Communicating Processes

$c!\,2 \;\|\; (c?.\; x := c)$

Communicating Processes

$$c!\,2 \;\|\; (c?.\; x{:=}\,c)$$

$$=\quad \mathcal{M}_{\boldsymbol{w}} = 2 \;\land\; (\boldsymbol{w}{:=}\,\boldsymbol{w}{+}1) \;\|\; (\boldsymbol{r}{:=}\,\boldsymbol{r}{+}1.\; x{:=}\,\mathcal{M}_{\boldsymbol{r}-1})$$

Communicating Processes

$$c!\ 2 \ \| \ (c?.\ \ x{:=}\ c)$$

$$= \quad \mathcal{M}_{\boldsymbol{w}} = 2 \ \land \ (\boldsymbol{w}{:=}\ \boldsymbol{w}{+}1) \,\|\, (\boldsymbol{r}{:=}\ \boldsymbol{r}{+}1.\ \ x{:=}\ \mathcal{M}_{\boldsymbol{r}-1})$$

$$= \quad \mathcal{M}_{\boldsymbol{w}} = 2 \ \land \ \boldsymbol{w}' = \boldsymbol{w}{+}1 \ \land \ \boldsymbol{r}' = \boldsymbol{r}{+}1 \ \land \ x' = \mathcal{M}_{\boldsymbol{r}}$$

Communicating Processes

$$c!\ 2\ \|\ (c?.\ x{:=}\ c)$$

$$=\quad \mathcal{M}_{\boldsymbol{w}} = 2\ \wedge\ (\boldsymbol{w}{:=}\ \boldsymbol{w}{+}1)\ \|\ (\boldsymbol{r}{:=}\ \boldsymbol{r}{+}1.\ x{:=}\ \mathcal{M}_{\boldsymbol{r}-1})$$

$$=\quad \mathcal{M}_{\boldsymbol{w}} = 2\ \wedge\ \boldsymbol{w}' = \boldsymbol{w}{+}1\ \wedge\ \boldsymbol{r}' = \boldsymbol{r}{+}1\ \wedge\ x' = \mathcal{M}_{\boldsymbol{r}}$$

$$c!\ 1.\ (c!\ 2\ \|\ (c?.\ x{:=}\ c)).\ c?$$

Communicating Processes

$$c!\,2 \;\|\; (c?.\; x{:=}\, c)$$

$$= \quad \mathcal{M}_{\boldsymbol{w}} = 2 \;\wedge\; (\boldsymbol{w}{:=}\, \boldsymbol{w}{+}1) \,\|\, (\boldsymbol{r}{:=}\, \boldsymbol{r}{+}1.\; x{:=}\, \mathcal{M}_{\boldsymbol{r}-1}\,)$$

$$= \quad \mathcal{M}_{\boldsymbol{w}} = 2 \;\wedge\; \boldsymbol{w}' = \boldsymbol{w}{+}1 \;\wedge\; \boldsymbol{r}' = \boldsymbol{r}{+}1 \;\wedge\; x' = \mathcal{M}_{\boldsymbol{r}}$$

$$c!\,1.\; (c!\,2 \;\|\; (c?.\; x{:=}\, c)).\; c?$$

channel declaration

Communicating Processes

$$c!\ 2\ \|\ (c?.\ x{:=}\ c)$$

$$=\quad \mathcal{M}_{\boldsymbol{w}} = 2\ \wedge\ (\boldsymbol{w}{:=}\ \boldsymbol{w}{+}1)\ \|\ (\boldsymbol{r}{:=}\ \boldsymbol{r}{+}1.\ x{:=}\ \mathcal{M}_{\boldsymbol{r}-1}\ )$$

$$=\quad \mathcal{M}_{\boldsymbol{w}} = 2\ \wedge\ \boldsymbol{w}' = \boldsymbol{w}{+}1\ \wedge\ \boldsymbol{r}' = \boldsymbol{r}{+}1\ \wedge\ x' = \mathcal{M}_{\boldsymbol{r}}$$

$$c!\ 1.\ (c!\ 2\ \|\ (c?.\ x{:=}\ c)).\ c?$$

channel declaration

new $c?\ T\cdot\ P$

Communicating Processes

$$c!\,2 \;\|\; (c?.\; x{:=}\,c)$$

$$= \quad \mathcal{M}_{\boldsymbol{w}} = 2 \;\wedge\; (\boldsymbol{w}{:=}\,\boldsymbol{w}{+}1) \,\|\, (\boldsymbol{r}{:=}\,\boldsymbol{r}{+}1.\; x{:=}\,\mathcal{M}_{\boldsymbol{r}-1})$$

$$= \quad \mathcal{M}_{\boldsymbol{w}} = 2 \;\wedge\; \boldsymbol{w}' = \boldsymbol{w}{+}1 \;\wedge\; \boldsymbol{r}' = \boldsymbol{r}{+}1 \;\wedge\; x' = \mathcal{M}_{\boldsymbol{r}}$$

$$c!\,1.\; (c!\,2 \;\|\; (c?.\; x{:=}\,c)).\; c?$$

channel declaration

$$\mathbf{new}\; c?\; T\cdot\; P$$

$$= \quad \exists \mathcal{M}c\colon \infty{*}T\cdot\; \exists \mathcal{T}c\colon \infty{*}xnat\cdot\; \mathbf{new}\; \boldsymbol{r}c\,,\, \boldsymbol{w}c\colon xnat := 0\cdot\; P$$

ignoring time

$\textbf{new}\ c?\ int\cdot\ \ c!\ 2\ \ \|\ \ (c?.\ \ x{:=}\ c)$

ignoring time

$$\textbf{new } c?\ int\cdot\ \ c!\ 2\ \ \|\ \ (c?.\ \ x{:=}\ c)$$

$=$

$$\exists \mathcal{M}\colon \infty{*}int\cdot\ \textbf{new } \boldsymbol{r}, \boldsymbol{w}\colon xnat := 0\cdot$$

$$x' = \mathcal{M}_{\boldsymbol{r}}\ \wedge\ \mathcal{M}_{\boldsymbol{w}} = 2\ \wedge\ \boldsymbol{r}' = \boldsymbol{r}{+}1\ \wedge\ \boldsymbol{w}' = \boldsymbol{w}{+}1\ \wedge\ \text{(other variables unchanged)}$$

ignoring time

new $c?\ int\cdot\ \ c!\ 2\ \ \|\ \ (c?.\ \ x{:=}\ c)$

$=$ $\exists\mathcal{M}{:}\ \infty{*}int\cdot$ **new** $\boldsymbol{r}, \boldsymbol{w}{:}\ xnat := 0\cdot$

$x' = \mathcal{M}_{\boldsymbol{r}}\ \wedge\ \mathcal{M}_{\boldsymbol{w}} = 2\ \wedge\ \boldsymbol{r}' = \boldsymbol{r}{+}1\ \wedge\ \boldsymbol{w}' = \boldsymbol{w}{+}1\ \wedge$ (other variables unchanged)

$=$ $\exists\mathcal{M}{:}\ \infty{*}int\cdot$ **new** $\boldsymbol{r}, \boldsymbol{w}{:}\ xnat\cdot$

$x' = \mathcal{M}_0\ \wedge\ \mathcal{M}_0 = 2\ \wedge\ \boldsymbol{r}'{=}1\ \wedge\ \boldsymbol{w}'{=}1\ \wedge$ (other variables unchanged)

ignoring time

$\textbf{new } c?\ int\cdot\ \ c!\ 2\ \ \|\ \ (c?.\ \ x{:=}\ c)$

$=\qquad \exists\mathcal{M}{:}\ \infty{*}int\cdot\ \textbf{new } \boldsymbol{r}, \boldsymbol{w}{:}\ xnat := 0\cdot$

$x' = \mathcal{M}_{\boldsymbol{r}}\ \land\ \mathcal{M}_{\boldsymbol{w}} = 2\ \land\ \boldsymbol{r}' = \boldsymbol{r}{+}1\ \land\ \boldsymbol{w}' = \boldsymbol{w}{+}1\ \land$ (other variables unchanged)

$=\qquad \exists\mathcal{M}{:}\ \infty{*}int\cdot\ \textbf{new } \boldsymbol{r}, \boldsymbol{w}{:}\ xnat\cdot$

$x' = \mathcal{M}_0\ \land\ \mathcal{M}_0 = 2\ \land\ \boldsymbol{r}'{=}1\ \land\ \boldsymbol{w}'{=}1\ \land$ (other variables unchanged)

$=\qquad x'{=}2\ \land$ (other variables unchanged)

ignoring time

$\textbf{new } c?\ int\cdot\ \ c!\ 2\ \|\ (c?.\ \ x{:=}\ c)$

$=\quad \exists\mathcal{M}{:}\ \infty{*}int\cdot\ \textbf{new } \boldsymbol{r}, \boldsymbol{w}{:}\ xnat := 0\cdot$

$x' = \mathcal{M}_{\boldsymbol{r}}\ \land\ \mathcal{M}_{\boldsymbol{w}} = 2\ \land\ \boldsymbol{r}' = \boldsymbol{r}{+}1\ \land\ \boldsymbol{w}' = \boldsymbol{w}{+}1\ \land$ (other variables unchanged)

$=\quad \exists\mathcal{M}{:}\ \infty{*}int\cdot\ \textbf{new } \boldsymbol{r}, \boldsymbol{w}{:}\ xnat\cdot$

