

325 Here is a nest of loops. All **exits** are shown. What refinements need to be proven in order to prove that this nest of loops refines specification S ?

(a)✓

```
do A.
  do B.
    exit 2 when c.
    D. od.
  E od
```

(d)

```
do A.
  do B.
    exit 2 when u.
    C.
    exit 1 when v.
    D od.
  E.
  exit 1 when w.
  F.
  do G.
    do H.
      exit 1 when x.
      I.
      exit 2 when y.
      J od.
    K.
    exit 1 when z.
    L od.
  M od
```

(b)✓

```
do A.
  exit 1 when b.
  C.
  do D.
    exit 2 when e.
    F.
    exit 1 when g.
    H od.
  I od
```

(c)

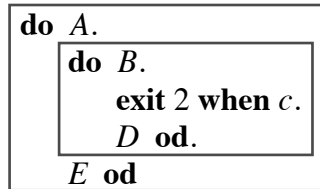
```
do A.
  do B.
    do C.
      exit 1 when u.
      exit 2 when v.
      exit 3 when w.
      D od.
    E od.
  F od
```

(e)

```
do A.
  exit 1 when u.
  do B.
    exit 2 when v.
    exit 1 when  $\top$  od od
```

After trying the question, scroll down to the solution.

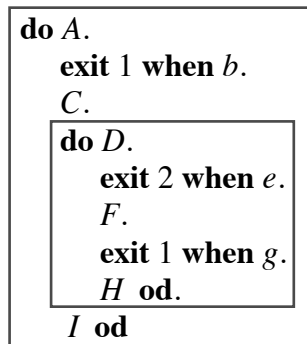
(a)✓



§ Using R as specification for the inner loop,

$S \Leftarrow A. R$
 $R \Leftarrow B. \text{ if } c \text{ then } ok \text{ else } D. R \text{ fi}$

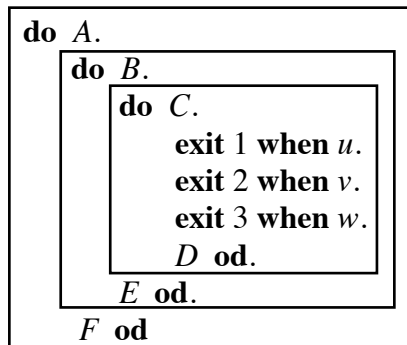
(b)✓



§ Using R as specification for the inner loop,

$S \Leftarrow A. \text{ if } b \text{ then } ok \text{ else } C. R \text{ fi}$
 $R \Leftarrow D. \text{ if } e \text{ then } ok \text{ else } F. \text{ if } g \text{ then } I. S \text{ else } H. R \text{ fi fi}$

(c)

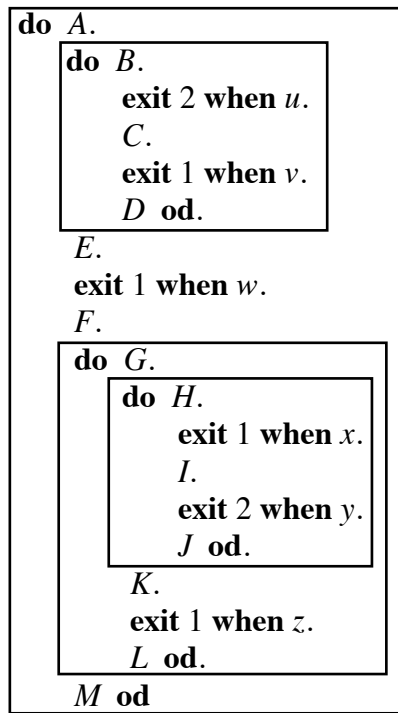


§ Using R and Q as specifications for the inner loops,

$S \Leftarrow A. R$
 $R \Leftarrow B. Q$
 $Q \Leftarrow C. \text{ if } u \text{ then } E. R$
 $\text{ else if } v \text{ then } F. S$
 $\text{ else if } w \text{ then } ok$
 $\text{ else } D. Q \text{ fi fi fi}$

Specification R isn't necessary; its two uses could be replaced by $B.Q$.

(d)

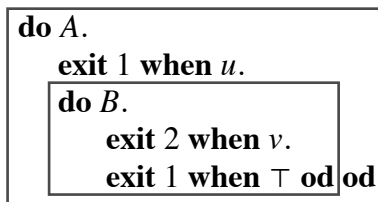


§ Using P , Q , and R as specifications for the inner loops,

```
S  $\Leftarrow$  A. P  
P  $\Leftarrow$  B. if u then ok  
           else C. if v then E. if w then ok  
                    else F. Q fi  
           else D. P fi fi  
Q  $\Leftarrow$  G. R  
R  $\Leftarrow$  H. if x then K. if z then M. S  
           else L. Q fi  
           else I. if y then M. S  
           else J. R fi fi
```

Specification Q isn't necessary; its two uses could be replaced by $G.R$.

(e)



§ Using R as specification for the inner loop,

```
S  $\Leftarrow$  A. if u then ok else R fi  
R  $\Leftarrow$  B. if v then ok else S fi
```

or we can combine these two refinements into one, not requiring any new specification:

```
S  $\Leftarrow$  A. if u then ok else B. if v then ok else S fi fi
```