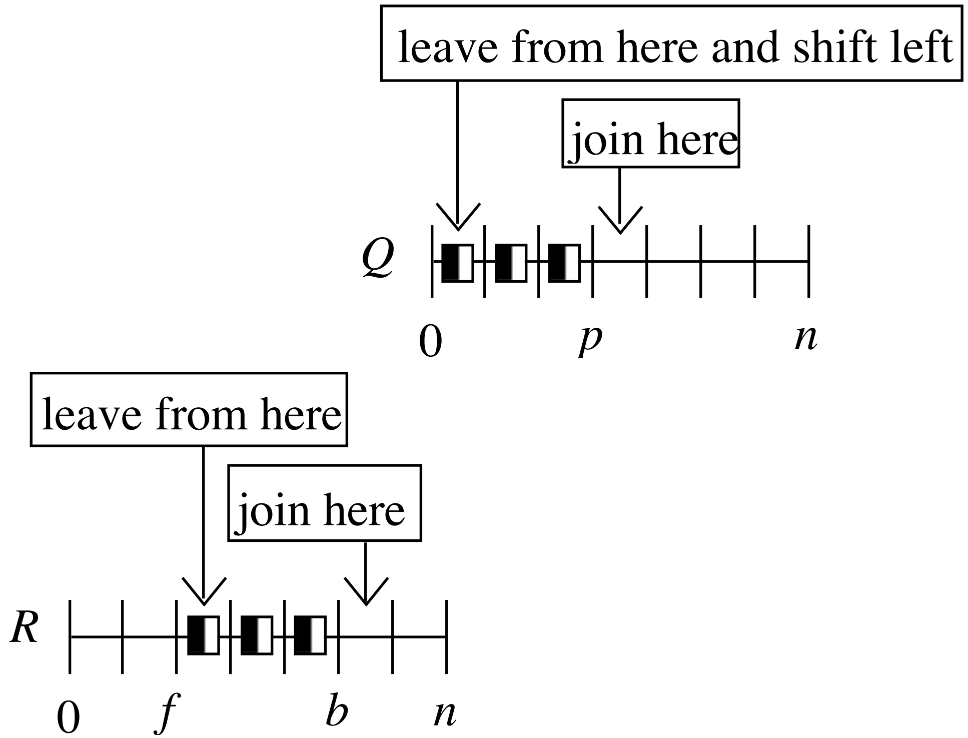

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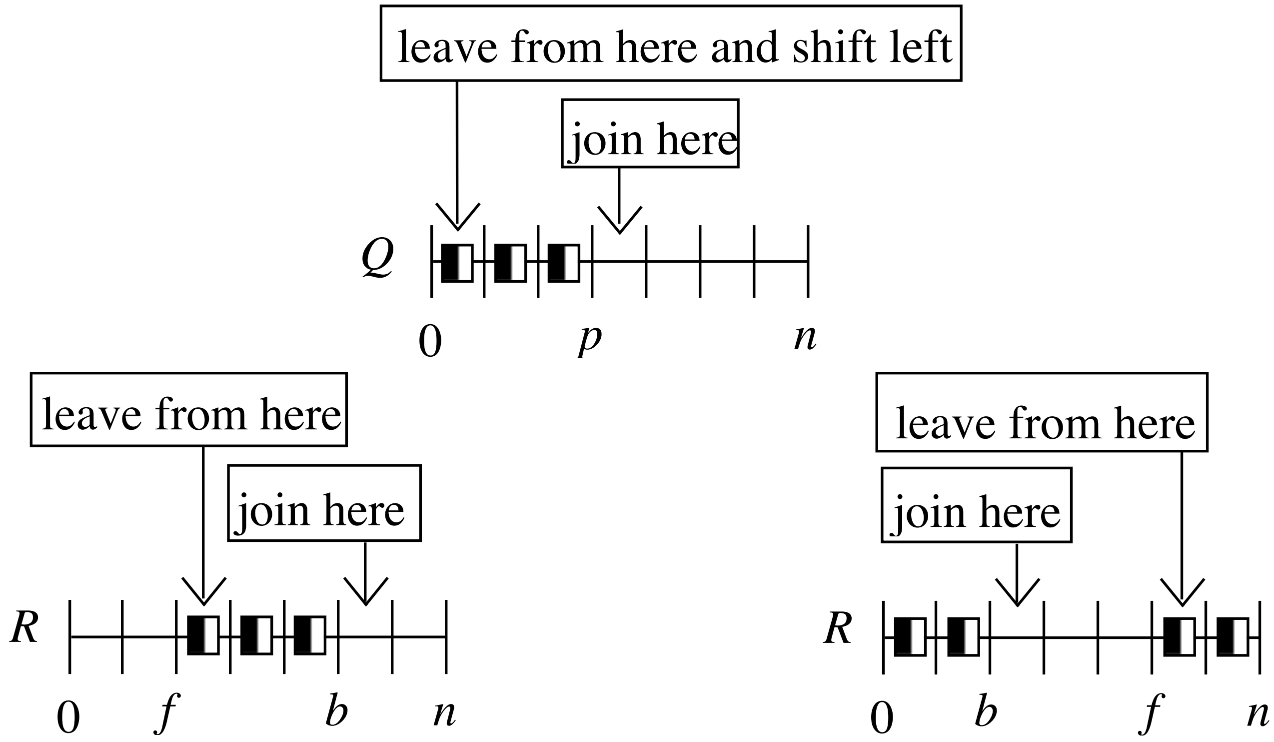