$x' = \mathcal{M}_0\ \land\ \mathcal{M}_0 = 2\ \land\ \boldsymbol{r}'{=}1\ \land\ \boldsymbol{w}'{=}1\ \land$ (other variables unchanged)

$=\quad x'{=}2\ \land$ (other variables unchanged)

$=\quad x{:=}\ 2$

ignoring time

$\textbf{new } c?\ int\cdot\ \ c!\ 2\ \|\ (c?.\ \ x{:=}\ c)$

$=\quad \exists\mathcal{M}{:}\ \infty{*}int\cdot\ \textbf{new } \boldsymbol{r}, \boldsymbol{w}{:}\ xnat := 0\cdot$

$x' = \mathcal{M}_{\boldsymbol{r}}\ \land\ \mathcal{M}_{\boldsymbol{w}} = 2\ \land\ \boldsymbol{r}' = \boldsymbol{r}{+}1\ \land\ \boldsymbol{w}' = \boldsymbol{w}{+}1\ \land$ (other variables unchanged)

$=\quad \exists\mathcal{M}{:}\ \infty{*}int\cdot\ \textbf{new } \boldsymbol{r}, \boldsymbol{w}{:}\ xnat\cdot$

$x' = \mathcal{M}_0\ \land\ \mathcal{M}_0 = 2\ \land\ \boldsymbol{r}'{=}1\ \land\ \boldsymbol{w}'{=}1\ \land$ (other variables unchanged)

$=\quad x'{=}2\ \land$ (other variables unchanged)

$=\quad x{:=}\ 2$

including time

$\textbf{new } c?\ int\cdot\ \ c!\ 2\ \|\ (t{:=}\ t\uparrow(\mathcal{T}_{\boldsymbol{r}} + 1).\ \ c?.\ \ x{:=}\ c)$

$=\quad x'{=}2\ \land\ t' = t{+}1\ \land$ (other variables unchanged)

Deadlock

Deadlock

$\mathbf{new}\ c?\ int\cdot\ \ t{:=}\ t\uparrow(\mathcal{T}_{\boldsymbol{r}}+1).\ \ c?.\ \ c!\ 5$

Deadlock

$\textbf{new } c?\ int\cdot\ \ t{:=}\ t\uparrow(\mathcal{T}_{\boldsymbol{r}} + 1).\ \ c?.\ \ c!\ 5$

$=\quad \exists\mathcal{M}{:}\ \infty{*}int\cdot\ \exists\mathcal{T}{:}\ \infty{*}xnat\cdot\ \textbf{new } \boldsymbol{r}, \boldsymbol{w}{:}\ xnat := 0\cdot$

$t{:=}\ t\uparrow(\mathcal{T}_{\boldsymbol{r}} + 1).\ \ \boldsymbol{r}{:=}\ \boldsymbol{r}{+}1.\ \ \mathcal{M}_{\boldsymbol{w}} = 5\ \ \land\ \ \mathcal{T}_{\boldsymbol{w}} = t\ \ \land\ \ (\boldsymbol{w}{:=}\ \boldsymbol{w}{+}1)$

Deadlock

$\textbf{new}\ c?\ int\cdot\ \ t{:=}\ t\uparrow(\mathcal{T}_{\boldsymbol{r}}+1).\ \ c?.\ \ c!\ 5$

$=\quad \exists\mathcal{M}{:}\ \infty{*}int\cdot\ \exists\mathcal{T}{:}\ \infty{*}xnat\cdot\ \textbf{new}\ \boldsymbol{r},\boldsymbol{w}{:}\ xnat := 0\cdot$

$t{:=}\ t\uparrow(\mathcal{T}_{\boldsymbol{r}}+1).\ \ \boldsymbol{r}{:=}\ \boldsymbol{r}{+}1.\ \ \mathcal{M}_{\boldsymbol{w}}=5\ \wedge\ \mathcal{T}_{\boldsymbol{w}}=t\ \wedge\ (\boldsymbol{w}{:=}\ \boldsymbol{w}{+}1)$

$=\quad \exists\mathcal{M}{:}\ \infty{*}int\cdot\ \exists\mathcal{T}{:}\ \infty{*}xnat\cdot\ \exists\boldsymbol{r},\boldsymbol{r}',\boldsymbol{w},\boldsymbol{w}'{:}\ xnat\cdot$

$\boldsymbol{r}{:=}\ 0.\ \ \boldsymbol{w}{:=}\ 0.\ \ t{:=}\ t\uparrow(\mathcal{T}_{\boldsymbol{r}}+1).\ \ \boldsymbol{r}{:=}\ \boldsymbol{r}{+}1.$

$\mathcal{M}_{\boldsymbol{w}}=5\ \wedge\ \mathcal{T}_{\boldsymbol{w}}=t\ \wedge\ \boldsymbol{r}'{=}\boldsymbol{r}\ \wedge\ \boldsymbol{w}'=\boldsymbol{w}{+}1\ \wedge\ t'{=}t$

Deadlock

$\textbf{new } c?\ int\cdot\ \ t{:=}\ t\uparrow(\mathcal{T}_{\boldsymbol{r}}+1).\ \ c?.\ \ c!\ 5$

$=\quad \exists\mathcal{M}\colon \infty{*}int\cdot\ \exists\mathcal{T}\colon \infty{*}xnat\cdot\ \textbf{new } \boldsymbol{r}, \boldsymbol{w}\colon xnat := 0\cdot$

$t{:=}\ t\uparrow(\mathcal{T}_{\boldsymbol{r}}+1).\ \ \boldsymbol{r}{:=}\ \boldsymbol{r}{+}1.\ \ \mathcal{M}_{\boldsymbol{w}}=5\ \ \land\ \ \mathcal{T}_{\boldsymbol{w}}=t\ \ \land\ \ (\boldsymbol{w}{:=}\ \boldsymbol{w}{+}1)$

$=\quad \exists\mathcal{M}\colon \infty{*}int\cdot\ \exists\mathcal{T}\colon \infty{*}xnat\cdot\ \exists\boldsymbol{r}, \boldsymbol{r}', \boldsymbol{w}, \boldsymbol{w}'\colon xnat\cdot$

$\boldsymbol{r}{:=}\ 0.\ \ \boldsymbol{w}{:=}\ 0.\ \ t{:=}\ t\uparrow(\mathcal{T}_{\boldsymbol{r}}+1).\ \ \boldsymbol{r}{:=}\ \boldsymbol{r}{+}1.$

$\mathcal{M}_{\boldsymbol{w}}=5\ \ \land\ \ \mathcal{T}_{\boldsymbol{w}}=t\ \ \land\ \ \boldsymbol{r}'{=}\boldsymbol{r}\ \ \land\ \ \boldsymbol{w}'=\boldsymbol{w}{+}1\ \ \land\ \ t'{=}t$

$=\quad \exists\mathcal{M}\colon \infty{*}int\cdot\ \exists\mathcal{T}\colon \infty{*}xnat\cdot\ \exists\boldsymbol{r}, \boldsymbol{r}', \boldsymbol{w}, \boldsymbol{w}'\colon xnat\cdot$

$\mathcal{M}_0=5\ \ \land\ \ \mathcal{T}_0=t\uparrow(\mathcal{T}_0+1)\ \ \land\ \ \boldsymbol{r}'{=}1\ \ \land\ \ \boldsymbol{w}'{=}1\ \ \land\ \ t'=t\uparrow(\mathcal{T}_0+1)$

Deadlock

$$\textbf{new } c?\ int\cdot\ \ t{:=}\ t\uparrow(\mathcal{T}_{\boldsymbol{r}}+1).\ \ c?.\ \ c!\ 5$$

$$=\quad \exists\mathcal{M}{:}\ \infty{*}int\cdot\ \exists\mathcal{T}{:}\ \infty{*}xnat\cdot\ \textbf{new } \boldsymbol{r}, \boldsymbol{w}{:}\ xnat := 0\cdot$$

$$t{:=}\ t\uparrow(\mathcal{T}_{\boldsymbol{r}}+1).\ \ \boldsymbol{r}{:=}\ \boldsymbol{r}{+}1.\ \ \mathcal{M}_{\boldsymbol{w}} = 5\ \wedge\ \mathcal{T}_{\boldsymbol{w}} = t\ \wedge\ (\boldsymbol{w}{:=}\ \boldsymbol{w}{+}1)$$

$$=\quad \exists\mathcal{M}{:}\ \infty{*}int\cdot\ \exists\mathcal{T}{:}\ \infty{*}xnat\cdot\ \exists\boldsymbol{r}, \boldsymbol{r}', \boldsymbol{w}, \boldsymbol{w}'{:}\ xnat\cdot$$

$$\boldsymbol{r}{:=}\ 0.\ \ \boldsymbol{w}{:=}\ 0.\ \ t{:=}\ t\uparrow(\mathcal{T}_{\boldsymbol{r}}+1).\ \ \boldsymbol{r}{:=}\ \boldsymbol{r}{+}1.$$

$$\mathcal{M}_{\boldsymbol{w}} = 5\ \wedge\ \mathcal{T}_{\boldsymbol{w}} = t\ \wedge\ \boldsymbol{r}'{=}\boldsymbol{r}\ \wedge\ \boldsymbol{w}' = \boldsymbol{w}{+}1\ \wedge\ t'{=}t$$

$$=\quad \exists\mathcal{M}{:}\ \infty{*}int\cdot\ \exists\mathcal{T}{:}\ \infty{*}xnat\cdot\ \exists\boldsymbol{r}, \boldsymbol{r}', \boldsymbol{w}, \boldsymbol{w}'{:}\ xnat\cdot$$

$$\mathcal{M}_0 = 5\ \wedge\ \mathcal{T}_0 = t\uparrow(\mathcal{T}_0+1)\ \wedge\ \boldsymbol{r}'{=}1\ \wedge\ \boldsymbol{w}'{=}1\ \wedge\ t' = t\uparrow(\mathcal{T}_0+1)$$

