

43 We defined bunch *null* with the axiom *null: A* . Is there any harm in defining bunch *all* with the axiom *A: all* ?

After trying the question, scroll down to the solution.

§ With just Binary Theory, Number Theory, Character Theory, and Bunch Theory, there is no harm (inconsistency) in defining all with the axiom $A: all$. Even when we add Set Theory (in this book; we don't yet have set comprehension) there is no harm. But when we add Function Theory, specifically the \S quantifier, we have an inconsistency known as "Russell's Paradox". Let

$$R = \{s: \not all \cdot \neg s \in s\}$$

Then R is the set of all sets that are not members of themselves. Or, without abbreviation,

$$R = \{\S\langle s: \not all \cdot \neg s \in s \rangle\}$$

Then

$R \in R$	definition of R
$= R \in \{s: \not all \cdot \neg s \in s\}$	\in axiom
$= R: \S s: \not all \cdot \neg s \in s$	solution law
$= R: \not all \wedge \neg R \in R$	definition of R
$= \{s: \not all \cdot \neg s \in s\}: \not all \wedge \neg R \in R$	\S axiom
$= (\S s: \not all \cdot \neg s \in s): all \wedge \neg R \in R$	all axiom
$= \top \wedge \neg R \in R$	identity law
$= \neg R \in R$	

and we have inconsistency.

It might be nice to have all , and to weaken the solution law to accommodate it. But I have stayed with standard mathematics, excluding all and including the strong form of solution law.

Even without all , we still have a benign form of Russell's Paradox (Exercise 48); it is not an inconsistency, but it may disturb some people.