Limited Queue

user's variables: c: *bin* and x: X

old implementer's variables: Q: $[n*X]$ and p: *nat*

operations

$$mkemptyq \;=\; p:= 0$$

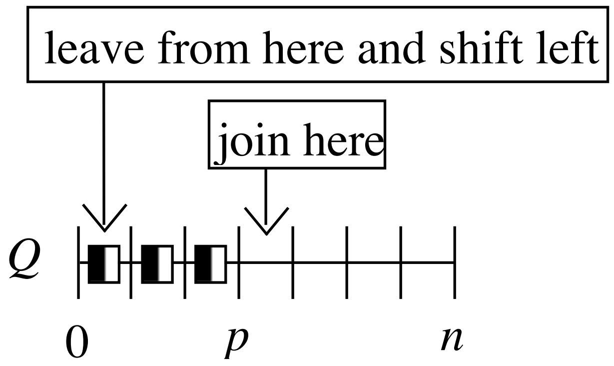
$$isemptyq \;=\; c:= p=0$$

$$isfullq \;=\; c:= p=n$$

$$join \;=\; Q\,p:= x.\;\; p:= p+1$$

$$leave \;=\; \textbf{for } i:= 1;..p \textbf{ do } Q(i-1):= Q\,i \textbf{ od}.\;\; p:= p-1$$

$$front \;=\; x:= Q\,0$$



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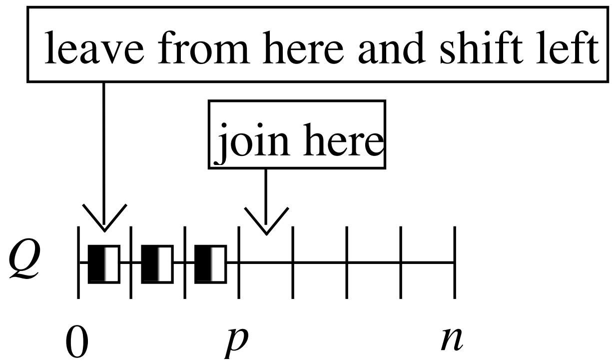

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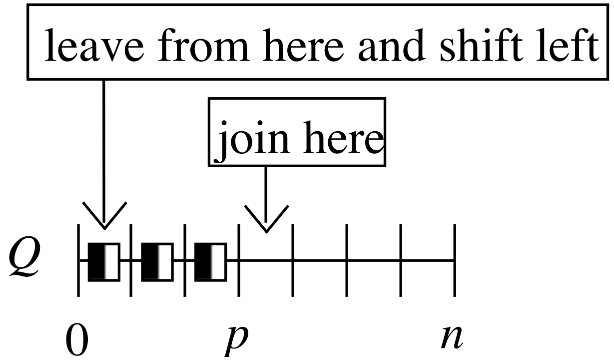

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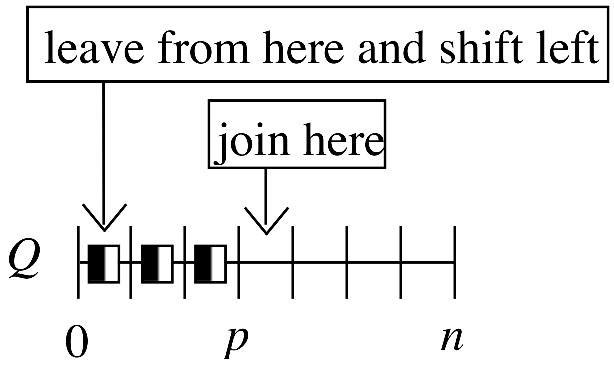

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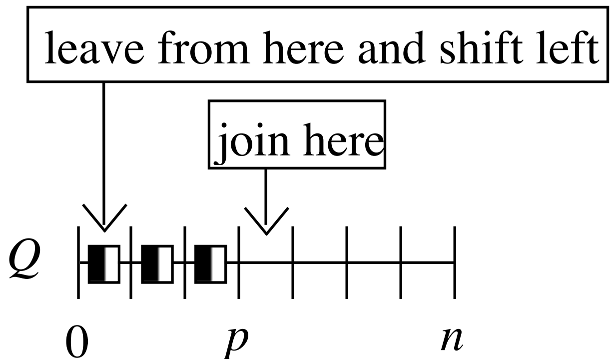

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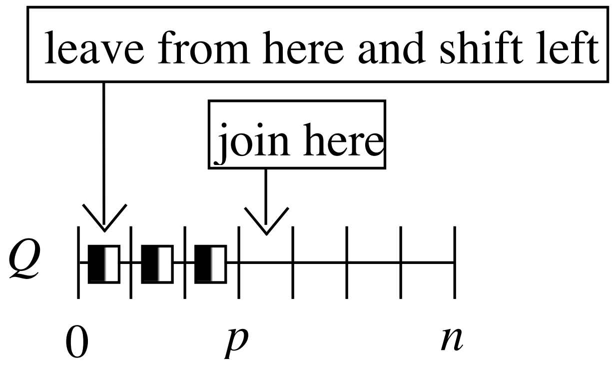

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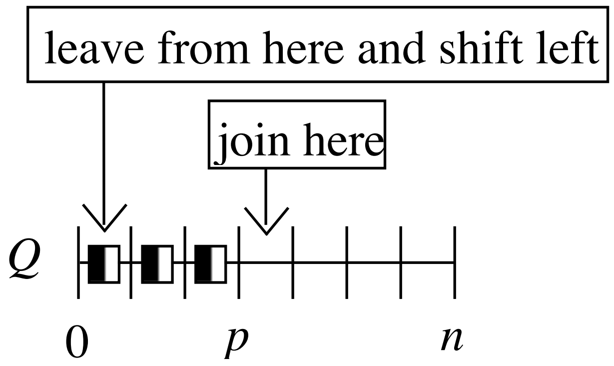

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new implementer's variables: R: $[n*X]$ and f, b: $0,..n$

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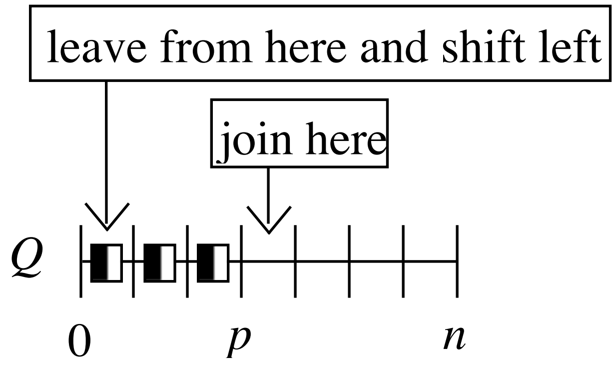