Deadlock

$\textbf{new } c?\ int\cdot\ \ t{:=}\ t\uparrow(\mathcal{T}_{\boldsymbol{r}}+1).\ \ c?.\ \ c!\ 5$

$=\quad \exists\mathcal{M}{:}\ \infty{*}int\cdot\ \exists\mathcal{T}{:}\ \infty{*}xnat\cdot\ \textbf{new } \boldsymbol{r}, \boldsymbol{w}{:}\ xnat := 0\cdot$

$t{:=}\ t\uparrow(\mathcal{T}_{\boldsymbol{r}}+1).\ \ \boldsymbol{r}{:=}\ \boldsymbol{r}{+}1.\ \ \mathcal{M}_{\boldsymbol{w}} = 5\ \ \wedge\ \ \mathcal{T}_{\boldsymbol{w}} = t\ \ \wedge\ \ (\boldsymbol{w}{:=}\ \boldsymbol{w}{+}1)$

$=\quad \exists\mathcal{M}{:}\ \infty{*}int\cdot\ \exists\mathcal{T}{:}\ \infty{*}xnat\cdot\ \exists\boldsymbol{r}, \boldsymbol{r}', \boldsymbol{w}, \boldsymbol{w}'{:}\ xnat\cdot$

$\boldsymbol{r}{:=}\ 0.\ \ \boldsymbol{w}{:=}\ 0.\ \ t{:=}\ t\uparrow(\mathcal{T}_{\boldsymbol{r}}+1).\ \ \boldsymbol{r}{:=}\ \boldsymbol{r}{+}1.$

$\mathcal{M}_{\boldsymbol{w}} = 5\ \ \wedge\ \ \mathcal{T}_{\boldsymbol{w}} = t\ \ \wedge\ \ \boldsymbol{r}'{=}\boldsymbol{r}\ \ \wedge\ \ \boldsymbol{w}' = \boldsymbol{w}{+}1\ \ \wedge\ \ t'{=}t$

$=\quad \exists\mathcal{M}{:}\ \infty{*}int\cdot\ \exists\mathcal{T}{:}\ \infty{*}xnat\cdot\ \exists\boldsymbol{r}, \boldsymbol{r}', \boldsymbol{w}, \boldsymbol{w}'{:}\ xnat\cdot$

$\mathcal{M}_0 = 5\ \ \wedge\ \ \mathcal{T}_0 = t\uparrow(\mathcal{T}_0+1)\ \ \wedge\ \ \boldsymbol{r}'{=}1\ \ \wedge\ \ \boldsymbol{w}'{=}1\ \ \wedge\ \ t' = t\uparrow(\mathcal{T}_0+1)$

Deadlock

$\textbf{new}\ c?\ int\cdot\ \ t{:=}\ t\uparrow(\mathcal{T}_{\boldsymbol{r}}+1).\ \ c?.\ \ c!\ 5$

$=\quad \exists\mathcal{M}{:}\ \infty{*}int\cdot\ \exists\mathcal{T}{:}\ \infty{*}xnat\cdot\ \textbf{new}\ \boldsymbol{r},\boldsymbol{w}{:}\ xnat:=0\cdot$

$t{:=}\ t\uparrow(\mathcal{T}_{\boldsymbol{r}}+1).\ \ \boldsymbol{r}{:=}\ \boldsymbol{r}{+}1.\ \ \mathcal{M}_{\boldsymbol{w}}=5\ \ \wedge\ \ \mathcal{T}_{\boldsymbol{w}}=t\ \ \wedge\ \ (\boldsymbol{w}{:=}\ \boldsymbol{w}{+}1)$

$=\quad \exists\mathcal{M}{:}\ \infty{*}int\cdot\ \exists\mathcal{T}{:}\ \infty{*}xnat\cdot\ \exists\boldsymbol{r},\boldsymbol{r}',\boldsymbol{w},\boldsymbol{w}'{:}\ xnat\cdot$

$\boldsymbol{r}{:=}\ 0.\ \ \boldsymbol{w}{:=}\ 0.\ \ t{:=}\ t\uparrow(\mathcal{T}_{\boldsymbol{r}}+1).\ \ \boldsymbol{r}{:=}\ \boldsymbol{r}{+}1.$

$\mathcal{M}_{\boldsymbol{w}}=5\ \ \wedge\ \ \mathcal{T}_{\boldsymbol{w}}=t\ \ \wedge\ \ \boldsymbol{r}'{=}\boldsymbol{r}\ \ \wedge\ \ \boldsymbol{w}'=\boldsymbol{w}{+}1\ \ \wedge\ \ t'{=}t$

$=\quad \exists\mathcal{M}{:}\ \infty{*}int\cdot\ \exists\mathcal{T}{:}\ \infty{*}xnat\cdot\ \exists\boldsymbol{r},\boldsymbol{r}',\boldsymbol{w},\boldsymbol{w}'{:}\ xnat\cdot$

$\mathcal{M}_0=5\ \ \wedge\ \ \mathcal{T}_0=t\uparrow(\mathcal{T}_0+1)\ \ \wedge\ \ \boldsymbol{r}'{=}1\ \ \wedge\ \ \boldsymbol{w}'{=}1\ \ \wedge\ \ t'=t\uparrow(\mathcal{T}_0+1)$

$=\quad t'{=}\infty$

Deadlock

$\mathbf{new}\ c, d ?\ int \cdot\ (c?.\ \ d!\ 6)\ \|\ (d?.\ \ c!\ 7)$

Deadlock

$$\mathbf{new}\ c, d?\ int\cdot\ \ (c?.\ \ d!\ 6)\ \ \|\ \ (d?.\ \ c!\ 7)$$

$$\mathbf{new}\ c, d?\ int\cdot\ \ (t{:=}\ t\uparrow(\mathcal{T}c_{\ \boldsymbol{r}c}+1).\ \ c?.\ \ d!\ 6)\ \ \|\ \ (t{:=}\ t\uparrow(\mathcal{T}d_{\ \boldsymbol{r}d}+1).\ \ d?.\ \ c!\ 7)$$

Deadlock

$\textbf{new } c, d?\ int\cdot\ (c?.\ d!\ 6)\ \|\ (d?.\ c!\ 7)$

$\textbf{new } c, d?\ int\cdot\ (t{:=}\ t\uparrow(\mathcal{T}c_{\boldsymbol{r}c} + 1).\ c?.\ d!\ 6)\ \|\ (t{:=}\ t\uparrow(\mathcal{T}d_{\boldsymbol{r}d} + 1).\ d?.\ c!\ 7)$

$$
\begin{aligned}
= \quad & \exists \mathcal{M}c, \mathcal{M}d\colon \infty{*}int\cdot \exists \mathcal{T}c, \mathcal{T}d\colon \infty{*}xnat\cdot \exists \boldsymbol{r}c, \boldsymbol{r}c', \boldsymbol{w}c, \boldsymbol{w}c', \boldsymbol{r}d, \boldsymbol{r}d', \boldsymbol{w}d, \boldsymbol{w}d'\colon xnat\cdot \\
& \quad \mathcal{M}d_0 = 6\ \wedge\ \mathcal{M}c_0 = 7\ \wedge\ \boldsymbol{r}c' = \boldsymbol{w}c' = \boldsymbol{r}d' = \boldsymbol{w}d' = 1 \\
& \wedge \quad \mathcal{T}c_0 = t\uparrow(\mathcal{T}d_0 + 1)\ \wedge\ \mathcal{T}d_0 = t\uparrow(\mathcal{T}c_0 + 1) \\
& \wedge \quad t' = t\uparrow(\mathcal{T}d_0 + 1)\uparrow(\mathcal{T}c_0 + 1)
\end{aligned}
$$

Deadlock

$\mathbf{new}\ c, d?\ int\cdot\ \ (c?.\ \ d!\ 6)\ \|\ (d?.\ \ c!\ 7)$

$\mathbf{new}\ c, d?\ int\cdot\ \ (t{:}=t\uparrow(\mathcal{T}c_{\boldsymbol{r}c}+1).\ \ c?.\ \ d!\ 6)\ \|\ (t{:}=t\uparrow(\mathcal{T}d_{\boldsymbol{r}d}+1).\ \ d?.\ \ c!\ 7)$

$=\quad \exists\mathcal{M}c, \mathcal{M}d{:}\ \infty^*int\cdot\ \exists\mathcal{T}c, \mathcal{T}d{:}\ \infty^*xnat\cdot\ \exists\boldsymbol{r}c, \boldsymbol{r}c', \boldsymbol{w}c, \boldsymbol{w}c', \boldsymbol{r}d, \boldsymbol{r}d', \boldsymbol{w}d, \boldsymbol{w}d'{:}\ xnat\cdot$

$\qquad \mathcal{M}d_0=6\ \wedge\ \mathcal{M}c_0=7\ \wedge\ \boldsymbol{r}c'=\boldsymbol{w}c'=\boldsymbol{r}d'=\boldsymbol{w}d'=1$

$\wedge\quad \mathcal{T}c_0=t\uparrow(\mathcal{T}d_0+1)\ \wedge\ \mathcal{T}d_0=t\uparrow(\mathcal{T}c_0+1)\ \longleftarrow$