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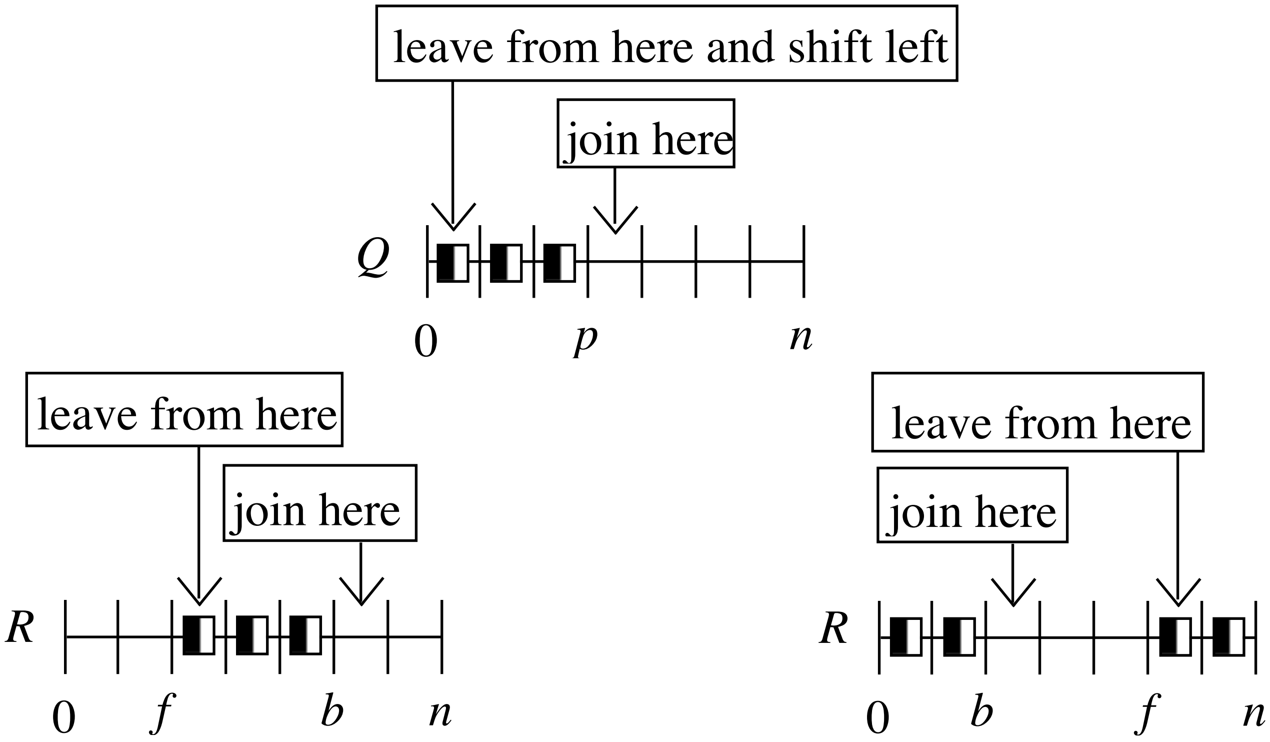

data transformer D :

$$Q[0;..p] = R[f;..b] \lor Q[0;..p] = R[(f;..n); (0;..b)]$$

Limited Queue

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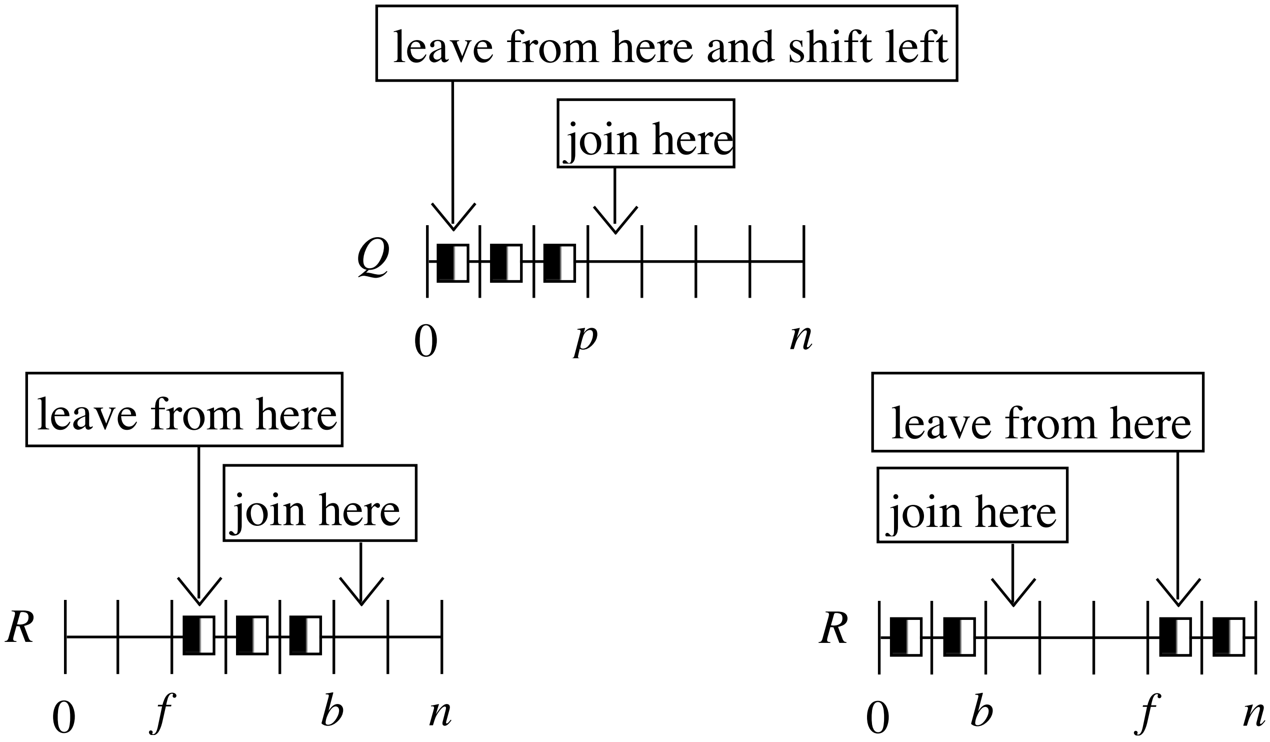

data transformer D :

