

421 Suppose we define **while** b **do** P **od** by fixed-point construction and induction, ignoring time.

while b **do** P **od** = **if** b **then** P . **while** b **do** P **od** **else** ok **fi**

$\forall \sigma, \sigma'. W = \text{if } b \text{ then } P. W \text{ else } ok \text{ fi} \implies \forall \sigma, \sigma'. \text{while } b \text{ do } P \text{ od} \Leftarrow W$

Prove that ordinary construction and induction

if b **then** P . **while** b **do** P **od** **else** ok **fi** \Leftarrow **while** b **do** P **od**

$\forall \sigma, \sigma'. \text{if } b \text{ then } P. W \text{ else } ok \text{ fi} \Leftarrow W \implies \forall \sigma, \sigma'. \text{while } b \text{ do } P \text{ od} \Leftarrow W$

are theorems.

§ Ordinary construction is implied immediately by fixed-point construction. What remains is to prove ordinary induction from both fixed-point construction and fixed-point induction.