$\wedge\quad t'=t\uparrow(\mathcal{T}d_0+1)\uparrow(\mathcal{T}c_0+1)$

Deadlock

$\textbf{new}\ c, d?\ int\cdot\ \ (c?.\ \ d!\ 6)\ \|\ (d?.\ \ c!\ 7)$

$\textbf{new}\ c, d?\ int\cdot\ \ (t{:=}\ t\uparrow(\mathcal{T}c_{\boldsymbol{r}c} + 1).\ \ c?.\ \ d!\ 6)\ \|\ (t{:=}\ t\uparrow(\mathcal{T}d_{\boldsymbol{r}d} + 1).\ \ d?.\ \ c!\ 7)$

$$
\begin{aligned}
= \quad & \exists \mathcal{M}c, \mathcal{M}d\colon \infty{*}int\cdot\ \exists \mathcal{T}c, \mathcal{T}d\colon \infty{*}xnat\cdot\ \exists \boldsymbol{r}c, \boldsymbol{r}c', \boldsymbol{w}c, \boldsymbol{w}c', \boldsymbol{r}d, \boldsymbol{r}d', \boldsymbol{w}d, \boldsymbol{w}d'\colon xnat\cdot \\
& \quad \mathcal{M}d_0 = 6\ \wedge\ \mathcal{M}c_0 = 7\ \wedge\ \boldsymbol{r}c' = \boldsymbol{w}c' = \boldsymbol{r}d' = \boldsymbol{w}d' = 1 \\
\wedge \quad & \quad \mathcal{T}c_0 = t\uparrow(\mathcal{T}d_0 + 1)\ \wedge\ \mathcal{T}d_0 = t\uparrow(\mathcal{T}c_0 + 1) \\
\wedge \quad & \quad t' = t\uparrow(\mathcal{T}d_0 + 1)\uparrow(\mathcal{T}c_0 + 1) \quad \longleftarrow
\end{aligned}
$$

Deadlock

$$\textbf{new } c, d?\ int\cdot\ (c?.\ d!\ 6)\ \|\ (d?.\ c!\ 7)$$

$$\textbf{new } c, d?\ int\cdot\ (t{:=}\ t\uparrow(\mathcal{T}c_{\boldsymbol{r}c}+1).\ c?.\ d!\ 6)\ \|\ (t{:=}\ t\uparrow(\mathcal{T}d_{\boldsymbol{r}d}+1).\ d?.\ c!\ 7)$$

$$= \quad \exists\mathcal{M}c, \mathcal{M}d\colon \infty{*}int\cdot \exists\mathcal{T}c, \mathcal{T}d\colon \infty{*}xnat\cdot \exists\boldsymbol{r}c, \boldsymbol{r}c', \boldsymbol{w}c, \boldsymbol{w}c', \boldsymbol{r}d, \boldsymbol{r}d', \boldsymbol{w}d, \boldsymbol{w}d'\colon xnat\cdot$$

$$\mathcal{M}d_0 = 6\ \wedge\ \mathcal{M}c_0 = 7\ \wedge\ \boldsymbol{r}c' = \boldsymbol{w}c' = \boldsymbol{r}d' = \boldsymbol{w}d' = 1$$

$$\wedge \quad \mathcal{T}c_0 = t\uparrow(\mathcal{T}d_0+1)\ \wedge\ \mathcal{T}d_0 = t\uparrow(\mathcal{T}c_0+1)$$

$$\wedge \quad t' = t\uparrow(\mathcal{T}d_0+1)\uparrow(\mathcal{T}c_0+1)$$

$$= \quad t'{=}\infty$$

Power Series Multiplication

Power Series Multiplication

Input on channel a : $a_0\ a_1\ a_2$ ...

Power Series Multiplication

Input on channel a : $a_0\ a_1\ a_2$ ... $A = a_0 + a_1 \times x + a_2 \times x^2 + ...$

Power Series Multiplication

Input on channel a : $a_0\ a_1\ a_2$ ... $A = a_0 + a_1 \times x + a_2 \times x^2 + ...$

Input on channel b : $b_0\ b_1\ b_2$ ...

Power Series Multiplication

Input on channel a : $a_0\ a_1\ a_2$ ... $A = a_0 + a_1 \times x + a_2 \times x^2 + \ldots$

Input on channel b : $b_0\ b_1\ b_2$ ... $B = b_0 + b_1 \times x + b_2 \times x^2 + \ldots$

Power Series Multiplication

Input on channel a : $a_0\ a_1\ a_2$ ...

Input on channel b : $b_0\ b_1\ b_2$ ...

Output on channel c : $c_0\ c_1\ c_2$ ...

$A = a_0 + a_1 \times x + a_2 \times x^2 + \ldots$

$B = b_0 + b_1 \times x + b_2 \times x^2 + \ldots$

Power Series Multiplication

Input on channel a : $a_0\ a_1\ a_2$ ... $A = a_0 + a_1 \times x + a_2 \times x^2 + \ldots$

Input on channel b : $b_0\ b_1\ b_2$ ... $B = b_0 + b_1 \times x + b_2 \times x^2 + \ldots$

Output on channel c : $c_0\ c_1\ c_2$ ... $C = c_0 + c_1 \times x + c_2 \times x^2 + \ldots$

Power Series Multiplication

Input on channel a : $a_0\ a_1\ a_2\ ...$

Input on channel b : $b_0\ b_1\ b_2\ ...$

Output on channel c : $c_0\ c_1\ c_2\ ...$

$A = a_0 + a_1 \times x + a_2 \times x^2 + ...$

$B = b_0 + b_1 \times x + b_2 \times x^2 + ...$

$C = c_0 + c_1 \times x + c_2 \times x^2 + ...$

$C = A \times B$

Power Series Multiplication

Input on channel a : $a_0\ a_1\ a_2$ ... $\quad A = a_0 + a_1 \times x + a_2 \times x^2 + \dots$

Input on channel b : $b_0\ b_1\ b_2$ ... $\quad B = b_0 + b_1 \times x + b_2 \times x^2 + \dots$

Output on channel c : $c_0\ c_1\ c_2$ ... $\quad C = c_0 + c_1 \times x + c_2 \times x^2 + \dots$

$$C = A \times B = a_0 \times b_0 + (a_0 \times b_1 + a_1 \times b_0)x + (a_0 \times b_2 + a_1 \times b_1 + a_2 \times b_0) \times x^2 + \dots$$

Power Series Multiplication

Input on channel a : $a_0\ a_1\ a_2$ ...

Input on channel b : $b_0\ b_1\ b_2$ ...

Output on channel c : $c_0\ c_1\ c_2$ ...

$A = a_0 + a_1 \times x + a_2 \times x^2 + \ldots$

$B = b_0 + b_1 \times x + b_2 \times x^2 + \ldots$

$C = c_0 + c_1 \times x + c_2 \times x^2 + \ldots$

$A_1 = a_1 + a_2 \times x + a_3 \times x^2 + \ldots$

$A_2 = a_2 + a_3 \times x + a_4 \times x^2 + \ldots$

$B_1 = b_1 + b_2 \times x + b_3 \times x^2 + \ldots$

$B_2 = b_2 + b_3 \times x + b_4 \times x^2 + \ldots$

$$C = A \times B = a_0 \times b_0 + (a_0 \times b_1 + a_1 \times b_0)x + (a_0 \times b_2 + a_1 \times b_1 + a_2 \times b_0) \times x^2 + \ldots$$

Power Series Multiplication

Input on channel a : $a_0\ a_1\ a_2\ \ldots$ $\quad A = a_0 + a_1 \times x + a_2 \times x^2 + \ldots$

Input on channel b : $b_0\ b_1\ b_2\ \ldots$ $\quad B = b_0 + b_1 \times x + b_2 \times x^2 + \ldots$

Output on channel c : $c_0\ c_1\ c_2\ \ldots$ $\quad C = c_0 + c_1 \times x + c_2 \times x^2 + \ldots$

$A_1 = a_1 + a_2 \times x + a_3 \times x^2 + \ldots$ $\quad B_1 = b_1 + b_2 \times x + b_3 \times x^2 + \ldots$

$A_2 = a_2 + a_3 \times x + a_4 \times x^2 + \ldots$ $\quad B_2 = b_2 + b_3 \times x + b_4 \times x^2 + \ldots$

$$C = A \times B = a_0 \times b_0 + (a_0 \times b_1 + a_1 \times b_0)x + (a_0 \times b_2 + a_1 \times b_1 + a_2 \times b_0) \times x^2 + \ldots$$

Power Series Multiplication

Input on channel a : $a_0\ a_1\ a_2\ \ldots$

Input on channel b : $b_0\ b_1\ b_2\ \ldots$

Output on channel c : $c_0\ c_1\ c_2\ \ldots$

$$A = a_0 + a_1 \times x + a_2 \times x^2 + \ldots$$

$$B = b_0 + b_1 \times x + b_2 \times x^2 + \ldots$$

$$C = c_0 + c_1 \times x + c_2 \times x^2 + \ldots$$

$$A_1 = a_1 + a_2 \times x + a_3 \times x^2 + \ldots$$

$$A_2 = a_2 + a_3 \times x + a_4 \times x^2 + \ldots$$

$$B_1 = b_1 + b_2 \times x + b_3 \times x^2 + \ldots$$

$$B_2 = b_2 + b_3 \times x + b_4 \times x^2 + \ldots$$

$$C = A \times B = a_0 \times b_0 + (a_0 \times b_1 + a_1 \times b_0)x + (a_0 \times b_2 + a_1 \times b_1 + a_2 \times b_0) \times x^2 + \ldots$$