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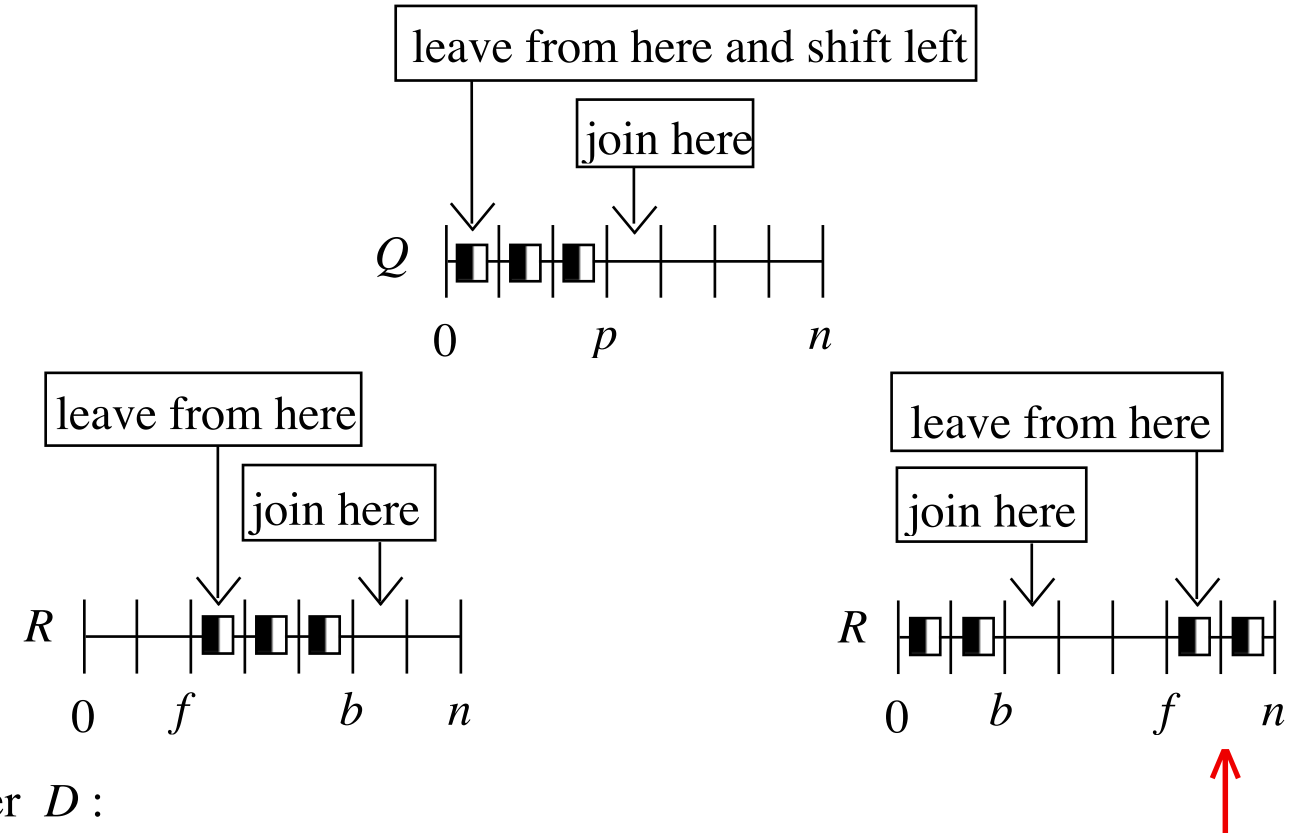

data transformer D :

$$Q[0;..p] = R[f;..b] \lor Q[0;..p] = R[(f;..n); (0;..b)]$$

Limited Queue

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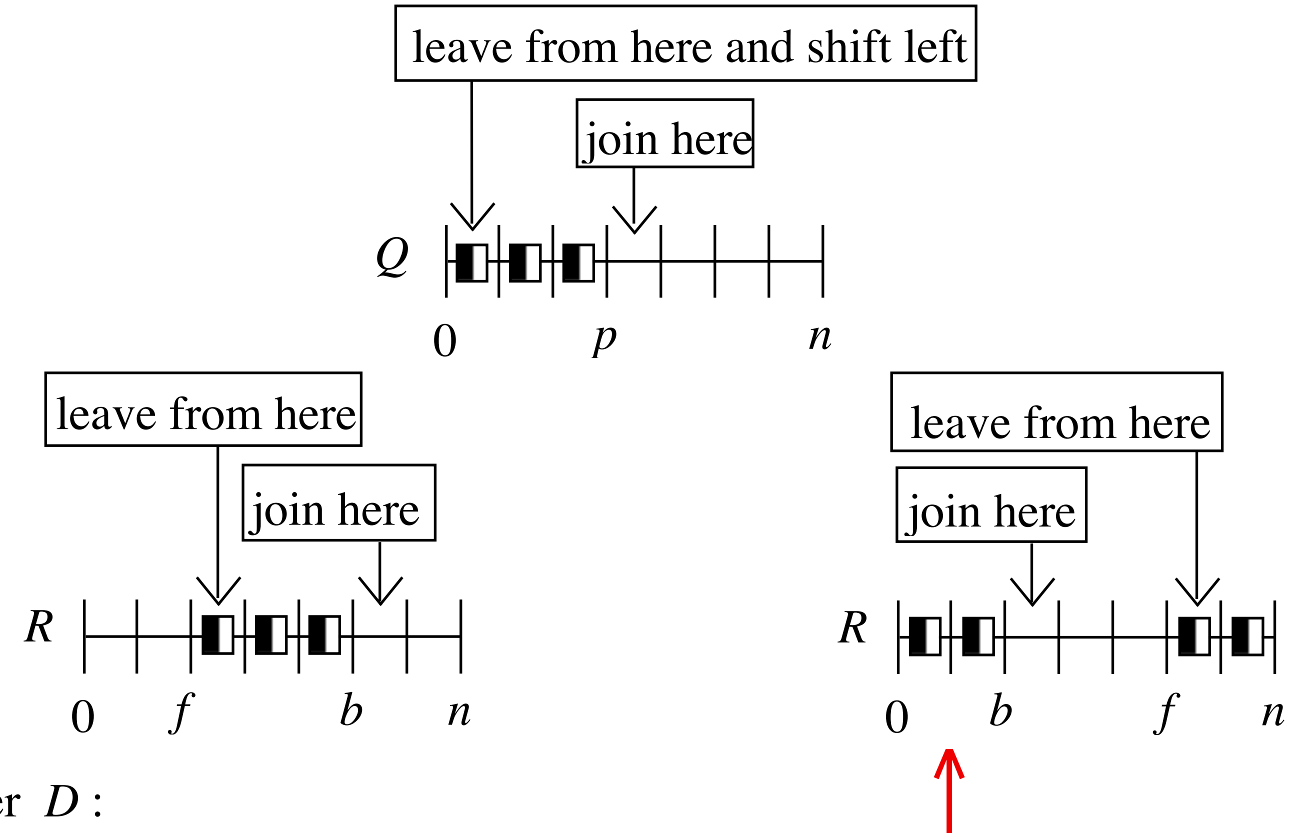