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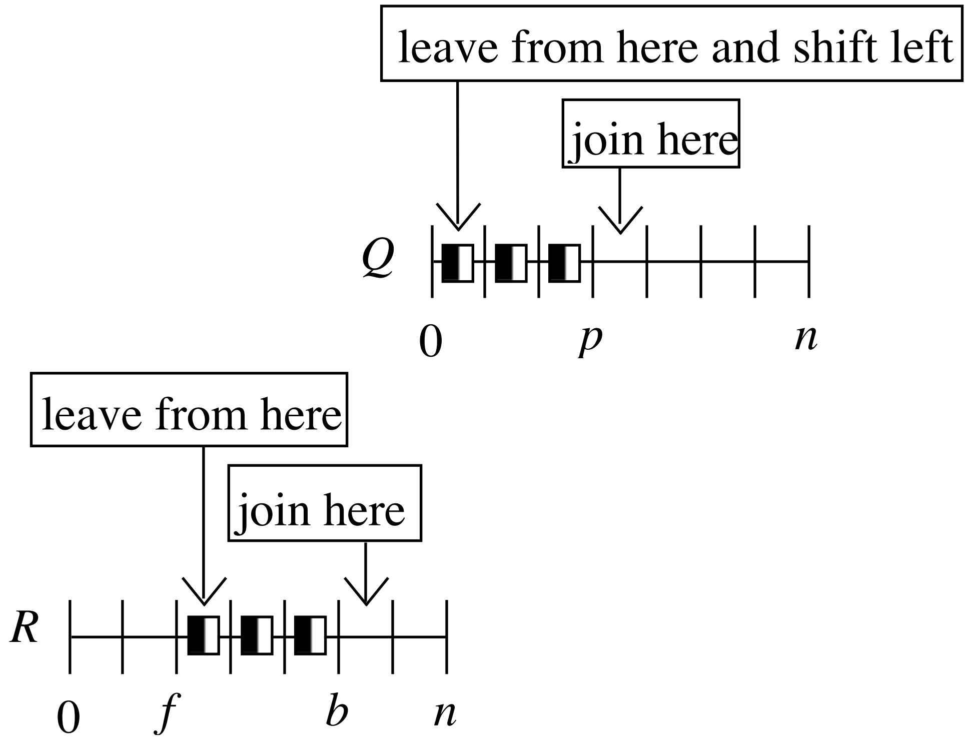

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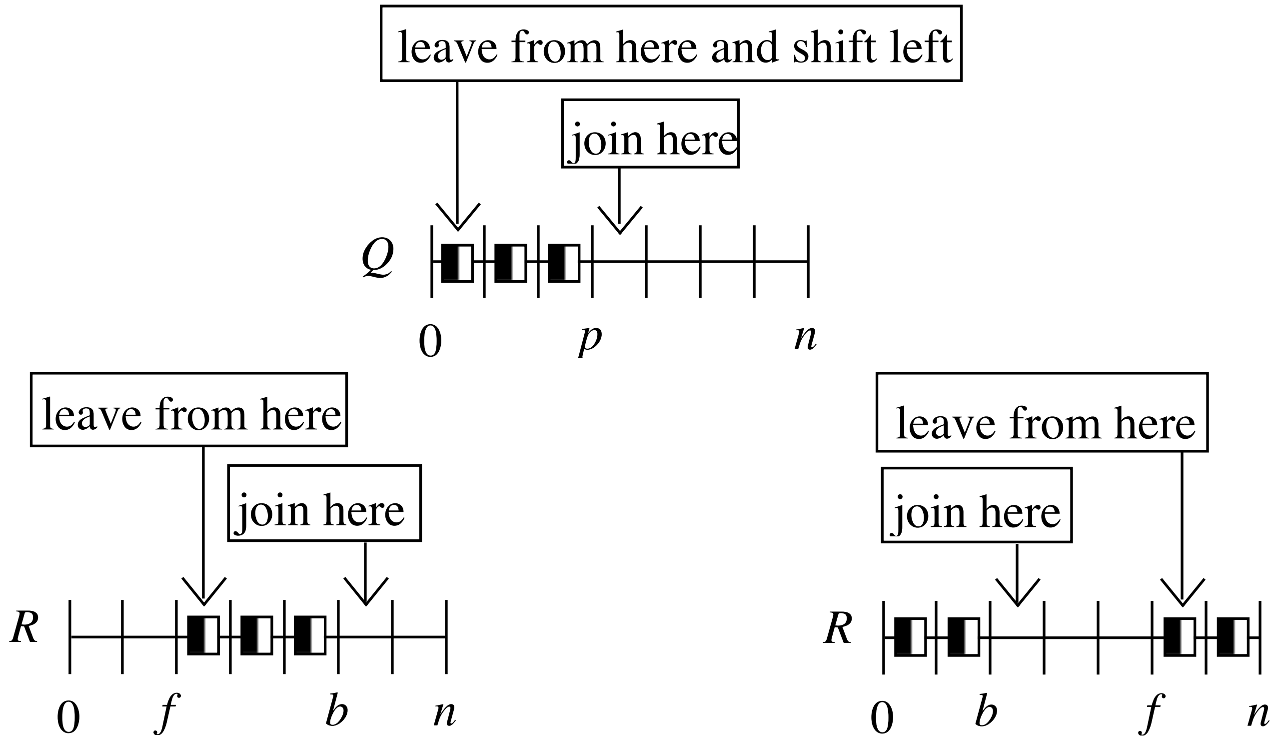