Power Series Multiplication

Input on channel a : $a_0\ a_1\ a_2\ \ldots$

Input on channel b : $b_0\ b_1\ b_2\ \ldots$

Output on channel c : $c_0\ c_1\ c_2\ \ldots$

$$A = a_0 + a_1 \times x + a_2 \times x^2 + \ldots$$

$$B = b_0 + b_1 \times x + b_2 \times x^2 + \ldots$$

$$C = c_0 + c_1 \times x + c_2 \times x^2 + \ldots$$

$$A_1 = a_1 + a_2 \times x + a_3 \times x^2 + \ldots$$

$$A_2 = a_2 + a_3 \times x + a_4 \times x^2 + \ldots$$

$$B_1 = b_1 + b_2 \times x + b_3 \times x^2 + \ldots$$

$$B_2 = b_2 + b_3 \times x + b_4 \times x^2 + \ldots$$

$$C = A \times B = a_0 \times b_0 + (a_0 \times b_1 + a_1 \times b_0)x + (a_0 \times B_2 + A_1 \times B_1 + A_2 \times b_0) \times x^2$$

Power Series Multiplication

Input on channel a : $a_0\ a_1\ a_2\ ...$

Input on channel b : $b_0\ b_1\ b_2\ ...$

Output on channel c : $c_0\ c_1\ c_2\ ...$

$A = a_0 + a_1 \times x + a_2 \times x^2 + ...$

$B = b_0 + b_1 \times x + b_2 \times x^2 + ...$

$C = c_0 + c_1 \times x + c_2 \times x^2 + ...$

$A_1 = a_1 + a_2 \times x + a_3 \times x^2 + ...$

$A_2 = a_2 + a_3 \times x + a_4 \times x^2 + ...$

$B_1 = b_1 + b_2 \times x + b_3 \times x^2 + ...$

$B_2 = b_2 + b_3 \times x + b_4 \times x^2 + ...$

$$C = A \times B = a_0 \times b_0 + (a_0 \times b_1 + a_1 \times b_0)x + (a_0 \times B_2 + A_1 \times B_1 + A_2 \times b_0) \times x^2$$

↑ ↑

Power Series Multiplication

Input on channel a : $a_0\ a_1\ a_2\ ...$

Input on channel b : $b_0\ b_1\ b_2\ ...$

Output on channel c : $c_0\ c_1\ c_2\ ...$

$$A = a_0 + a_1 \times x + a_2 \times x^2 + ...$$

$$B = b_0 + b_1 \times x + b_2 \times x^2 + ...$$

$$C = c_0 + c_1 \times x + c_2 \times x^2 + ...$$

$$A_1 = a_1 + a_2 \times x + a_3 \times x^2 + ...$$

$$A_2 = a_2 + a_3 \times x + a_4 \times x^2 + ...$$

$$B_1 = b_1 + b_2 \times x + b_3 \times x^2 + ...$$

$$B_2 = b_2 + b_3 \times x + b_4 \times x^2 + ...$$

$$C = A \times B = a_0 \times b_0 + (a_0 \times b_1 + a_1 \times b_0)x + (a_0 \times B_2 + A_1 \times B_1 + A_2 \times b_0) \times x^2$$

Power Series Multiplication

Input on channel a : $a_0\ a_1\ a_2$ ...

Input on channel b : $b_0\ b_1\ b_2$ ...

Output on channel c : $c_0\ c_1\ c_2$ ...

$A = a_0 + a_1 \times x + a_2 \times x^2 + \ldots$

$B = b_0 + b_1 \times x + b_2 \times x^2 + \ldots$

$C = c_0 + c_1 \times x + c_2 \times x^2 + \ldots$

$A_1 = a_1 + a_2 \times x + a_3 \times x^2 + \ldots$

$A_2 = a_2 + a_3 \times x + a_4 \times x^2 + \ldots$

$B_1 = b_1 + b_2 \times x + b_3 \times x^2 + \ldots$

$B_2 = b_2 + b_3 \times x + b_4 \times x^2 + \ldots$

$$C = A \times B = a_0 \times b_0 + (a_0 \times b_1 + a_1 \times b_0)x + (a_0 \times B_2 + A_1 \times B_1 + A_2 \times b_0) \times x^2$$

$C = A \times B$

Power Series Multiplication

Input on channel a : $a_0\ a_1\ a_2\ ...$ $\qquad A = a_0 + a_1{\times}x + a_2{\times}x^2 + ...$

Input on channel b : $b_0\ b_1\ b_2\ ...$ $\qquad B = b_0 + b_1{\times}x + b_2{\times}x^2 + ...$

Output on channel c : $c_0\ c_1\ c_2\ ...$ $\qquad C = c_0 + c_1{\times}x + c_2{\times}x^2 + ...$

$A_1 = a_1 + a_2{\times}x + a_3{\times}x^2 + ...$ $\qquad B_1 = b_1 + b_2{\times}x + b_3{\times}x^2 + ...$

$A_2 = a_2 + a_3{\times}x + a_4{\times}x^2 + ...$ $\qquad B_2 = b_2 + b_3{\times}x + b_4{\times}x^2 + ...$

$$C = A \times B = a_0{\times}b_0 + (a_0{\times}b_1 + a_1{\times}b_0)x + (a_0{\times}B_2 + A_1{\times}B_1 + A_2{\times}b_0){\times}x^2$$

$\langle c?\ rat\cdot\ C = A{\times}B\rangle$

Power Series Multiplication

Input on channel a : $a_0\ a_1\ a_2\ ...$

Input on channel b : $b_0\ b_1\ b_2\ ...$

Output on channel c : $c_0\ c_1\ c_2\ ...$

$A = a_0 + a_1{\times}x + a_2{\times}x^2 + ...$

$B = b_0 + b_1{\times}x + b_2{\times}x^2 + ...$

$C = c_0 + c_1{\times}x + c_2{\times}x^2 + ...$

$A_1 = a_1 + a_2{\times}x + a_3{\times}x^2 + ...$

$A_2 = a_2 + a_3{\times}x + a_4{\times}x^2 + ...$

$B_1 = b_1 + b_2{\times}x + b_3{\times}x^2 + ...$

$B_2 = b_2 + b_3{\times}x + b_4{\times}x^2 + ...$

$$C = A \times B = a_0{\times}b_0 + (a_0{\times}b_1 + a_1{\times}b_0)x + (a_0{\times}B_2 + A_1{\times}B_1 + A_2{\times}b_0){\times}x^2$$

$\langle c?\ rat\cdot\ C = A{\times}B \rangle\ c$

Power Series Multiplication

Input on channel a : $a_0\ a_1\ a_2$ ...

Input on channel b : $b_0\ b_1\ b_2$ ...

Output on channel c : $c_0\ c_1\ c_2$ ...

$A = a_0 + a_1{\times}x + a_2{\times}x^2 + ...$

$B = b_0 + b_1{\times}x + b_2{\times}x^2 + ...$

$C = c_0 + c_1{\times}x + c_2{\times}x^2 + ...$

$A_1 = a_1 + a_2{\times}x + a_3{\times}x^2 + ...$

$A_2 = a_2 + a_3{\times}x + a_4{\times}x^2 + ...$

$B_1 = b_1 + b_2{\times}x + b_3{\times}x^2 + ...$

$B_2 = b_2 + b_3{\times}x + b_4{\times}x^2 + ...$

$$C = A \times B = a_0{\times}b_0 + (a_0{\times}b_1 + a_1{\times}b_0)x + (a_0{\times}B_2 + A_1{\times}B_1 + A_2{\times}b_0){\times}x^2$$

$\langle c?\ rat\cdot\ C = A{\times}B\rangle\ c \quad \Leftarrow \quad (a? \,\|\, b?).\ c!\ a{\times}b.$

new $a0$: $rat := a\cdot$ **new** $b0$: $rat := b\cdot$ **new** $d?\ rat\cdot$

$\langle c?\ rat\cdot\ C = A{\times}B\rangle\ d$

$\|\ ((a? \,\|\, b?).\ c!\ a0{\times}b + a{\times}b0.\ C = a0{\times}B + D + A{\times}b0)$

$C = a0{\times}B + D + A{\times}b0 \quad \Leftarrow \quad (a? \,\|\, b? \,\|\, d?).\ c!\ a0{\times}b + d + a{\times}b0.\ C = a0{\times}B + D + A{\times}b0$

Power Series Multiplication

Input on channel a : $a_0\ a_1\ a_2\ ...$ $\qquad A\ =\ a_0 + a_1{\times}x + a_2{\times}x^2 + ...$

Input on channel b : $b_0\ b_1\ b_2\ ...$ $\qquad B\ =\ b_0 + b_1{\times}x + b_2{\times}x^2 + ...$

Output on channel c : $c_0\ c_1\ c_2\ ...$ $\qquad C\ =\ c_0 + c_1{\times}x + c_2{\times}x^2 + ...$

$A_1\ =\ a_1 + a_2{\times}x + a_3{\times}x^2 + ...$ $\qquad B_1\ =\ b_1 + b_2{\times}x + b_3{\times}x^2 + ...$

