398 (pure sets) A pure set is a set all of whose elements are pure sets. For example

$$\{\{null\}, \{\{null\}\}\}$$

First, $\{null\}$ is a pure set because all zero of its elements are pure sets. So $\{\{null\}\}$ is a pure set because its one element $\{null\}$ is a pure set. So $\{\{null\}, \{\{null\}\}\}$ is a pure set because both its elements are pure sets. All of mathematics can be implemented as pure sets. Define the bunch of pure sets.

After trying the question, scroll down to the solution.

§ Define *puresets* as the bunch of all the pure sets by one construction axiom

A: *puresets* ⇒ {A}: *puresets*

We know that *null* is a bunch of pure sets by a bunch axiom. We just need to say that if A is a bunch of pure sets, then {A} is a pure set. To define *puresets* as only the pure sets, we need an induction axiom. This is a bit tricky. We want something like

(A: B ⇒ {A}: B) ⇒ *puresets*: B

except that (A: B ⇒ {A}: B) should say

for all bunches A of pure sets, (A: B ⇒ {A}: B)

But we cannot quantify over bunches. We can quantify over sets, so the induction axiom is

(∀S: *puresets*· ~S: B ⇒ S: B) ⇒ *puresets*: B

If *puresets* is the bunch of all pure sets, then {*puresets*} is the set of all pure sets. And it is a pure set. So it is a member of itself.

Let S be the set of all pure sets that are not members of themselves. If $S \in S$ then $\neg S \in S$, and if $\neg S \in S$ then $S \in S$. We have an inconsistency.

But there is no inconsistency in defining the bunch of all pure set expressions.

"*null*": *pse*

pse; ","; *pse*: *pse*

"{"; *pse*; "}": *pse*

"*null*", B; ","; B , "{"; B; "}" : B ⇒ *pse*: B