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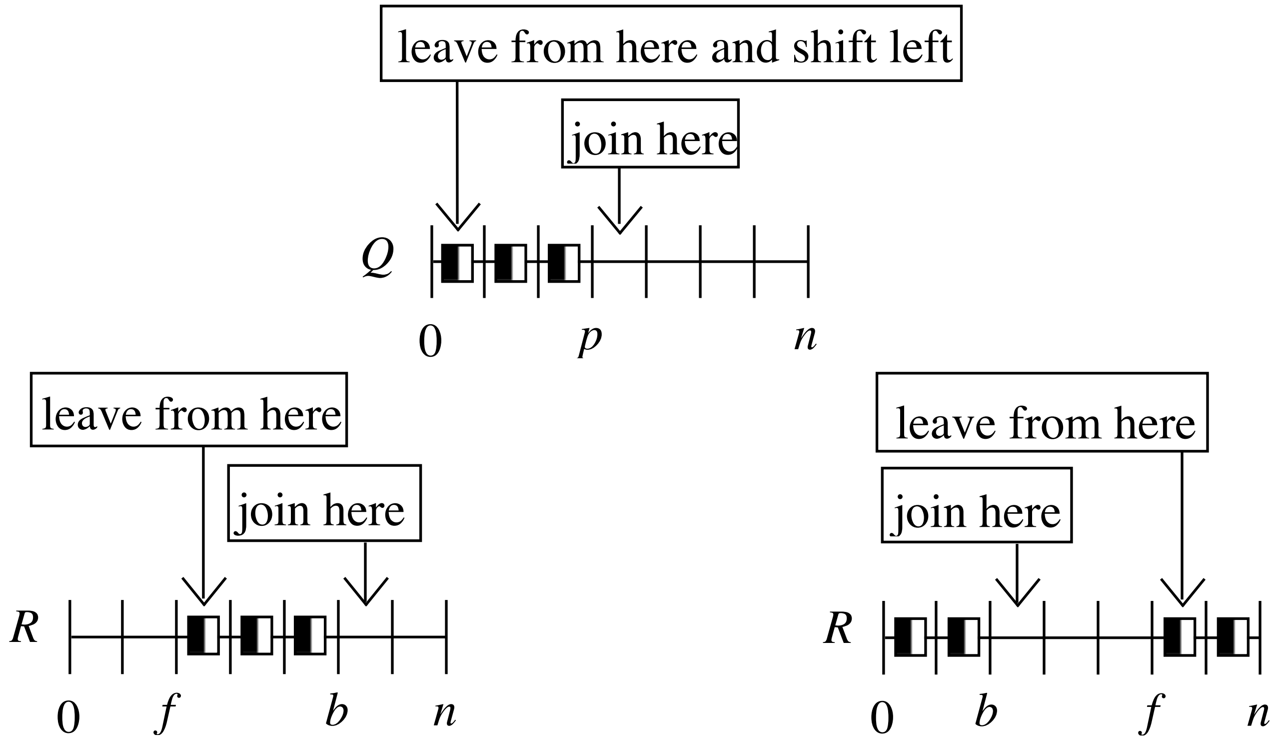

data transformer D :

$$0 \le p = b-f < n \;\wedge\; Q[0;..p] = R[f;..b]$$

$$\vee \quad 0 < p = n-f+b \le n \;\wedge\; Q[0;..p] = R[(f;..n);\,(0;..b)]$$

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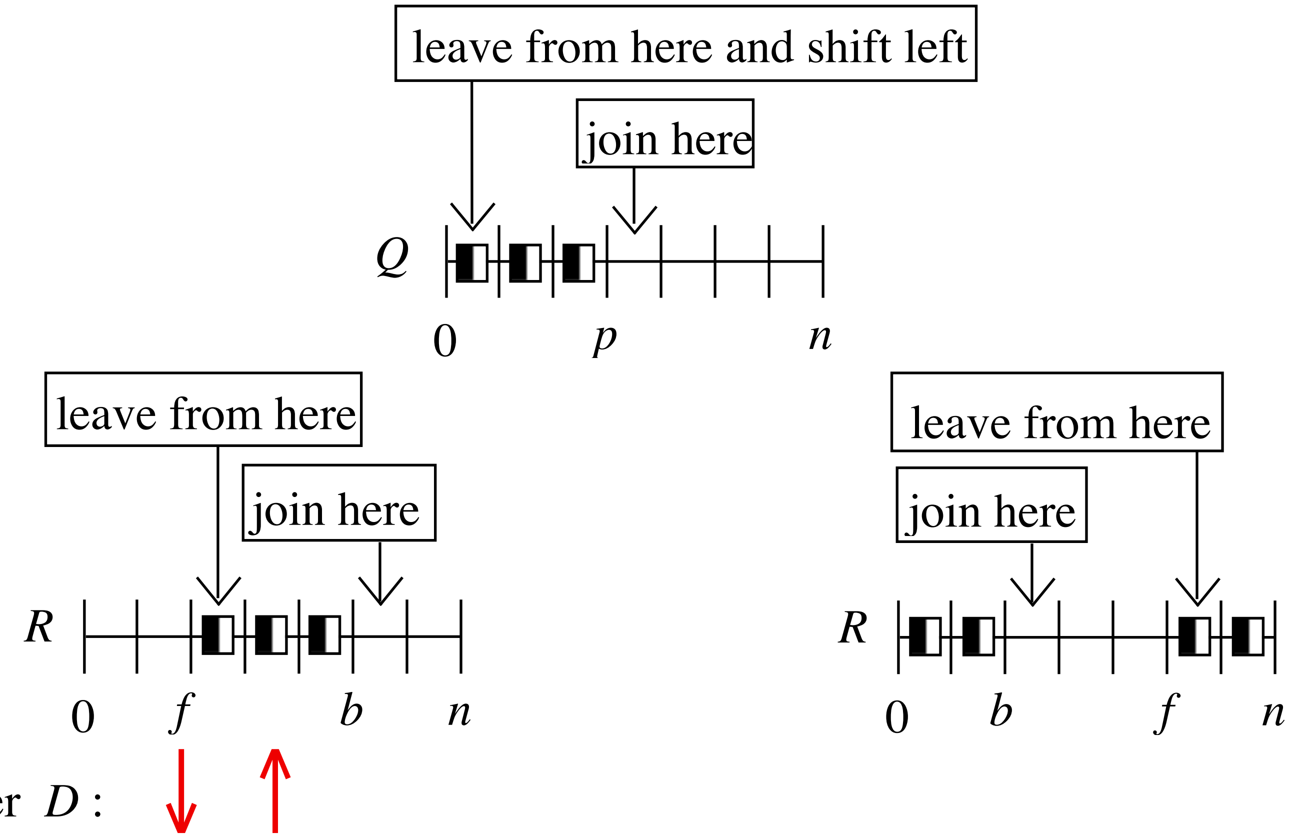