$A_2\ =\ a_2 + a_3{\times}x + a_4{\times}x^2 + ...$ $\qquad B_2\ =\ b_2 + b_3{\times}x + b_4{\times}x^2 + ...$

$$C\ =\ A \times B\ =\ a_0{\times}b_0 + (a_0{\times}b_1 + a_1{\times}b_0)x + (a_0{\times}B_2 + A_1{\times}B_1 + A_2{\times}b_0){\times}x^2$$

$\langle c?\ rat\cdot\ C = A{\times}B\rangle\ c \quad \Leftarrow \quad (a?\ \|\ b?).\ \ c!\ a{\times}b.$

$\quad$ **new** $a0$: $rat := a\cdot$ **new** $b0$: $rat := b\cdot$ **new** $d?\ rat\cdot$

$\quad\quad \langle c?\ rat\cdot\ C = A{\times}B\rangle\ d$

$\quad \|\ \ ((a?\ \|\ b?).\ \ c!\ a0{\times}b + a{\times}b0.\ \ C = a0{\times}B + D + A{\times}b0)$

$C = a0{\times}B + D + A{\times}b0 \quad \Leftarrow \quad (a?\ \|\ b?\ \|\ d?).\ \ c!\ a0{\times}b + d + a{\times}b0.\ \ C = a0{\times}B + D + A{\times}b0$

Power Series Multiplication

Input on channel a : $a_0\ a_1\ a_2\ \ldots$ $\qquad A = a_0 + a_1{\times}x + a_2{\times}x^2 + \ldots$

Input on channel b : $b_0\ b_1\ b_2\ \ldots$ $\qquad B = b_0 + b_1{\times}x + b_2{\times}x^2 + \ldots$

Output on channel c : $c_0\ c_1\ c_2\ \ldots$ $\qquad C = c_0 + c_1{\times}x + c_2{\times}x^2 + \ldots$

$A_1 = a_1 + a_2{\times}x + a_3{\times}x^2 + \ldots \qquad B_1 = b_1 + b_2{\times}x + b_3{\times}x^2 + \ldots$

$A_2 = a_2 + a_3{\times}x + a_4{\times}x^2 + \ldots \qquad B_2 = b_2 + b_3{\times}x + b_4{\times}x^2 + \ldots$

$$C = A \times B = a_0{\times}b_0 + (a_0{\times}b_1 + a_1{\times}b_0)x + (a_0{\times}B_2 + A_1{\times}B_1 + A_2{\times}b_0){\times}x^2$$

$\langle c?\ rat\cdot\ C = A{\times}B\rangle\ c \quad \Leftarrow \quad (a? \parallel b?).\ \ c!\ a{\times}b.$

$\qquad$ **new** $a0$: $rat := a\cdot$ **new** $b0$: $rat := b\cdot$ **new** $d?\ rat\cdot$

$\qquad\quad \langle c?\ rat\cdot\ C = A{\times}B\rangle\ d$

$\qquad \parallel\ \ ((a? \parallel b?).\ \ c!\ a0{\times}b + a{\times}b0.\ \ C = a0{\times}B + D + A{\times}b0)$

$C = a0{\times}B + D + A{\times}b0 \quad \Leftarrow \quad (a? \parallel b? \parallel d?).\ \ c!\ a0{\times}b + d + a{\times}b0.\ \ C = a0{\times}B + D + A{\times}b0$

Power Series Multiplication

Input on channel a : $a_0\ a_1\ a_2$ ...

Input on channel b : $b_0\ b_1\ b_2$ ...

Output on channel c : $c_0\ c_1\ c_2$ ...

$A = a_0 + a_1{\times}x + a_2{\times}x^2 + \ldots$

$B = b_0 + b_1{\times}x + b_2{\times}x^2 + \ldots$

$C = c_0 + c_1{\times}x + c_2{\times}x^2 + \ldots$

$A_1 = a_1 + a_2{\times}x + a_3{\times}x^2 + \ldots$

$A_2 = a_2 + a_3{\times}x + a_4{\times}x^2 + \ldots$

$B_1 = b_1 + b_2{\times}x + b_3{\times}x^2 + \ldots$

$B_2 = b_2 + b_3{\times}x + b_4{\times}x^2 + \ldots$

$$C = A \times B = a_0{\times}b_0 + (a_0{\times}b_1 + a_1{\times}b_0)x + (a_0{\times}B_2 + A_1{\times}B_1 + A_2{\times}b_0){\times}x^2$$

$\langle c?\ rat\cdot\ C = A{\times}B\rangle\ c \quad \Leftarrow \quad (a? \parallel b?).\ \ c!\ a{\times}b.$

$\longrightarrow$ $\textbf{new}\ a0{:}\ rat := a\cdot\ \ \textbf{new}\ b0{:}\ rat := b\cdot\ \ \textbf{new}\ d?\ rat\cdot$

$\langle c?\ rat\cdot\ C = A{\times}B\rangle\ d$

$\parallel\ \ ((a? \parallel b?).\ \ c!\ a0{\times}b + a{\times}b0.\ \ C = a0{\times}B + D + A{\times}b0)$

$C = a0{\times}B + D + A{\times}b0 \quad \Leftarrow \quad (a? \parallel b? \parallel d?).\ \ c!\ a0{\times}b + d + a{\times}b0.\ \ C = a0{\times}B + D + A{\times}b0$

Power Series Multiplication

Input on channel a : $a_0\ a_1\ a_2$ ... $\qquad A = a_0 + a_1{\times}x + a_2{\times}x^2 + ...$

Input on channel b : $b_0\ b_1\ b_2$ ... $\qquad B = b_0 + b_1{\times}x + b_2{\times}x^2 + ...$

Output on channel c : $c_0\ c_1\ c_2$ ... $\qquad C = c_0 + c_1{\times}x + c_2{\times}x^2 + ...$

$A_1 = a_1 + a_2{\times}x + a_3{\times}x^2 + ...$ $\qquad B_1 = b_1 + b_2{\times}x + b_3{\times}x^2 + ...$

$A_2 = a_2 + a_3{\times}x + a_4{\times}x^2 + ...$ $\qquad B_2 = b_2 + b_3{\times}x + b_4{\times}x^2 + ...$

$$C = A \times B = a_0{\times}b_0 + (a_0{\times}b_1 + a_1{\times}b_0)x + (a_0{\times}B_2 + A_1{\times}B_1 + A_2{\times}b_0){\times}x^2$$

$\langle c?\ rat\cdot\ C = A{\times}B\rangle\ c \quad \Leftarrow \quad (a?\ \|\ b?).\ \ c!\ a{\times}b.$

new $a0$: $rat := a\cdot$ **new** $b0$: $rat := b\cdot$ **new** $d?\ rat\cdot$

$\rightarrow\ \langle c?\ rat\cdot\ C = A{\times}B\rangle\ d$

$\|\ \ ((a?\ \|\ b?).\ \ c!\ a0{\times}b + a{\times}b0.\ \ C = a0{\times}B + D + A{\times}b0)$

$C = a0{\times}B + D + A{\times}b0 \quad \Leftarrow \quad (a?\ \|\ b?\ \|\ d?).\ \ c!\ a0{\times}b + d + a{\times}b0.\ \ C = a0{\times}B + D + A{\times}b0$

Power Series Multiplication

Input on channel a : $a_0\ a_1\ a_2$ ... $\quad A = a_0 + a_1{\times}x + a_2{\times}x^2 + ...$

Input on channel b : $b_0\ b_1\ b_2$ ... $\quad B = b_0 + b_1{\times}x + b_2{\times}x^2 + ...$

Output on channel c : $c_0\ c_1\ c_2$ ... $\quad C = c_0 + c_1{\times}x + c_2{\times}x^2 + ...$

$A_1 = a_1 + a_2{\times}x + a_3{\times}x^2 + ...$ $\quad B_1 = b_1 + b_2{\times}x + b_3{\times}x^2 + ...$

$A_2 = a_2 + a_3{\times}x + a_4{\times}x^2 + ...$ $\quad B_2 = b_2 + b_3{\times}x + b_4{\times}x^2 + ...$

$$C = A \times B = a_0{\times}b_0 + (a_0{\times}b_1 + a_1{\times}b_0)x + (a_0{\times}B_2 + A_1{\times}B_1 + A_2{\times}b_0){\times}x^2$$

$$\uparrow$$

$\langle c?\ rat\cdot\ C = A{\times}B\rangle\ c \quad \Leftarrow \quad (a? \parallel b?).\ c!\ a{\times}b.$

$\quad$ **new** $a0: rat := a\cdot$ **new** $b0: rat := b\cdot$ **new** $d?\ rat\cdot$

$\longrightarrow \quad \langle c?\ rat\cdot\ C = A{\times}B\rangle\ d$

$\quad \parallel \quad ((a? \parallel b?).\ c!\ a0{\times}b + a{\times}b0.\ C = a0{\times}B + D + A{\times}b0)$

$C = a0{\times}B + D + A{\times}b0 \quad \Leftarrow \quad (a? \parallel b? \parallel d?).\ c!\ a0{\times}b + d + a{\times}b0.\ C = a0{\times}B + D + A{\times}b0$

Power Series Multiplication

Input on channel a : $a_0\ a_1\ a_2\ ...$ $\qquad A = a_0 + a_1{\times}x + a_2{\times}x^2 + ...$

Input on channel b : $b_0\ b_1\ b_2\ ...$ $\qquad B = b_0 + b_1{\times}x + b_2{\times}x^2 + ...$

Output on channel c : $c_0\ c_1\ c_2\ ...$ $\qquad C = c_0 + c_1{\times}x + c_2{\times}x^2 + ...$

