138 (factorial) In natural variables n and f prove

$$f{:=}\ n! \ \Leftarrow\ \textbf{if}\ n{=}0\ \textbf{then}\ f{:=}\ 1\ \textbf{else}\ n{:=}\ n-1.\ f{:=}\ n!.\ n{:=}\ n+1.\ f{:=}\ f\times n\ \textbf{fi}$$

where $n! = 1\times2\times3\times...\times n$. The occurrence of $f{:=}\ n!$ on the right side is a recursive call.

After trying the question, scroll down to the solution.

§ Proof by cases.

	$n=0 \land (f:=1)$	expand assignment
$=$	$n=0 \land f'=1 \land n'=n$	use context from left conjunct to change middle conjunct
$=$	$n=0 \land f'=n! \land n'=n$	contract assignment
$=$	$n=0 \land (f:=n!)$	specialization
$\Longrightarrow$	$f:=n!$	

	$n\neq 0 \land (n:=n-1.\ f:=n!.\ n:=n+1.\ f:=f\times n)$	
		specialize (drop conjunct) and expand last assignment
$\Longrightarrow$	$n:=n-1.\ f:=n!.\ n:=n+1.\ f'=f\times n \land n'=n$	substitution
$=$	$n:=n-1.\ f:=n!.\ f'=f\times(n+1) \land n'=n+1$	substitution
$=$	$n:=n-1.\ f'=n!\times(n+1) \land n'=n+1$	simplify
$=$	$n:=n-1.\ f'=(n+1)! \land n'=n+1$	substitution
$=$	$f'=n! \land n'=n$	definition of assignment
$=$	$f:=n!$	