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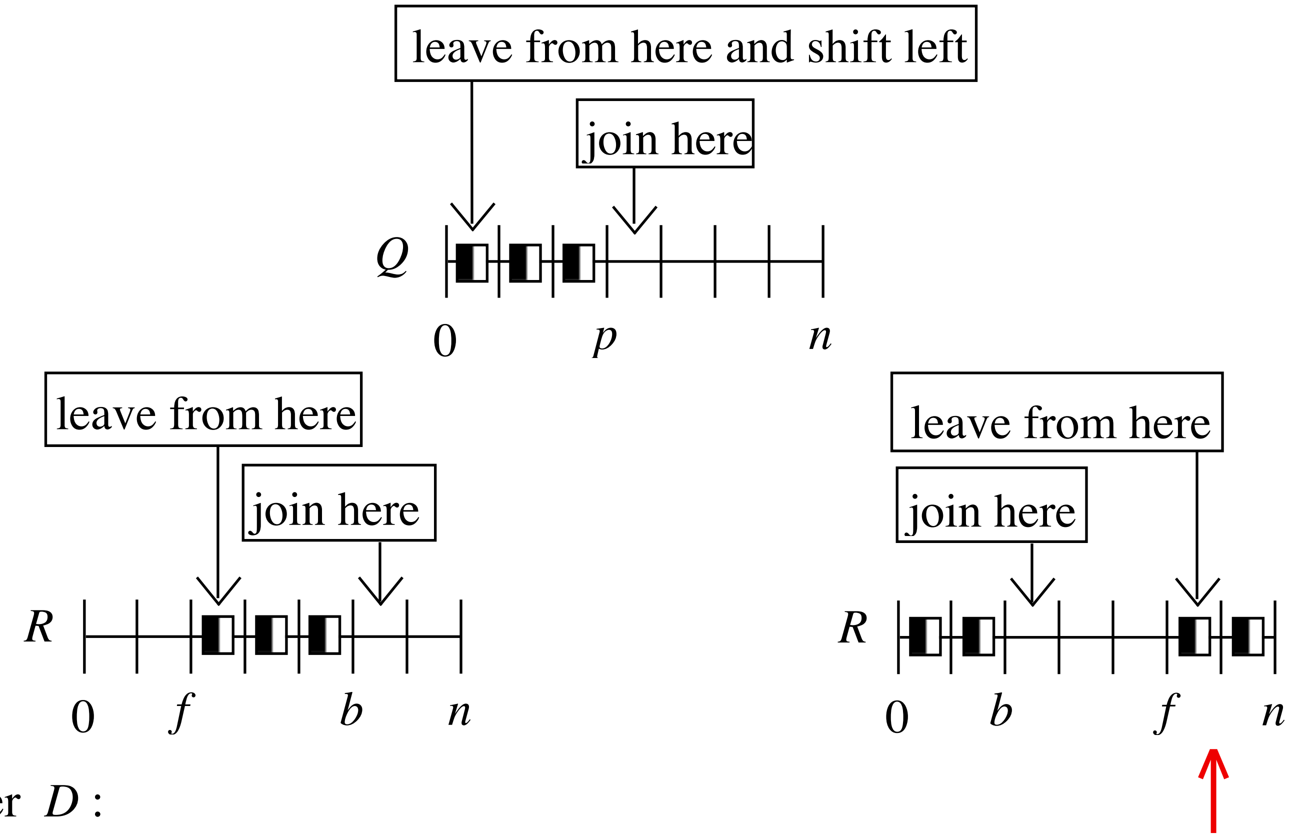

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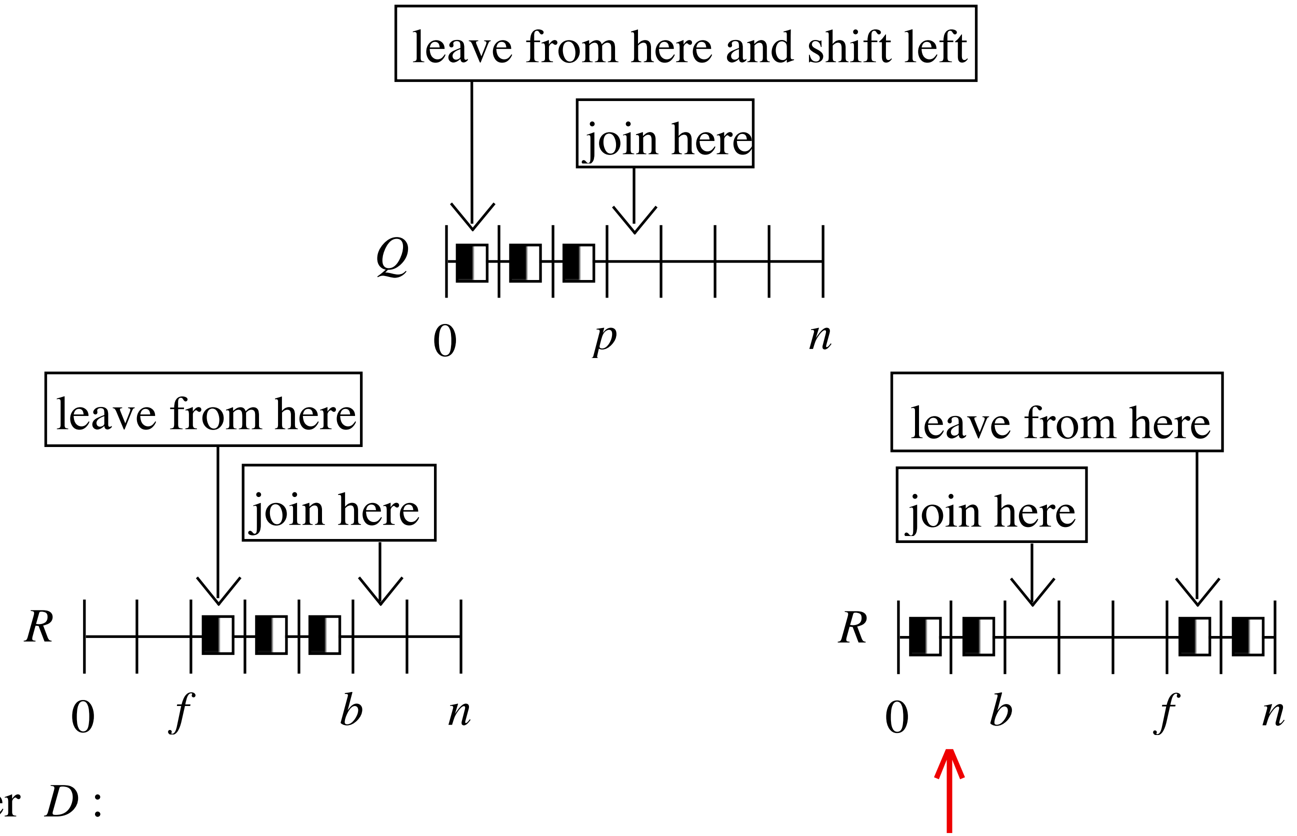