$A_1 = a_1 + a_2{\times}x + a_3{\times}x^2 + ...$ $\qquad B_1 = b_1 + b_2{\times}x + b_3{\times}x^2 + ...$

$A_2 = a_2 + a_3{\times}x + a_4{\times}x^2 + ...$ $\qquad B_2 = b_2 + b_3{\times}x + b_4{\times}x^2 + ...$

$$C = A \times B = a_0{\times}b_0 + (a_0{\times}b_1 + a_1{\times}b_0)x + (a_0{\times}B_2 + A_1{\times}B_1 + A_2{\times}b_0){\times}x^2$$

$\langle c?\ rat\cdot\ C = A{\times}B\rangle\ c \quad \Leftarrow \quad (a? \parallel b?).\ \ c!\ a{\times}b.$

$\qquad$ **new** $a0$: $rat := a\cdot$ **new** $b0$: $rat := b\cdot$ **new** $d?\ rat\cdot$

$\qquad\quad \langle c?\ rat\cdot\ C = A{\times}B\rangle\ d$

$\qquad \rightarrow \parallel\ ((a? \parallel b?).\ \ c!\ a0{\times}b + a{\times}b0.\ \ C = a0{\times}B + D + A{\times}b0)$

$C = a0{\times}B + D + A{\times}b0 \quad \Leftarrow \quad (a? \parallel b? \parallel d?).\ \ c!\ a0{\times}b + d + a{\times}b0.\ \ C = a0{\times}B + D + A{\times}b0$

Power Series Multiplication

Input on channel a : $a_0\ a_1\ a_2$ ... $A\ =\ a_0 + a_1{\times}x + a_2{\times}x^2 + \ldots$

Input on channel b : $b_0\ b_1\ b_2$ ... $B\ =\ b_0 + b_1{\times}x + b_2{\times}x^2 + \ldots$

Output on channel c : $c_0\ c_1\ c_2$ ... $C\ =\ c_0 + c_1{\times}x + c_2{\times}x^2 + \ldots$

$A_1\ =\ a_1 + a_2{\times}x + a_3{\times}x^2 + \ldots$ $B_1\ =\ b_1 + b_2{\times}x + b_3{\times}x^2 + \ldots$

$A_2\ =\ a_2 + a_3{\times}x + a_4{\times}x^2 + \ldots$ $B_2\ =\ b_2 + b_3{\times}x + b_4{\times}x^2 + \ldots$

$$C\ =\ A \times B\ =\ a_0{\times}b_0 + (a_0{\times}b_1 + a_1{\times}b_0)x + (a_0{\times}B_2 + A_1{\times}B_1 + A_2{\times}b_0){\times}x^2$$

$\langle c?\ rat\cdot\ C = A{\times}B\rangle\ c \quad \Leftarrow \quad (a? \parallel b?).\ \ c!\ a{\times}b.$

$\quad$ **new** $a0$: $rat := a\cdot$ **new** $b0$: $rat := b\cdot$ **new** $d?\ rat\cdot$

$\quad\quad \langle c?\ rat\cdot\ C = A{\times}B\rangle\ d$

$\quad \parallel\ \ ((a? \parallel b?).\ \ c!\ a0{\times}b + a{\times}b0.\ \ C = a0{\times}B + D + A{\times}b0)$

$\uparrow \quad \uparrow$

$C = a0{\times}B + D + A{\times}b0 \quad \Leftarrow \quad (a? \parallel b? \parallel d?).\ \ c!\ a0{\times}b + d + a{\times}b0.\ \ C = a0{\times}B + D + A{\times}b0$

Power Series Multiplication

Input on channel a : $a_0\ a_1\ a_2\ ...$ $\quad A = a_0 + a_1{\times}x + a_2{\times}x^2 + ...$

Input on channel b : $b_0\ b_1\ b_2\ ...$ $\quad B = b_0 + b_1{\times}x + b_2{\times}x^2 + ...$

Output on channel c : $c_0\ c_1\ c_2\ ...$ $\quad C = c_0 + c_1{\times}x + c_2{\times}x^2 + ...$

$A_1 = a_1 + a_2{\times}x + a_3{\times}x^2 + ...$ $\quad B_1 = b_1 + b_2{\times}x + b_3{\times}x^2 + ...$

$A_2 = a_2 + a_3{\times}x + a_4{\times}x^2 + ...$ $\quad B_2 = b_2 + b_3{\times}x + b_4{\times}x^2 + ...$

$$C = A \times B = a_0{\times}b_0 + (a_0{\times}b_1 + a_1{\times}b_0)x + (a_0{\times}B_2 + A_1{\times}B_1 + A_2{\times}b_0){\times}x^2$$

$\uparrow$

$\langle c?\ rat\cdot\ C = A{\times}B\rangle\ c \quad \Leftarrow \quad (a?\ \|\ b?).\ \ c!\ a{\times}b.$

new $a0$: $rat := a\cdot$ **new** $b0$: $rat := b\cdot$ **new** $d?\ rat\cdot$

$\langle c?\ rat\cdot\ C = A{\times}B\rangle\ d$

$\|\ \ ((a?\ \|\ b?).\ \ c!\ a0{\times}b + a{\times}b0.\ \ C = a0{\times}B + D + A{\times}b0)$

$\uparrow$

$C = a0{\times}B + D + A{\times}b0 \quad \Leftarrow \quad (a?\ \|\ b?\ \|\ d?).\ \ c!\ a0{\times}b + d + a{\times}b0.\ \ C = a0{\times}B + D + A{\times}b0$

Power Series Multiplication

Input on channel a : $a_0\ a_1\ a_2$ ... $\quad A = a_0 + a_1{\times}x + a_2{\times}x^2 + ...$

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$A_1 = a_1 + a_2{\times}x + a_3{\times}x^2 + ... \qquad B_1 = b_1 + b_2{\times}x + b_3{\times}x^2 + ...$

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$$C = A \times B = a_0{\times}b_0 + (a_0{\times}b_1 + a_1{\times}b_0)x + (a_0{\times}B_2 + A_1{\times}B_1 + A_2{\times}b_0){\times}x^2$$

$\langle c?\ rat\cdot\ C = A{\times}B\rangle\ c \quad \Leftarrow \quad (a? \parallel b?).\ \ c!\ a{\times}b.$

$\quad$ **new** $a0$: $rat := a\cdot$ **new** $b0$: $rat := b\cdot$ **new** $d?\ rat\cdot$

$\qquad \langle c?\ rat\cdot\ C = A{\times}B\rangle\ d$

$\quad \parallel \ ((a? \parallel b?).\ \ c!\ a0{\times}b + a{\times}b0.\ \ C = a0{\times}B + D + A{\times}b0)$

$C = a0{\times}B + D + A{\times}b0 \quad \Leftarrow \quad (a? \parallel b? \parallel d?).\ \ c!\ a0{\times}b + d + a{\times}b0.\ \ C = a0{\times}B + D + A{\times}b0$

Power Series Multiplication

Input on channel a : $a_0\ a_1\ a_2\ ...$ $\qquad A = a_0 + a_1{\times}x + a_2{\times}x^2 + ...$

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$$C = A \times B = a_0{\times}b_0 + (a_0{\times}b_1 + a_1{\times}b_0)x + (a_0{\times}B_2 + A_1{\times}B_1 + A_2{\times}b_0){\times}x^2$$

$\langle c?\ rat \cdot\ C = A{\times}B\rangle\ c \quad \Leftarrow \quad (a? \,\|\, b?).\ \ c!\ a{\times}b.$

$\quad$ **new** $a0$: $rat := a\cdot$ **new** $b0$: $rat := b\cdot$ **new** $d?\ rat\cdot$

$\quad\quad \langle c?\ rat\cdot\ C = A{\times}B\rangle\ d$

$\quad \|\ \ ((a? \,\|\, b?).\ \ c!\ a0{\times}b + a{\times}b0.\ \ C = a0{\times}B + D + A{\times}b0)$

$\downarrow \quad \downarrow$

$C = a0{\times}B + D + A{\times}b0 \quad \Leftarrow \quad (a? \,\|\, b? \,\|\, d?).\ \ c!\ a0{\times}b + d + a{\times}b0.\ \ C = a0{\times}B + D + A{\times}b0$

Power Series Multiplication

Input on channel a : $a_0\ a_1\ a_2$ ...

Input on channel b : $b_0\ b_1\ b_2$ ...