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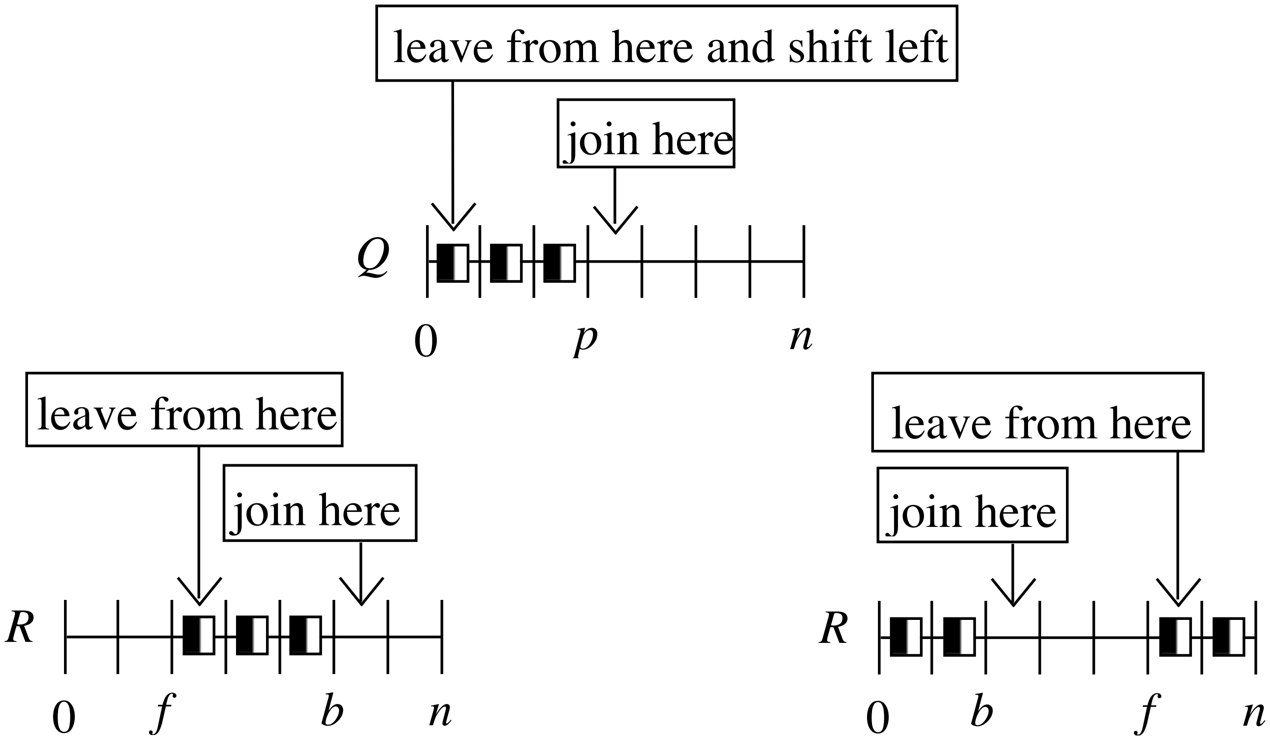