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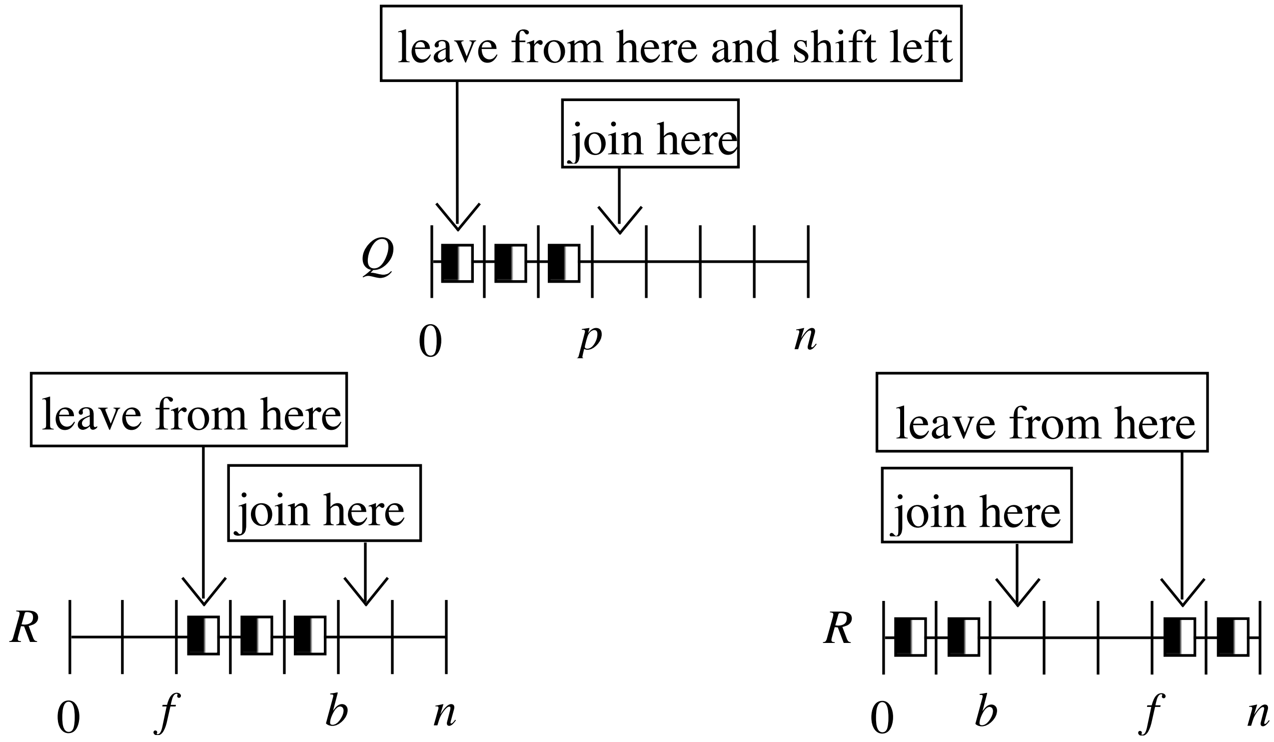

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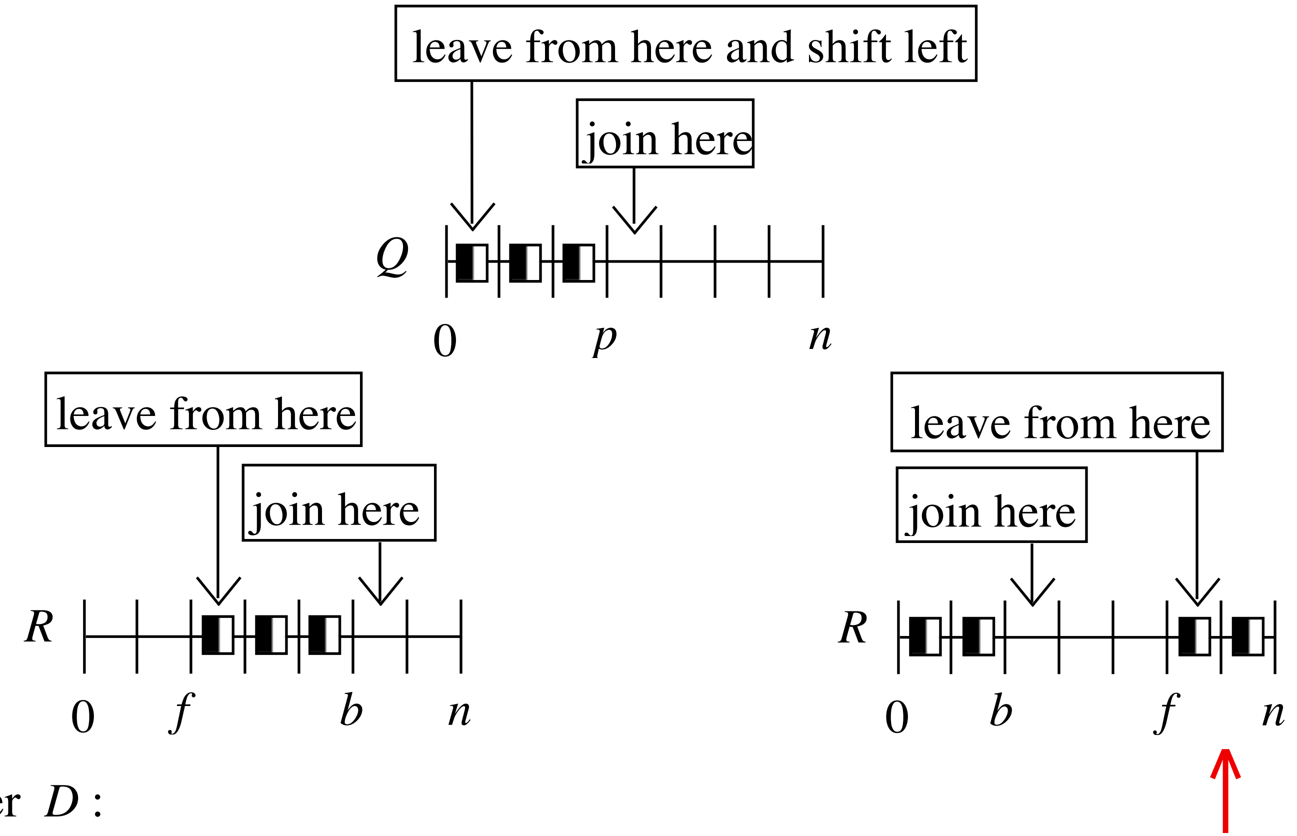

