

- 201 (duplicate count) Write a program to find how many items are duplicates (repeats) of earlier items
- (a) in a given sorted nonempty list.
 - (b) in a given list.

After trying the question, scroll down to the solution.

(a) in a given sorted nonempty list.

§ Let the list be L . Let n and j be natural state variables. The result will be reported as n' . Let's call the specification S , defined as

$$S = n' = \wp(\S i: 1..#L \cdot L i = L(i-1)) \wedge t' = t + \#L - 1$$

Also define

$$P = 1 \leq j \leq \#L \wedge n' = n + \wp(\S i: j..#L \cdot L i = L(i-1)) \wedge t' = t + \#L - j$$

The problem is solved by the refinements

$$S \Leftarrow n := 0. j := 1. P$$

$$P \Leftarrow \text{if } j = \#L \text{ then ok} \\ \text{else if } L j = L(j-1) \text{ then } n := n + 1 \\ \text{else ok fi.} \\ j := j + 1. t := t + 1. P \text{ fi}$$

Proof: NOT YET WRITTEN

(b) in a given list.

Maybe the best way is to check if the list is nonempty, sort it, then use the solution of part (a). Here is another way. If item $L k$ is a duplicate of earlier item $L i$, then $L i$ is a duplicate of later item $L k$. So it is equivalent to write a program to find how many items are duplicates (repeats) of later items.

Define:

$$S = n' = \wp(\S j: 0..#L \cdot \exists m: j+1..#L \cdot L j = L m) \wedge t' \leq t + (\#L)^2/2 \\ P = n' = n + \wp(\S j: i..#L \cdot \exists m: j+1..#L \cdot L j = L m) \wedge t' \leq t + (\#L - i)^2/2 \\ Q = n' = n + \text{if } \exists m: k..#L \cdot L i = L m \text{ then } 1 \text{ else } 0 \text{ fi} \\ + \wp(\S j: i+1..#L \cdot \exists m: j+1..#L \cdot L j = L m) \\ \wedge t' \leq t + (\#L - k) + (\#L - (i+1))^2/2$$

Refine:

$$S \Leftarrow n := 0. i := 0. P \\ P \Leftarrow \text{if } i = \#L \text{ then ok else } k := i + 1. t := t + 1. Q \text{ fi} \\ Q \Leftarrow \text{if } k = \#L \text{ then } i := i + 1. P \\ \text{else if } L i = L k \text{ then } n := n + 1. i := i + 1. P \\ \text{else } k := k + 1. t := t + 1. Q \text{ fi}$$

Prove:

$$\begin{aligned} & n := 0. i := 0. P && \text{expand } P \\ = & n := 0. i := 0. n' = n + \wp(\S j: i..#L \cdot \exists m: j+1..#L \cdot L j = L m) \wedge t' \leq t + (\#L - i)^2/2 && \text{substitution law twice} \\ = & n' = 0 + \wp(\S j: 0..#L \cdot \exists m: j+1..#L \cdot L j = L m) \wedge t' \leq t + (\#L - 0)^2/2 && \text{arithmetic} \\ = & n' = \wp(\S j: 0..#L \cdot \exists m: j+1..#L \cdot L j = L m) \wedge t' \leq t + (\#L)^2/2 && \text{contract } S \\ = & S \end{aligned}$$

$$\begin{aligned} & i = \#L \wedge \text{ok} && \text{expand } ok \\ = & i = \#L \wedge n' = n \wedge i' = i \wedge k' = k \wedge t' = t && \\ \Rightarrow & n' = n + \wp(\S j: i..#L \cdot \exists m: j+1..#L \cdot L j = L m) \wedge t' \leq t + (\#L - i)^2/2 && \text{contract } P \\ = & P \end{aligned}$$

$$\begin{aligned} & i \neq \#L \wedge (k := i + 1. Q) && \text{expand } Q \\ = & i \neq \#L \wedge (k := i + 1. t := t + 1. \\ & n' = n + \text{if } \exists m: k..#L \cdot L i = L m \text{ then } 1 \text{ else } 0 \text{ fi} \\ & + \wp(\S j: i+1..#L \cdot \exists m: j+1..#L \cdot L j = L m) \end{aligned}$$

