

120 Most arithmetic expressions can be evaluated. For example, $2+3$ evaluates to 5 . Our evaluation rules do not give us any answer for $1/0$. That's because there is no single answer that makes sense. We might look at $\uparrow n \cdot 1/(1/n)$, which is ∞ . But we might equally well look at $\uparrow n \cdot 1/(-1/n)$, which is $-\infty$. We already have some arithmetic expressions that evaluate to a bunch of answers. For example, $4^{1/2}$ evaluates to $2, -2$. So perhaps it makes sense to say $1/0 = \infty, -\infty$. What sense can we make of $0/0$?

After trying the question, scroll down to the solution.

§ A sports team that has played p games and has won w of them has a success rate of w/p . At the start of the season, before any games have been played, their success rate is $0/0$. So $0/0$ is a legitimate expression representing a real-world value.

As we saw in Exercise 39, if we consider only the number laws, not the bunch laws, we can consistently say $0/0 = 5$, and might try to justify it by noticing that

$$\uparrow n \cdot (5/n)/(1/n) = 5$$

But by the same reasoning we can say $0/0 = x$ for any x . So we could say $0/0 = xreal$. Or even $0/0 = xcom$ if we define $xcom$ to be the extended complex numbers.

A bunch law in the back of the book says

$$xreal: 0/0$$

Division by 0

leaving undecided whether $0/0$ includes more than $xreal$. Another bunch law says

$$\infty, -\infty: x/0$$

Division by 0

which tells us $\infty, -\infty: 1/0$, but leaves $1/0 = \infty, -\infty$ undecided.