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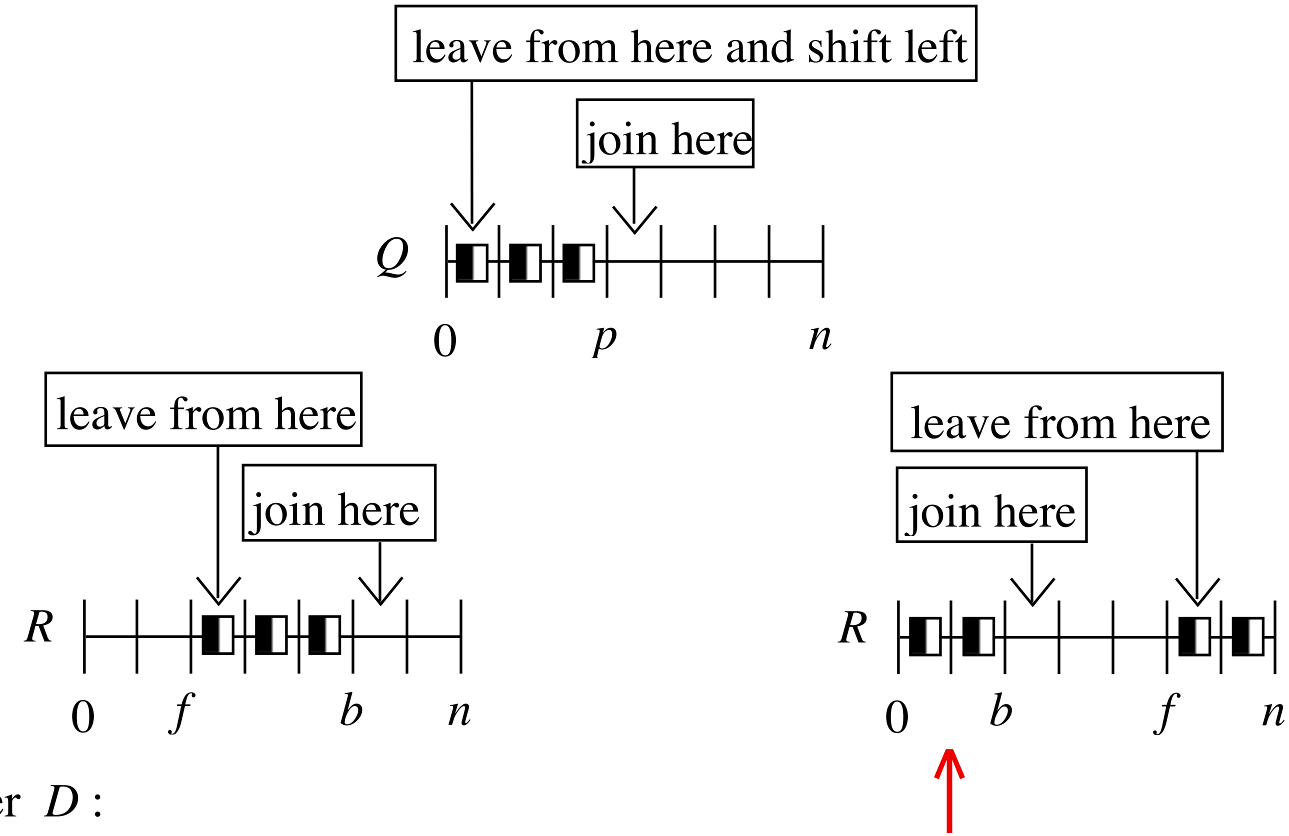

data transformer D :

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$$\lor \quad 0 < p = n\text{–}f\text{+}b \le n \ \land \ Q[0;..p] = R[(f;..n);\ (0;..b)]$$

Limited Queue

$\forall Q, p \cdot D \Rightarrow \exists Q', p' \cdot D' \land mkemptyq$

Limited Queue

$$\forall Q, p\cdot\ D \Rightarrow \exists Q', p'\cdot\ D' \wedge mkemptyq$$

$$= \quad \forall Q, p\cdot\ D \Rightarrow \exists Q', p'\cdot\ D' \wedge (p := 0)$$

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$$= \quad \forall Q, p \cdot D \Rightarrow \exists Q', p' \cdot D' \wedge (p := 0)$$

$$= \quad \forall Q, p \cdot D \Rightarrow \exists Q', p' \cdot D' \wedge p'=0 \wedge Q'=Q \wedge c'=c \wedge x'=x$$

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$$\forall Q, p\cdot D \Rightarrow \exists Q', p'\cdot D' \land mkemptyq$$

$$= \quad \forall Q, p\cdot D \Rightarrow \exists Q', p'\cdot D' \land (p{:=}\,0)$$

$$= \quad \forall Q, p\cdot D \Rightarrow \exists Q', p'\cdot D' \land p'{=}0 \land Q'{=}Q \land c'{=}c \land x'{=}x$$

$$= \quad f'{=}b' \land c'{=}c \land x'{=}x$$

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$$= \quad \forall Q, p\cdot D \Rightarrow \exists Q', p'\cdot D' \wedge (p := 0)$$

$$= \quad \forall Q, p\cdot D \Rightarrow \exists Q', p'\cdot D' \wedge p'=0 \wedge Q'=Q \wedge c'=c \wedge x'=x$$

$$= \quad f'=b' \wedge c'=c \wedge x'=x$$

$$\Leftarrow \quad f := 0.\ \ b := 0$$

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$\forall Q, p \cdot D \Rightarrow \exists Q', p' \cdot D' \wedge isemptyq$

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$$= \quad f{<}b \ \wedge\ f'{<}b' \ \wedge\ b{-}f = b'{-}f'$$
$$\wedge \quad R[f;..b] = R'[f';..b'] \ \wedge\ x'{=}x \ \wedge\ \neg c'$$
$$\vee \quad f{<}b \ \wedge\ f'{>}b' \ \wedge\ b{-}f = n{+}b'{-}f'$$
$$\wedge \quad R[f;..b] = R'[(f';..n);\ (0;..b')] \ \wedge\ x'{=}x \ \wedge\ \neg c'$$
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$$= \quad f<b \;\land\; f'<b' \;\land\; b-f = b'-f'$$

$$\land \quad R[f;..b] = R'[f';..b'] \;\land\; x'=x \;\land\; \neg c'$$

$$\lor \longrightarrow \quad f<b \;\land\; f'>b' \;\land\; b-f = n+b'-f'$$

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$$\land \quad R[f;..b] = R'[(f';..n);\,(0;..b')] \;\land\; x'=x \;\land\; \neg c'$$
$$\lor \longrightarrow \quad f>b \;\land\; f'<b' \;\land\; n+b-f = b'-f'$$
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$$\land \quad R[(f;..n);\,(0;..b)]=R'[(f';..n);\,(0;..b')] \;\land\; x'=x \;\land\; \neg c'$$

$f=b$ is missing!

Limited Queue

$$\forall Q, p\cdot D \Rightarrow \exists Q', p'\cdot D' \wedge isemptyq$$

$$= \quad \forall Q, p\cdot D \Rightarrow \exists Q', p'\cdot D' \wedge (c{:=}\ p{=}0)$$

$$= \quad \forall Q, p\cdot D \Rightarrow \exists Q', p'\cdot D' \wedge c'{=}(p{=}0) \wedge p'{=}p \wedge Q'{=}Q \wedge x'{=}x$$

$$= \quad \longrightarrow \quad f{<}b \ \wedge\ f'{<}b' \ \wedge\ b{-}f = b'{-}f'$$

$$\wedge \quad R[f;..b] = R'[f';..b'] \ \wedge\ x'{=}x \ \wedge\ \neg c'$$

$$\vee \longrightarrow \quad f{<}b \ \wedge\ f'{>}b' \ \wedge\ b{-}f = n{+}b'{-}f'$$

$$\wedge \quad R[f;..b] = R'[(f';..n);\ (0;..b')] \ \wedge\ x'{=}x \ \wedge\ \neg c'$$

$$\vee \longrightarrow \quad f{>}b \ \wedge\ f'{<}b' \ \wedge\ n{+}b{-}f = b'{-}f'$$

