295 Let P and Q be specifications. Let A be an assertion and let A' be the same as A but with primes on all the variables. Prove the assertion law

$$P.Q \Leftarrow P \wedge A'.\ A \Rightarrow Q$$

After trying the question, scroll down to the solution.

§ Let A'' be the same as A but with double-primed variables. Then

$$
\begin{array}{lll}
 & P \wedge A' .\ A \Rightarrow Q & \\
= & \exists \sigma'' \cdot \langle \sigma' \cdot P \wedge A' \rangle \sigma'' \ \wedge\ \langle \sigma \cdot A \Rightarrow Q \rangle \sigma'' & \\
= & \exists \sigma'' \cdot \langle \sigma' \cdot P \rangle \sigma'' \wedge A'' \ \wedge\ (A'' \Rightarrow \langle \sigma \cdot Q \rangle \sigma'') & \text{discharge} \\
= & \exists \sigma'' \cdot \langle \sigma' \cdot P \rangle \sigma'' \wedge A'' \ \wedge\ \langle \sigma \cdot Q \rangle \sigma'' & \text{specialize} \\
\Longrightarrow & \exists \sigma'' \cdot \langle \sigma' \cdot P \rangle \sigma'' \ \wedge\ \langle \sigma \cdot Q \rangle \sigma'' & \\
= & P . Q &
\end{array}
$$