$$\forall Q, p \cdot D \Rightarrow \exists Q', p' \cdot D' \wedge mkemptyq$$

Limited Queue

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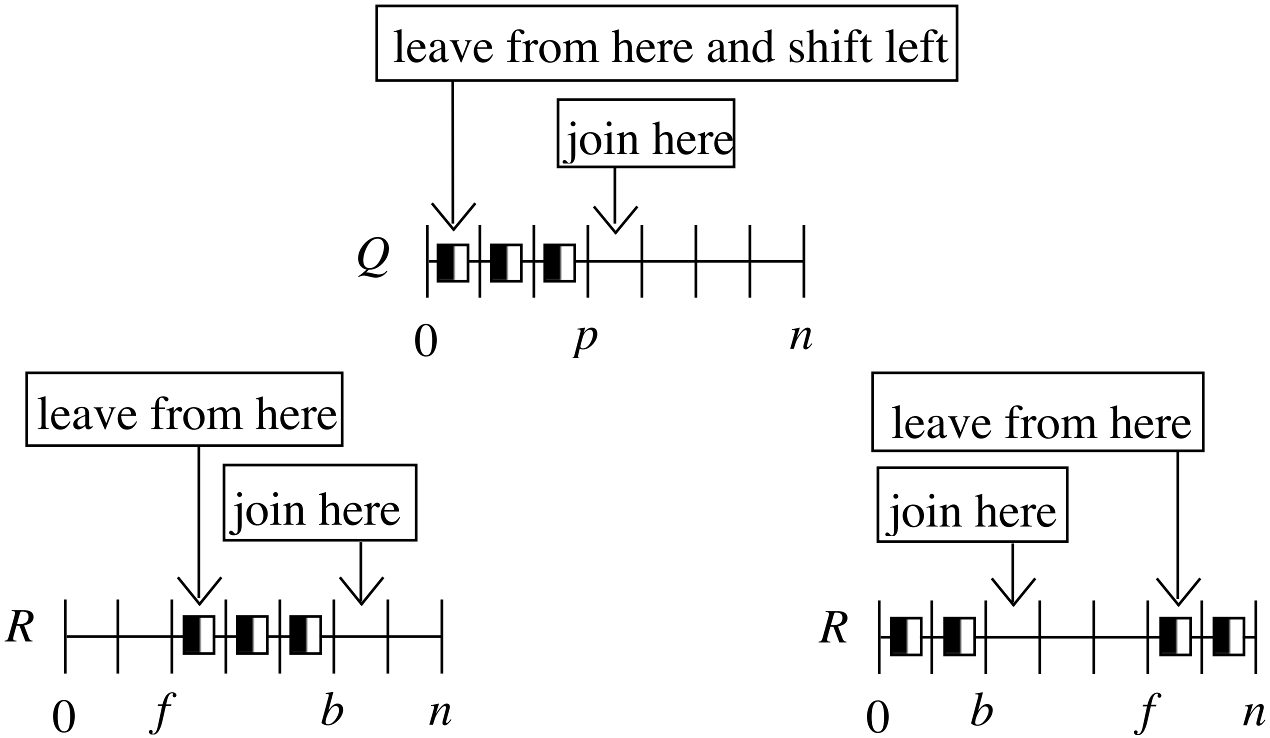
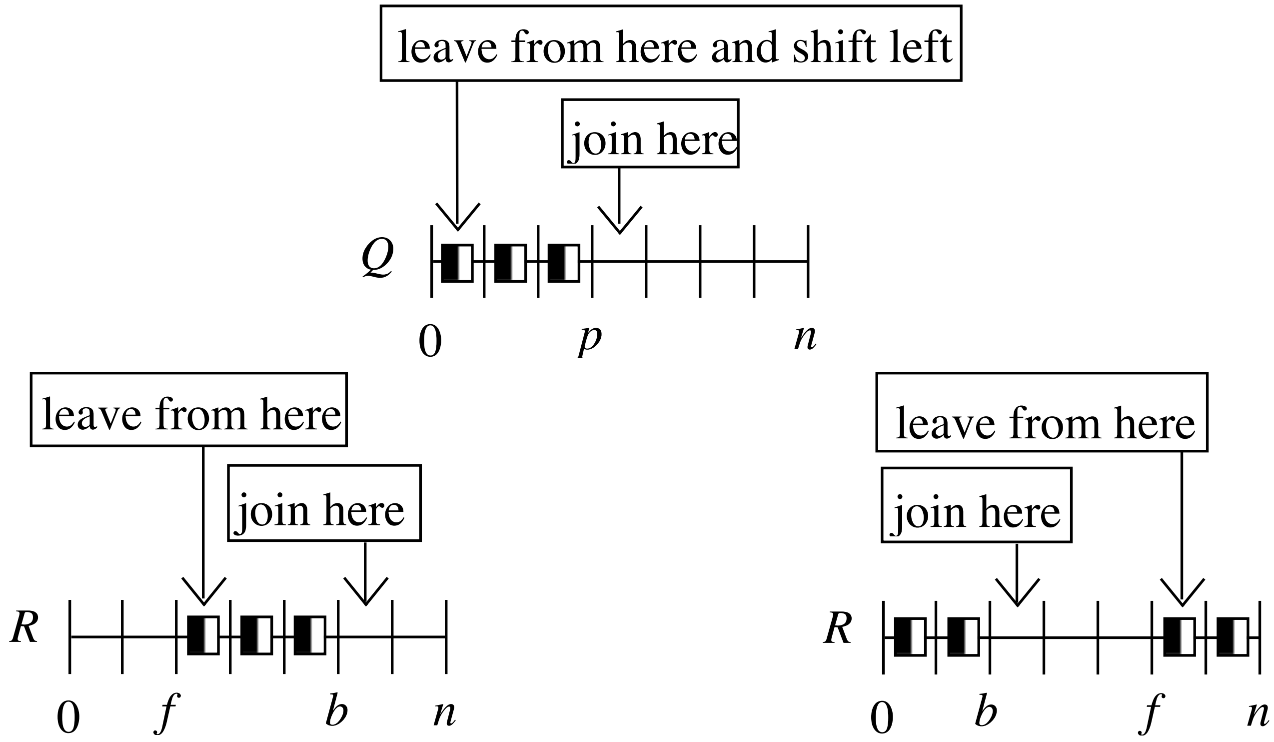


$$\forall Q, p \cdot D \Rightarrow \exists Q', p' \cdot D' \land mkemptyq$$

$$= \quad \forall Q, p \cdot D \Rightarrow \exists Q', p' \cdot D' \land (p := 0)$$

Limited Queue

new implementer's variables: R: $[n*X]$ and f, b: $0,..n+1$



$$\forall Q, p\cdot\ D \Rightarrow \exists Q', p'\cdot\ D' \land mkemptyq$$

$$= \quad \forall Q, p\cdot\ D \Rightarrow \exists Q', p'\cdot\ D' \land (p{:=}\ 0)$$

$$= \quad \forall Q, p\cdot\ D \Rightarrow \exists Q', p'\cdot\ D' \land p'{=}0 \land Q'{=}Q \land c'{=}c \land x'{=}x$$

Limited Queue

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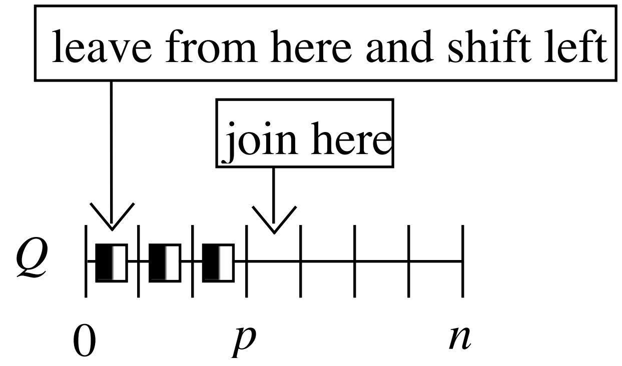