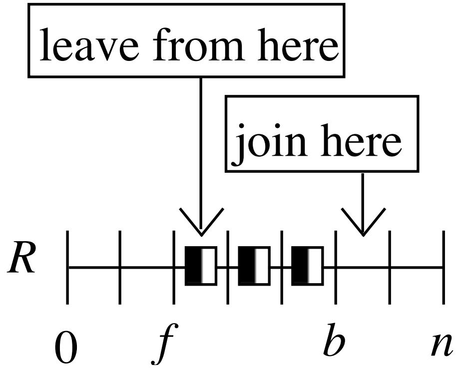
$$\wedge \quad R[(f;..n);\ (0;..b)] = R'[f';..b'] \ \wedge\ x'{=}x \ \wedge\ \neg c'$$

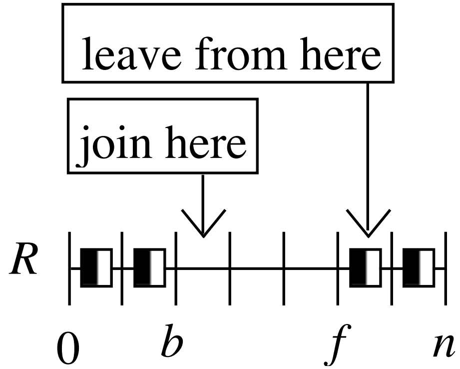
$$\vee \longrightarrow \quad f{>}b \ \wedge\ f'{>}b' \ \wedge\ b{-}f = b'{-}f'$$

$$\wedge \quad R[(f;..n);\ (0;..b)]{=}R'[(f';..n);\ (0;..b')] \ \wedge\ x'{=}x \ \wedge\ \neg c'$$

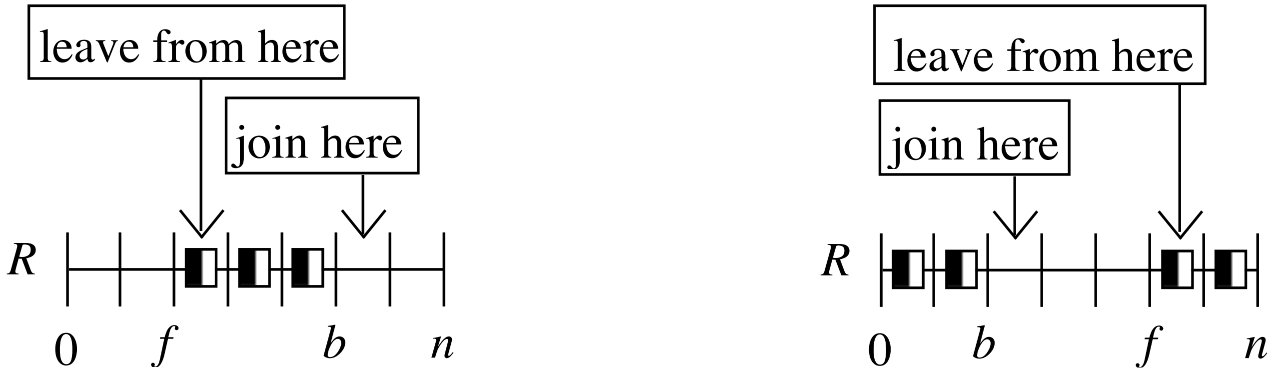
$f{=}b$ is missing! unimplementable!

Limited Queue





Limited Queue

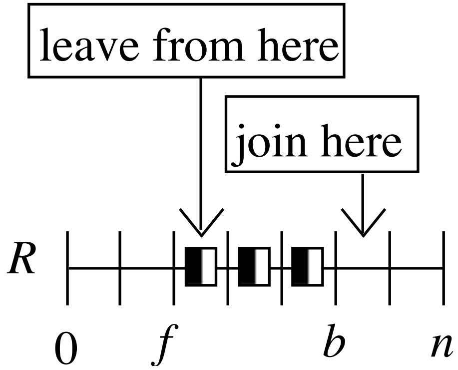


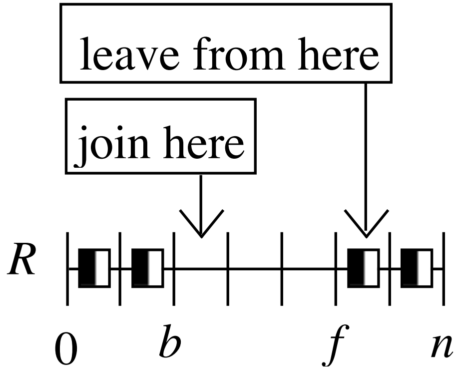
data transformer D :

$$0 \le p = b-f < n \;\wedge\; Q[0;..p] = R[f;..b]$$

$$\vee \quad 0 < p = n-f+b \le n \;\wedge\; Q[0;..p] = R[(f;..n);\ (0;..b)]$$

Limited Queue





data transformer D :

$$m \;\wedge\; 0 \le p = b-f < n \;\wedge\; Q[0;..p] = R[f;..b]$$

$$\vee\quad \neg m \;\wedge\; 0 < p = n-f+b \le n \;\wedge\; Q[0;..p] = R[(f;..n);\,(0;..b)]$$

Limited Queue

$$\forall Q, p\cdot D \Rightarrow \exists Q', p'\cdot D' \land mkemptyq$$

$$= \quad m' \land f'=b' \land c'=c \land x'=x$$

$$\Leftarrow \quad m:=\top.\ f:=0.\ b:=0$$

Limited Queue

$$\forall Q, p\cdot D \Rightarrow \exists Q', p'\cdot D' \land isemptyq$$

$= \quad m \land f<b \land m' \land f'<b' \land b-f = b'-f$

$\quad \land \quad R[f;..b] = R'[f';..b'] \land x'=x \land \neg c'$

$\lor \quad m \land f<b \land \neg m' \land f'>b' \land b-f = n+b'-f'$

$\quad \land \quad R[f;..b] = R'[(f';..n);\ (0;..b')] \land x'=x \land \neg c'$

$\lor \quad \neg m \land f>b \land m' \land f'<b' \land n+b-f = b'-f'$

$\quad \land \quad R[(f;..n);\ (0;..b)] = R'[f';..b'] \land x'=x \land \neg c'$

$\lor \quad \neg m \land f>b \land \neg m' \land f'>b' \land b-f = b'-f'$

$\quad \land \quad R[(f;..n);\ (0;..b)] = R'[(f';..n);\ (0;..b')] \land x'=x \land \neg c'$