Output on channel c : $c_0\ c_1\ c_2$ ...

$$A \;=\; a_0 + a_1{\times}x + a_2{\times}x^2 + \ldots$$

$$B \;=\; b_0 + b_1{\times}x + b_2{\times}x^2 + \ldots$$

$$C \;=\; c_0 + c_1{\times}x + c_2{\times}x^2 + \ldots$$

$$A_1 \;=\; a_1 + a_2{\times}x + a_3{\times}x^2 + \ldots$$

$$A_2 \;=\; a_2 + a_3{\times}x + a_4{\times}x^2 + \ldots$$

$$B_1 \;=\; b_1 + b_2{\times}x + b_3{\times}x^2 + \ldots$$

$$B_2 \;=\; b_2 + b_3{\times}x + b_4{\times}x^2 + \ldots$$

$$C \;=\; A \times B \;=\; a_0{\times}b_0 + (a_0{\times}b_1 + a_1{\times}b_0)x + (a_0{\times}B_2 + A_1{\times}B_1 + A_2{\times}b_0){\times}x^2$$

$$\langle c?\ rat\cdot\ C = A{\times}B\rangle\ c \quad \Leftarrow \quad (a? \parallel b?).\ \ c!\ a{\times}b.$$

$$\textbf{new}\ a0{:}\ rat := a\cdot\ \ \textbf{new}\ b0{:}\ rat := b\cdot\ \ \textbf{new}\ d?\ rat\cdot$$

$$\langle c?\ rat\cdot\ C = A{\times}B\rangle\ d$$

$$\parallel\ \ ((a? \parallel b?).\ \ c!\ a0{\times}b + a{\times}b0.\ \ C = a0{\times}B + D + A{\times}b0)$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$C = a0{\times}B + D + A{\times}b0 \quad \Leftarrow \quad (a? \parallel b? \parallel d?).\ \ c!\ a0{\times}b + d + a{\times}b0.\ \ C = a0{\times}B + D + A{\times}b0$$

Power Series Multiplication

Input on channel a : $a_0\ a_1\ a_2$ ... $\qquad A = a_0 + a_1{\times}x + a_2{\times}x^2 + ...$

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$$C = A \times B = a_0{\times}b_0 + (a_0{\times}b_1 + a_1{\times}b_0)x + (a_0{\times}B_2 + A_1{\times}B_1 + A_2{\times}b_0){\times}x^2$$

$$\uparrow$$

$\langle c?\ rat\cdot\ C = A{\times}B\rangle\ c \quad \Leftarrow \quad (a?\ \|\ b?).\ \ c!\ a{\times}b.$

$\quad$ **new** $a0$: $rat := a\cdot$ **new** $b0$: $rat := b\cdot$ **new** d? $rat\cdot$

$\quad\quad \langle c?\ rat\cdot\ C = A{\times}B\rangle\ d$

$\quad \|\ \ ((a?\ \|\ b?).\ \ c!\ a0{\times}b + a{\times}b0.\ \ C = a0{\times}B + D + A{\times}b0)$

$$\downarrow$$

$C = a0{\times}B + D + A{\times}b0 \quad \Leftarrow \quad (a?\ \|\ b?\ \|\ d?).\ \ c!\ a0{\times}b + d + a{\times}b0.\ \ C = a0{\times}B + D + A{\times}b0$

Power Series Multiplication

Input on channel a : $a_0\ a_1\ a_2$ ... $A\ =\ a_0 + a_1{\times}x + a_2{\times}x^2 + \ldots$

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$A_1\ =\ a_1 + a_2{\times}x + a_3{\times}x^2 + \ldots$ $B_1\ =\ b_1 + b_2{\times}x + b_3{\times}x^2 + \ldots$

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$$C\ =\ A \times B\ =\ a_0{\times}b_0 + (a_0{\times}b_1 + a_1{\times}b_0)x + (a_0{\times}B_2 + A_1{\times}B_1 + A_2{\times}b_0){\times}x^2$$

$\langle c?\ rat\cdot\ C = A{\times}B\rangle\ c\ \Leftarrow\ (a?\ \|\ b?).\ c!\ a{\times}b.$

new $a0$: $rat := a\cdot$ **new** $b0$: $rat := b\cdot$ **new** $d?\ rat\cdot$

$\langle c?\ rat\cdot\ C = A{\times}B\rangle\ d$

$\|\ \ ((a?\ \|\ b?).\ c!\ a0{\times}b + a{\times}b0.\ C = a0{\times}B + D + A{\times}b0)$

$C = a0{\times}B + D + A{\times}b0\ \Leftarrow\ (a?\ \|\ b?\ \|\ d?).\ c!\ a0{\times}b + d + a{\times}b0.\ C = a0{\times}B + D + A{\times}b0$

Power Series Multiplication

Input on channel a : $a_0\ a_1\ a_2$ ... $\quad A = a_0 + a_1 \times x + a_2 \times x^2 + \ldots$

Input on channel b : $b_0\ b_1\ b_2$ ... $\quad B = b_0 + b_1 \times x + b_2 \times x^2 + \ldots$

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$A_1 = a_1 + a_2 \times x + a_3 \times x^2 + \ldots \qquad B_1 = b_1 + b_2 \times x + b_3 \times x^2 + \ldots$

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$$C = A \times B = a_0 \times b_0 + (a_0 \times b_1 + a_1 \times b_0)x + (a_0 \times B_2 + A_1 \times B_1 + A_2 \times b_0) \times x^2$$

↑

$\langle c?\ rat \cdot\ C = A \times B \rangle\ c \quad \Leftarrow \quad (a? \parallel b?).\ c!\ a \times b.$

$\quad$ **new** $a0: rat := a \cdot$ **new** $b0: rat := b \cdot$ **new** $d?\ rat \cdot$

$\quad\quad \langle c?\ rat \cdot\ C = A \times B \rangle\ d$

$\quad \parallel \quad ((a? \parallel b?).\ c!\ a0 \times b + a \times b0.\ C = a0 \times B + D + A \times b0)$

↓

$C = a0 \times B + D + A \times b0 \quad \Leftarrow \quad (a? \parallel b? \parallel d?).\ c!\ a0 \times b + d + a \times b0.\ C = a0 \times B + D + A \times b0$

Power Series Multiplication

Input on channel a : $a_0\ a_1\ a_2$ ... $A\ =\ a_0 + a_1{\times}x + a_2{\times}x^2 + ...$

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$\langle c?\ rat\cdot\ C = A{\times}B\rangle\ c \quad \Leftarrow \quad (a? \parallel b?).\ \ c!\ a{\times}b.$

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$\quad\quad \langle c?\ rat\cdot\ C = A{\times}B\rangle\ d$

$\quad \parallel\ \ ((a? \parallel b?).\ \ c!\ a0{\times}b + a{\times}b0.\ \ C = a0{\times}B + D + A{\times}b0)$

$\downarrow$

$C = a0{\times}B + D + A{\times}b0 \quad \Leftarrow \quad (a? \parallel b? \parallel d?).\ \ c!\ a0{\times}b + d + a{\times}b0.\ \ C = a0{\times}B + D + A{\times}b0$

Power Series Multiplication

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$$C = A \times B = a_0{\times}b_0 + (a_0{\times}b_1 + a_1{\times}b_0)x + (a_0{\times}B_2 + A_1{\times}B_1 + A_2{\times}b_0){\times}x^2$$

$\langle c?\ rat\cdot\ C = A{\times}B\rangle\ c \quad \Leftarrow \quad (a? \,\|\, b?).\ c!\ a{\times}b.$

$\quad$ **new** $a0: rat := a\cdot$ **new** $b0: rat := b\cdot$ **new** $d?\ rat\cdot$

$\quad\quad \langle c?\ rat\cdot\ C = A{\times}B\rangle\ d$

$\quad \|\ \ ((a? \,\|\, b?).\ c!\ a0{\times}b + a{\times}b0.\ C = a0{\times}B + D + A{\times}b0)$

$C = a0{\times}B + D + A{\times}b0 \quad \Leftarrow \quad (a? \,\|\, b? \,\|\, d?).\ c!\ a0{\times}b + d + a{\times}b0.\ C = a0{\times}B + D + A{\times}b0$

Power Series Multiplication

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$$\langle c?\ rat\cdot\ C = A{\times}B\rangle\ c \quad \Leftarrow \quad (a? \parallel b?).\ c!\ a{\times}b.$$

$\rightarrow$ **new** $a0$: $rat := a\cdot$ **new** $b0$: $rat := b\cdot$ **new** $d?\ rat\cdot$

$\langle c?\ rat\cdot\ C = A{\times}B\rangle\ d$

$\parallel\ ((a? \parallel b?).\ c!\ a0{\times}b + a{\times}b0.\ C = a0{\times}B + D + A{\times}b0)$

$$C = a0{\times}B + D + A{\times}b0 \quad \Leftarrow \quad (a? \parallel b? \parallel d?).\ c!\ a0{\times}b + d + a{\times}b0.\ C = a0{\times}B + D + A{\times}b0$$

Power Series Multiplication

Input on channel a : $a_0\ a_1\ a_2$ ... $A = a_0 + a_1{\times}x + a_2{\times}x^2 + ...$

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$A_1 = a_1 + a_2{\times}x + a_3{\times}x^2 + ...$ $B_1 = b_1 + b_2{\times}x + b_3{\times}x^2 + ...$

$A_2 = a_2 + a_3{\times}x + a_4{\times}x^2 + ...$ $B_2 = b_2 + b_3{\times}x + b_4{\times}x^2 + ...$

$$C = A \times B = a_0{\times}b_0 + (a_0{\times}b_1 + a_1{\times}b_0)x + (a_0{\times}B_2 + A_1{\times}B_1 + A_2{\times}b_0){\times}x^2$$

$\langle c?\ rat\cdot\ C = A{\times}B\rangle\ c \quad \Leftarrow \quad (a? \parallel b?).\ \ c!\ a{\times}b.$

$\quad$ **new** $a0$: $rat := a\cdot$ **new** $b0$: $rat := b\cdot$ **new** $d?\ rat\cdot$

$\longrightarrow \quad \langle c?\ rat\cdot\ C = A{\times}B\rangle\ d$

$\longrightarrow \quad \parallel\ ((a? \parallel b?).\ \ c!\ a0{\times}b + a{\times}b0.\ \ C = a0{\times}B + D + A{\times}b0)$

$C = a0{\times}B + D + A{\times}b0 \quad \Leftarrow \quad (a? \parallel b? \parallel d?).\ \ c!\ a0{\times}b + d + a{\times}b0.\ \ C = a0{\times}B + D + A{\times}b0$