$$\forall Q, p\cdot\ D \Rightarrow \exists Q', p'\cdot\ D' \land mkemptyq$$

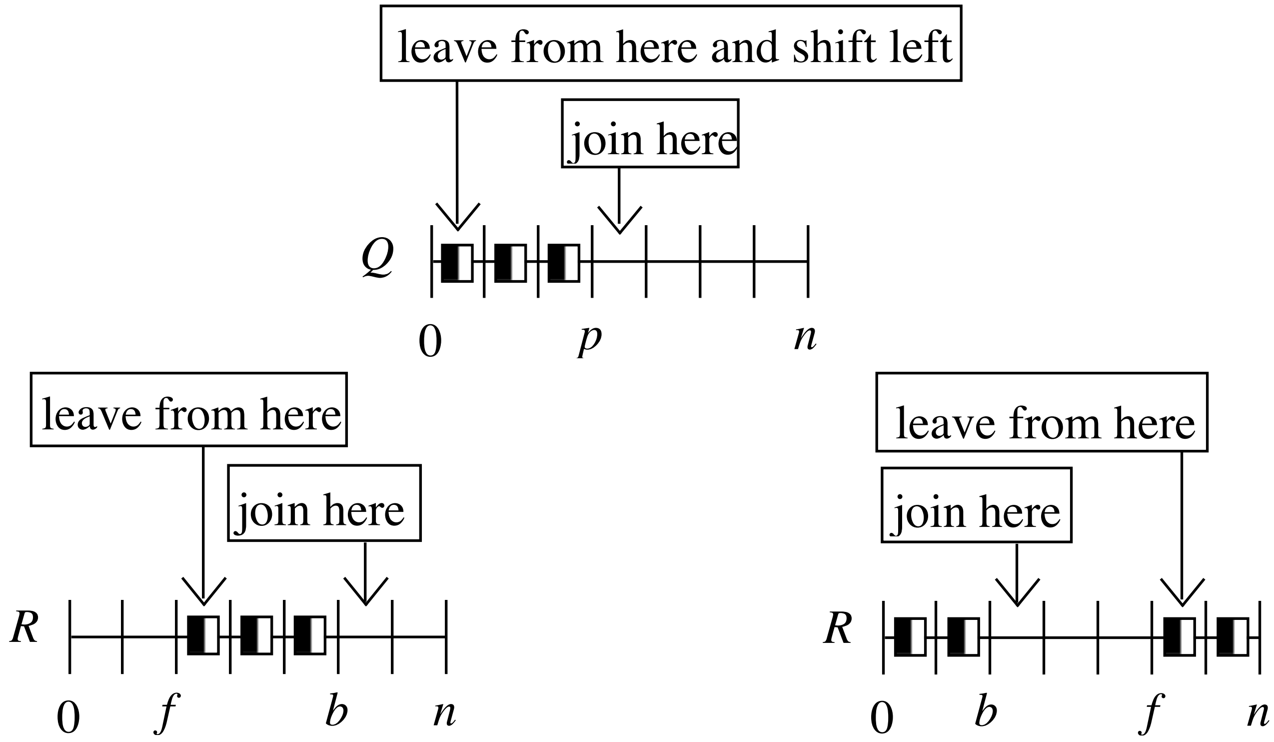
$$= \quad \forall Q, p\cdot\ D \Rightarrow \exists Q', p'\cdot\ D' \land (p{:=}\ 0)$$

$$= \quad \forall Q, p\cdot\ D \Rightarrow \exists Q', p'\cdot\ D' \land p'{=}0 \land Q'{=}Q \land c'{=}c \land x'{=}x$$

$$= \quad (f'{=}b' \lor f'{=}0 \land b'{=}n) \land c'{=}c \land x'{=}x$$

Limited Queue

new implementer's variables: R: $[n*X]$ and f, b: $0,..n+1$



$\forall Q, p \cdot D \Rightarrow \exists Q', p' \cdot D' \wedge mkemptyq$

$= \quad \forall Q, p \cdot D \Rightarrow \exists Q', p' \cdot D' \wedge (p := 0)$

$= \quad \forall Q, p \cdot D \Rightarrow \exists Q', p' \cdot D' \wedge p'=0 \wedge Q'=Q \wedge c'=c \wedge x'=x$

$= \quad (f'=b' \vee f'=0 \wedge b'=n) \wedge c'=c \wedge x'=x$

$\Leftarrow \quad f:= 0.\ \ b:= 0$

Limited Queue

$\forall Q, p \cdot D \Rightarrow \exists Q', p' \cdot D' \wedge isemptyq$

Limited Queue

$$\forall Q, p \cdot D \Rightarrow \exists Q', p' \cdot D' \land isemptyq$$

$$= \quad \forall Q, p \cdot D \Rightarrow \exists Q', p' \cdot D' \land (c := p{=}0)$$

Limited Queue

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$$= \quad \forall Q, p\cdot D \Rightarrow \exists Q', p'\cdot D' \wedge c'{=}(p{=}0) \wedge p'{=}p \wedge Q'{=}Q \wedge x'{=}x$$

$$= \quad f{<}b \;\wedge\; f'{<}b' \;\wedge\; R[f;..b] = R'[f';..b'] \;\wedge\; x'{=}x \;\wedge\; \neg c'$$

$$\vee \quad f{<}b \;\wedge\; f'{>}b' \;\wedge\; R[f;..b] = R'[(f';..n);\ (0;..b')] \;\wedge\; x'{=}x \;\wedge\; \neg c'$$

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$= \longrightarrow \quad f{<}b \ \land\ f'{<}b' \ \land\ R[f;..b] = R'[f';..b'] \ \land\ x'{=}x \ \land\ \neg c'$

$\lor \quad f{<}b \ \land\ f'{>}b' \ \land\ R[f;..b] = R'[(f';..n);\ (0;..b')] \ \land\ x'{=}x \ \land\ \neg c'$

$\lor \quad f{>}b \ \land\ f'{<}b' \ \land\ R[(f;..n);\ (0;..b)] = R'[f';..b'] \ \land\ x'{=}x \ \land\ \neg c'$

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$$\rightarrow \quad \lor \quad f{>}b \land f'{<}b' \land R[(f;..n);\ (0;..b)] = R'[f';..b'] \land x'{=}x \land \neg c'$$

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$\longrightarrow \quad \vee \quad f{>}b \;\wedge\; f'{>}b' \;\wedge\; R[(f;..n);\ (0;..b)]{=}R'[(f';..n);\ (0;..b')] \;\wedge\; x'{=}x \;\wedge\; \neg c'$

Limited Queue

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$$= \quad \forall Q, p\cdot D \Rightarrow \exists Q', p'\cdot D' \wedge (c:= p=0)$$

$$= \quad \forall Q, p\cdot D \Rightarrow \exists Q', p'\cdot D' \wedge c'=(p=0) \wedge p'=p \wedge Q'=Q \wedge x'=x$$

$$= \quad f<b \;\wedge\; f'<b' \;\wedge\; R[f;..b] = R'[f';..b'] \;\wedge\; x'=x \;\wedge\; \neg c' \;\longleftarrow$$

$$\vee \quad f<b \;\wedge\; f'>b' \;\wedge\; R[f;..b] = R'[(f';..n);\, (0;..b')] \;\wedge\; x'=x \;\wedge\; \neg c' \;\longleftarrow$$

$$\vee \quad f>b \;\wedge\; f'<b' \;\wedge\; R[(f;..n);\, (0;..b)] = R'[f';..b'] \;\wedge\; x'=x \;\wedge\; \neg c' \;\longleftarrow$$

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$f{=}b$ is missing!