$\lor \quad m \land f=b \land m' \land f'=b' \land x'=x \land c'$

$\lor \quad \neg m \land f=b \land \neg m' \land f'=b'$

$\quad \land \quad R[(f;..n);\ (0;..b)]=R'[(f';..n);\ (0;..b')] \land x'=x \land \neg c'$

$\Leftarrow \quad c' = (m \land f=b) \land f'=f \land b'=b \land R'=R \land x'=x$

$= \quad c{:=}\ m \land f=b$

Limited Queue

$$\forall Q, p\cdot D \Rightarrow \exists Q', p'\cdot D' \wedge isemptyq$$

$= \quad m \wedge f<b \wedge m' \wedge f'<b' \wedge b-f = b'-f$

$\quad \wedge \quad R[f;..b] = R'[f';..b'] \wedge x'=x \wedge \neg c'$

$\vee \quad m \wedge f<b \wedge \neg m' \wedge f'>b' \wedge b-f = n+b'-f'$

$\quad \wedge \quad R[f;..b] = R'[(f';..n); (0;..b')] \wedge x'=x \wedge \neg c'$

$\vee \quad \neg m \wedge f>b \wedge m' \wedge f'<b' \wedge n+b-f = b'-f'$

$\quad \wedge \quad R[(f;..n); (0;..b)] = R'[f';..b'] \wedge x'=x \wedge \neg c'$

$\vee \quad \neg m \wedge f>b \wedge \neg m' \wedge f'>b' \wedge b-f = b'-f'$

$\quad \wedge \quad R[(f;..n); (0;..b)] = R'[(f';..n); (0;..b')] \wedge x'=x \wedge \neg c'$

$\vee \quad m \wedge f=b \wedge m' \wedge f'=b' \wedge x'=x \wedge c'$ ←

$\vee \quad \neg m \wedge f=b \wedge \neg m' \wedge f'=b'$

$\quad \wedge \quad R[(f;..n); (0;..b)]=R'[(f';..n); (0;..b')] \wedge x'=x \wedge \neg c'$

$\Leftarrow \quad c' = (m \wedge f=b) \wedge f'=f \wedge b'=b \wedge R'=R \wedge x'=x$

$= \quad c:= m \wedge f=b$

Limited Queue

$\forall Q, p \cdot D \Rightarrow \exists Q', p' \cdot D' \wedge isemptyq$

$= \quad m \wedge f<b \wedge m' \wedge f'<b' \wedge b-f = b'-f$

$\quad \wedge \quad R[f;..b] = R'[f';..b'] \wedge x'=x \wedge \neg c'$

$\vee \quad m \wedge f<b \wedge \neg m' \wedge f'>b' \wedge b-f = n+b'-f'$

$\quad \wedge \quad R[f;..b] = R'[(f';..n); (0;..b')] \wedge x'=x \wedge \neg c'$

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$\vee \quad \neg m \wedge f>b \wedge \neg m' \wedge f'>b' \wedge b-f = b'-f'$

$\quad \wedge \quad R[(f;..n); (0;..b)] = R'[(f';..n); (0;..b')] \wedge x'=x \wedge \neg c'$

$\vee \quad m \wedge f=b \wedge m' \wedge f'=b' \wedge x'=x \wedge c'$

$\vee \quad \neg m \wedge f=b \wedge \neg m' \wedge f'=b'$

$\quad \wedge \quad R[(f;..n); (0;..b)]=R'[(f';..n); (0;..b')] \wedge x'=x \wedge \neg c'$

$\Leftarrow \quad c' = (m \wedge f=b) \wedge f'=f \wedge b'=b \wedge R'=R \wedge x'=x$

$= \quad c{:=}\ m \wedge f=b \quad \leftarrow$

Limited Queue

$$\forall Q, p\cdot D \Rightarrow \exists Q', p'\cdot D' \wedge isfullq$$

$$\Leftarrow \quad c:= \neg m \wedge f=b$$

$$\forall Q, p\cdot D \Rightarrow \exists Q', p'\cdot D' \wedge join$$

$$\Leftarrow \quad R\ b:= x.\ \textbf{if}\ b+1=n\ \textbf{then}\ b:= 0.\ m:= \bot\ \textbf{else}\ b:= b+1\ \textbf{fi}$$

$$\forall Q, p\cdot D \Rightarrow \exists Q', p'\cdot D' \wedge leave$$

$$\Leftarrow \quad \textbf{if}\ f+1=n\ \textbf{then}\ f:= 0.\ m:= \top\ \textbf{else}\ f:= f+1\ \textbf{fi}$$

$$\forall Q, p\cdot D \Rightarrow \exists Q', p'\cdot D' \wedge front$$

$$\Leftarrow \quad x:= R\,f$$

Limited Queue

$\forall Q, p \cdot D \Rightarrow \exists Q', p' \cdot D' \wedge isfullq \cdot$

$\Leftarrow$ $c := \neg m \wedge f{=}b$ $\longleftarrow$

$\forall Q, p \cdot D \Rightarrow \exists Q', p' \cdot D' \wedge join$

$\Leftarrow$ $R\, b := x.$ **if** $b{+}1{=}n$ **then** $b := 0.\ m := \bot$ **else** $b := b{+}1$ **fi**

$\forall Q, p \cdot D \Rightarrow \exists Q', p' \cdot D' \wedge leave$

$\Leftarrow$ **if** $f{+}1{=}n$ **then** $f := 0.\ m := \top$ **else** $f := f{+}1$ **fi**

$\forall Q, p \cdot D \Rightarrow \exists Q', p' \cdot D' \wedge front$

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Limited Queue

$$\forall Q, p\cdot D \Rightarrow \exists Q', p'\cdot D' \wedge \mathit{isfullq}$$

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$$\forall Q, p\cdot D \Rightarrow \exists Q', p'\cdot D' \wedge \mathit{leave}$$

$$\Leftarrow \quad \mathbf{if}\ f{+}1{=}n\ \mathbf{then}\ f:= 0.\ \ m:= \top\ \mathbf{else}\ f:= f{+}1\ \mathbf{fi}$$

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Limited Queue

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$\forall Q, p \cdot D \Rightarrow \exists Q', p' \cdot D' \wedge leave$

$\Leftarrow \quad \textbf{if}\ f{+}1{=}n\ \textbf{then}\ f := 0.\ \ m := \top\ \textbf{else}\ f := f{+}1\ \textbf{fi}$ ⟵

$\forall Q, p \cdot D \Rightarrow \exists Q', p' \cdot D' \wedge front$

$\Leftarrow \quad x := R\ f$

Limited Queue

$$\forall Q, p\cdot D \Rightarrow \exists Q', p'\cdot D' \land isfullq$$

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$$\forall Q, p\cdot D \Rightarrow \exists Q', p'\cdot D' \land front$$

$\Leftarrow$ $x := R\,f$ ←

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They should

state the transformer and transform the operations ✔