$$\begin{aligned}
& \wedge t' \leq t + (\#L - k) + (\#L - (i+1))^2 / 2 && \text{substitution law twice} \\
= & i \neq \#L \wedge n' = n + \mathbf{if} \exists m: i+1, \dots, \#L \cdot Li = Lm \mathbf{then} 1 \mathbf{else} 0 \mathbf{fi} \\
& \quad + \wp(\S j: i+1, \dots, \#L \cdot \exists m: j+1, \dots, \#L \cdot Lj = Lm) \\
& \quad \wedge t' \leq t+1 + (\#L - (i+1)) + (\#L - (i+1))^2 / 2 \\
= & i \neq \#L \wedge n' = n + \wp(\S j: i \cdot \exists m: j+1, \dots, \#L \cdot Lj = Lm) \\
& \quad + \wp(\S j: i+1, \dots, \#L \cdot \exists m: j+1, \dots, \#L \cdot Lj = Lm) \\
& \quad \wedge t' \leq t + (\#L - i)^2 / 2 + 1/2 \\
& \quad \text{combine the two } \wp \text{ and time is } xnat \text{ so lose the } 1/2 \text{ and drop } i \neq \#L \\
\Rightarrow & n' = n + \wp(\S j: i, \dots, \#L \cdot \exists m: j+1, \dots, \#L \cdot Lj = Lm) \wedge t' \leq t + (\#L - i)^2 / 2 && \text{contract } P \\
= & P \\
& k = \#L \wedge (i := i+1. P) && \text{expand } P \\
= & k = \#L \wedge (i := i+1. \\
& \quad n' = n + \wp(\S j: i, \dots, \#L \cdot \exists m: j+1, \dots, \#L \cdot Lj = Lm) \wedge t' \leq t + (\#L - i)^2 / 2) \\
& \quad \text{substitution law} \\
= & k = \#L \wedge n' = n + \wp(\S j: i+1, \dots, \#L \cdot \exists m: j+1, \dots, \#L \cdot Lj = Lm) \wedge t' \leq t + (\#L - (i+1))^2 / 2 \\
& \quad \text{add 0 to } n \text{ and to time} \\
= & k = \#L \\
& \wedge n' = n + \mathbf{if} \perp \mathbf{then} 1 \mathbf{else} 0 \mathbf{fi} \\
& \quad + \wp(\S j: i+1, \dots, \#L \cdot \exists m: j+1, \dots, \#L \cdot Lj = Lm) \\
& \quad \wedge t' \leq t + (\#L - k) + (\#L - (i+1))^2 / 2 \\
& \quad \text{using context } k = \#L \text{ expand } \perp \\
= & k = \#L \\
& \wedge n' = n + \mathbf{if} \exists m: k, \dots, \#L \cdot Li = Lm \mathbf{then} 1 \mathbf{else} 0 \mathbf{fi} \\
& \quad + \wp(\S j: i+1, \dots, \#L \cdot \exists m: j+1, \dots, \#L \cdot Lj = Lm) \\
& \quad \wedge t' \leq t + (\#L - k) + (\#L - (i+1))^2 / 2 && \text{drop } k = \#L \\
\Rightarrow & n' = n + \mathbf{if} \exists m: k, \dots, \#L \cdot Li = Lm \mathbf{then} 1 \mathbf{else} 0 \mathbf{fi} \\
& \quad + \wp(\S j: i+1, \dots, \#L \cdot \exists m: j+1, \dots, \#L \cdot Lj = Lm) \\
& \quad \wedge t' \leq t + (\#L - k) + (\#L - (i+1))^2 / 2 && \text{contract } Q \\
= & Q \\
& k \neq \#L \wedge Li = Lk \wedge (n := n+1. i := i+1. P) && \text{expand } P \\
= & k \neq \#L \wedge Li = Lk \\
& \wedge (n := n+1. i := i+1. \\
& \quad n' = n + \wp(\S j: i, \dots, \#L \cdot \exists m: j+1, \dots, \#L \cdot Lj = Lm) \wedge t' \leq t + (\#L - i)^2 / 2) \\
& \quad \text{substitution law twice} \\
= & k \neq \#L \wedge Li = Lk \\
& \wedge n' = n + 1 + \wp(\S j: i+1, \dots, \#L \cdot \exists m: j+1, \dots, \#L \cdot Lj = Lm) \\
& \quad \wedge t' \leq t + (\#L - (i+1))^2 / 2 && \text{STEPS AND JUSTIFICATIONS NEEDED HERE} \\
\Rightarrow & n' = n + \mathbf{if} \exists m: k, \dots, \#L \cdot Li = Lm \mathbf{then} 1 \mathbf{else} 0 \mathbf{fi} \\
& \quad + \wp(\S j: i+1, \dots, \#L \cdot \exists m: j+1, \dots, \#L \cdot Lj = Lm) \\
& \quad \wedge t' \leq t + (\#L - k) + (\#L - (i+1))^2 / 2 && \text{contract } Q \\
= & Q \\
& k \neq \#L \wedge Li \neq Lk \wedge (k := k+1. t := t+1. Q) && \text{expand } Q \\
= & k \neq \#L \wedge Li \neq Lk \\
& \wedge (k := k+1. t := t+1. \\
& \quad n' = n + \mathbf{if} \exists m: k, \dots, \#L \cdot Li = Lm \mathbf{then} 1 \mathbf{else} 0 \mathbf{fi} \\
& \quad + \wp(\S j: i+1, \dots, \#L \cdot \exists m: j+1, \dots, \#L \cdot Lj = Lm) \\
& \quad \wedge t' \leq t + (\#L - k) + (\#L - (i+1))^2 / 2) && \text{substitution law twice} \\
= & k \neq \#L \wedge Li \neq Lk \\
& \wedge n' = n + \mathbf{if} \exists m: k+1, \dots, \#L \cdot Li = Lm \mathbf{then} 1 \mathbf{else} 0 \mathbf{fi}
\end{aligned}$$

$$+ \wp(\S j: i+1, ..\#L \cdot \exists m: j+1, ..\#L \cdot L j = L m)$$

$$\wedge t' \leq t+1+(\#L-(k+1))+(\#L-(i+1))^2/2)$$

MORE STEPS AND JUSTIFICATIONS NEEDED HERE

$$\Rightarrow n' = n + \mathbf{if} \exists m: k, ..\#L \cdot L i = L m \mathbf{then} 1 \mathbf{else} 0 \mathbf{fi}$$

$$+ \wp(\S j: i+1, ..\#L \cdot \exists m: j+1, ..\#L \cdot L j = L m)$$

$$\wedge t' \leq t+(\#L-k)+(\#L-(i+1))^2/2$$

contract Q

$$= Q$$