Limited Queue

$$\forall Q, p\cdot D \Rightarrow \exists Q', p'\cdot D' \wedge \mathit{isemptyq}$$

$$= \quad \forall Q, p\cdot D \Rightarrow \exists Q', p'\cdot D' \wedge (c{:=}\ p{=}0)$$

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$$= \rightarrow \quad f<b \;\wedge\; f'<b' \;\wedge\; R[f;..b] = R'[f';..b'] \;\wedge\; x'{=}x \;\wedge\; \neg c'$$

$$\rightarrow \;\vee\; f<b \;\wedge\; f'>b' \;\wedge\; R[f;..b] = R'[(f';..n);\ (0;..b')] \;\wedge\; x'{=}x \;\wedge\; \neg c'$$

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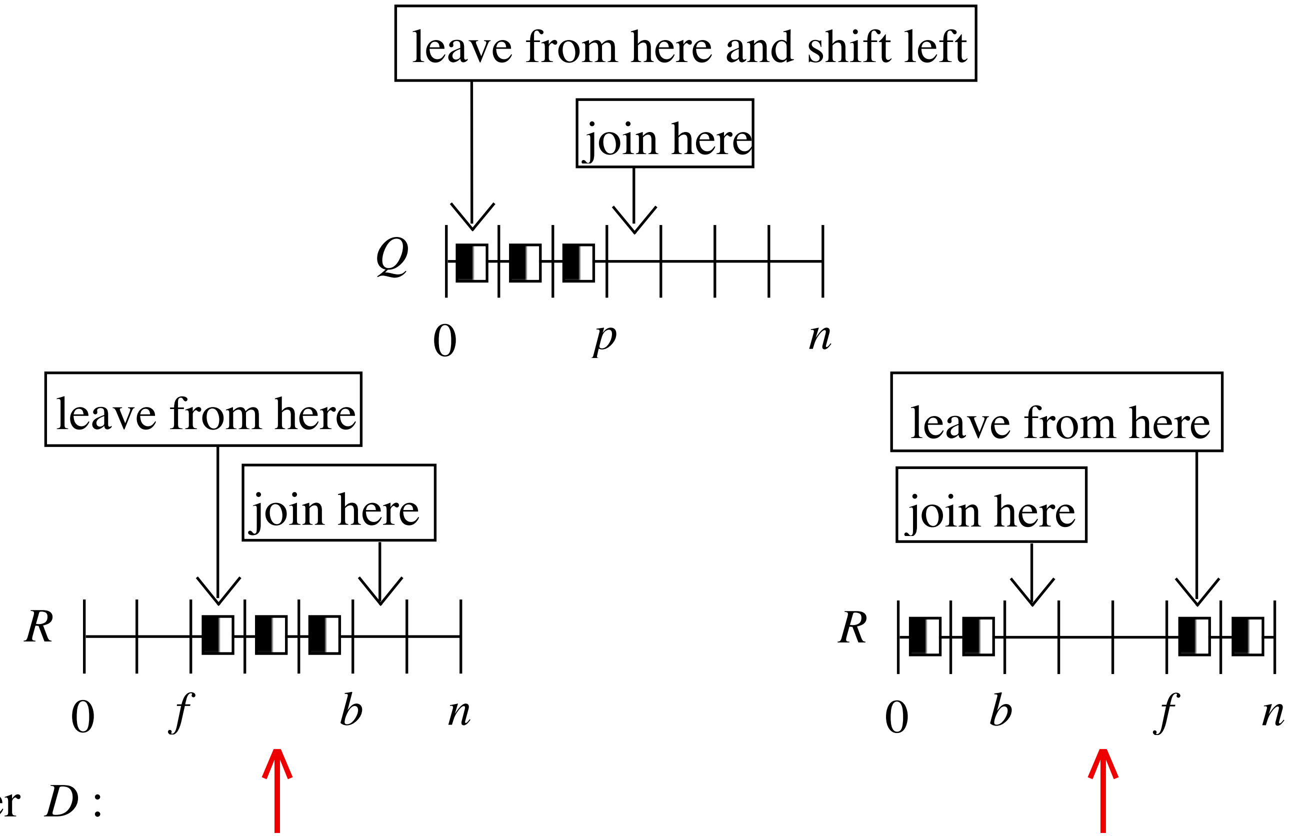
$f{=}b$ is missing! unimplementable!

Limited Queue

new implementer's variables: R: $[n*X]$ and f, b: $0,..n+1$



data transformer D :

$$Q[0;..p] = R[f;..b] \lor Q[0;..p] = R[(f;..n); (0;..b)]$$

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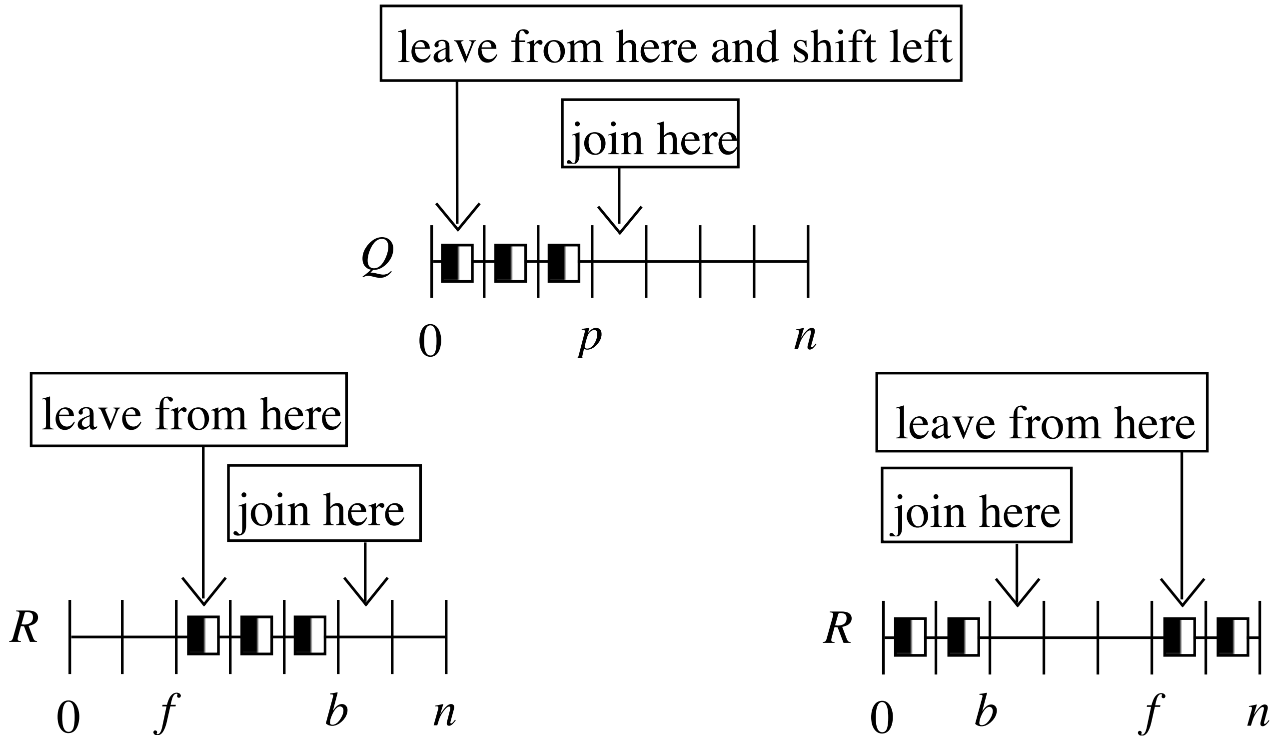
new implementer's variables: R: $[n*X]$ and f, b: $0,..n+1$ and m: bin



data transformer D :

$$m \land Q[0;..p] = R[f;..b] \quad \lor \quad \neg m \land Q[0;..p] = R[(f;..n);\ (0;..b)]$$

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$$\forall Q, p\cdot D \Rightarrow \exists Q', p'\cdot D' \wedge mkemptyq$$

$$= \quad m' \wedge f'=b' \wedge c'=c \wedge x'=x$$

$$\vee \quad \neg m' \wedge f'=n \wedge b'=0 \wedge c'=c \wedge x'=x$$

$$\Leftarrow \quad m:=\top.\ f:=0.\ b:=0$$

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$$\forall Q, p\cdot\ D \Rightarrow \exists Q', p'\cdot\ D' \wedge isemptyq$$

$$= \quad m \wedge m' \wedge R[f;..b] = R'[f';..b'] \wedge x'=x \wedge c'=(f=b)$$

$$\vee \quad m \wedge \neg m' \wedge R[f;..b] = R'[(f';..n); (0;..b')] \wedge x'=x \wedge c'=(f=b)$$

$$\vee \quad \neg m \wedge m' \wedge R[(f;..n); (0;..b)] = R'[f';..b'] \wedge x'=x \wedge c'=(b=0 \wedge f=n)$$

$$\vee \quad \neg m \wedge \neg m' \wedge R[(f;..n); (0;..b)] = R'[(f';..n); (0;..b')] \wedge x'=x \wedge c'=(b=0 \wedge f=n)$$

$$\Leftarrow \quad c' = (m \wedge f=b \vee \neg m \wedge b=0 \wedge f=n) \wedge f'=f \wedge b'=b \wedge R'=R \wedge x'=x \wedge m'=m$$

$$= \quad c:= \textbf{if } m \textbf{ then } f=b \textbf{ else } b=0 \wedge f=n \textbf{ fi}$$

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$$\forall Q, p\cdot D \Rightarrow \exists Q', p'\cdot D' \land isemptyq$$

$= \quad m \land m' \land R[f;..b] = R'[f';..b'] \land x'=x \land c'=(f=b)$ ⟵

$\lor \quad m \land \neg m' \land R[f;..b] = R'[(f';..n); (0;..b')] \land x'=x \land c'=(f=b)$ ⟵

$\lor \quad \neg m \land m' \land R[(f;..n); (0;..b)] = R'[f';..b'] \land x'=x \land c'=(b=0 \land f=n)$

$\lor \quad \neg m \land \neg m' \land R[(f;..n); (0;..b)] = R'[(f';..n); (0;..b')] \land x'=x \land c'=(b=0 \land f=n)$

$\Leftarrow \quad c' = (m \land f=b \lor \neg m \land b=0 \land f=n) \land f'=f \land b'=b \land R'=R \land x'=x \land m'=m$

$= \quad c:=$ **if** m **then** $f=b$ **else** $b=0 \land f=n$ **fi** $\quad \forall Q, p\cdot D \Rightarrow \exists Q', p'\cdot D' \land isemptyq$

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$$\forall Q, p\cdot D \Rightarrow \exists Q', p'\cdot D' \wedge isemptyq$$

$= \quad m \wedge m' \wedge R[f;..b] = R'[f';..b'] \wedge x'=x \wedge c'=(f=b)$

$\vee \quad m \wedge \neg m' \wedge R[f;..b] = R'[(f';..n);(0;..b')] \wedge x'=x \wedge c'=(f=b)$

$\vee \quad \neg m \wedge m' \wedge R[(f;..n);(0;..b)] = R'[f';..b'] \wedge x'=x \wedge c'=(b=0 \wedge f=n)$ ←

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$\Leftarrow \quad c' = (m \wedge f=b \vee \neg m \wedge b=0 \wedge f=n) \wedge f'=f \wedge b'=b \wedge R'=R \wedge x'=x \wedge m'=m$

$=$ $c:=$ **if** m **then** $f=b$ **else** $b=0 \wedge f=n$ **fi** $\longleftarrow$

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$\forall Q, p \cdot D \Rightarrow \exists Q', p' \cdot D' \land \mathit{isfullq}$

$\Leftarrow \quad c := \textbf{ if } m \textbf{ then } f{=}0 \land b{=}n \textbf{ else } f{=}b \textbf{ fi}$

$\forall Q, p \cdot D \Rightarrow \exists Q', p' \cdot D' \land \mathit{join}$

$\Leftarrow \quad \textbf{if } b{<}n \textbf{ then } R\ b := x.\ \ b := b{+}1 \textbf{ else } R\ 0 := x.\ \ b := 1.\ \ m := \bot \textbf{ fi}$

$\forall Q, p \cdot D \Rightarrow \exists Q', p' \cdot D' \land \mathit{leave}$

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They should

state the transformer and transform the operations ✔